ELEMENTS OF PHYSICS

EDWIN H. HALL

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Sir Isaac Newton

ELEMENTS OF PHYSICS

WITH

LABORATORY WORK FOR STUDENTS

THE SUCCESSOR OF HALL AND BERGEN'S
TEXT-BOOK OF PHYSICS

BY

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PREFACE

This book, when compared with the third edition of Hall and Bergen's *Text-book of Physics*, of which it is the natural successor, shows a considerable number of important changes.

Thus, though written for beginners in the systematic study of physics, it is intended for the later years of a school course or the first year of a college course, rather than for earlier use. Accordingly, certain laboratory *Exercises*, long familiar to teachers, have been replaced by lecture-room experiments or have been merged with other *Exercises;* a few have simply been dropped. On the other hand, certain new matter appears, notably in the chapter on *Molecular Attractions*, etc., which has been expanded from seven to nineteen pages, and in the treatment of heat engines, optics, and the applications of electromagnetism.

There are also several new *Exercises*, though, in consequence of the dropping and the consolidation already mentioned, the total number of *Exercise Titles* is reduced from sixty-one to fifty.

The next change to be mentioned, perhaps the most noticeable one of all, is the segregation of the *Directions* for the *Exercises* from the general text into a body by itself in the back of the book. When laboratory work for beginners in physics was a novel undertaking, from which many shrank, it seemed best to emphasize the importance of such work by giving it a very prominent place in a book which was intended to be a pioneer in the new way of instruction. But the critical stage of the enterprise is well passed; laboratory work is

now generally accepted, in America at least, as a necessary part of the best kind of elementary science-teaching; and therefore even the most strenuous apostles of the laboratory gospel may relax a little of their early rigor and take more thought of convenience and the appearance of things. No doubt the general text looks better and reads better without the interruption of frequently interpolated *Directions;* no doubt the teacher will find it more convenient not to have the *Exercises* introduced in such a way as to indicate that each must be done in a certain precise conjunction with the context. But even in this book the relation between experiment and discussion, between the laboratory work and the general text, is intended to be very close, much closer than it seems to be in most other physics books with which this one will naturally be compared. An attentive reading of the book will indicate in most cases approximately where, in the opinion of the author, each *Exercise,* if done at all, should be undertaken.

Moreover, a good deal of other matter which is suggestive, and perhaps important, for the teacher, but which need not be presented to the eye of the student or of the general reader, has been put into a special part, to be issued in a *Teachers' Edition* or under separate cover.

The *Exercises* which have not appeared before in *Hall and Bergen* or in the "*Harvard List*" are No. 9, on *Surface Tension,* No. 38, on the *Telescope and Microscope,* No. 46, on the *Electric Call-Bell,* and No. 50, on the *Efficiency of an Electric Motor.* One other, No. 17, on the *Measurement of "g,"* is a revival, in improved form, of an *Exercise* long ago discarded from the *Harvard List.* As now developed, by various experimenters, from the original ingenious but somewhat troublesome form devised by the late Doctor Harold Whiting, it is quite practicable for class use; and it is here given because some teachers earnestly desire to

have some laboratory work on gravity acceleration. It is the author's preference and practice to show the operation of this *Exercise* 17 apparatus on the lecture-table, and thus to make a hasty measurement of g; for the experiment goes well in a lecture, as a mere illustration of the way in which this constant, after the student has become acquainted with the "laws of falling bodies," can be found. The teacher who deals with this matter must take care lest the student get into his head the confused notion that the "laws of *uniform acceleration*" have to be discovered or investigated experimentally; it should be made plain that these laws follow from the very definition of uniform acceleration.

The book contains more matter in the *Exercises*, and probably in the general text, than is likely to be required in preparation for the examinations of the *College Entrance Board* or of any individual college. The teacher will naturally decide for himself, with due regard to the special circumstances of his teaching, what parts are to be taken and what parts omitted. So, too, in regard to the sequence or order of the work, there is opportunity for discretion. For example, the *Hall and Bergen*, for reasons which seemed good at the time and still have some force, interrupted *Mechanics* by the interpolation of *Light*, whereas the present book gives the *Mechanics* continuously; but the treatment of *Light* is still such that the teacher may without inconvenience follow the previous order if he prefers it.

It has been the aim of the author, and it will doubtless be the aim of the teacher, to choose such matter and to treat it in such a way as to make it vital, make it commend itself to the ordinary common-sense student and take root in his mind and grow with his experience. The unavoidable difficulties in the nurture of a sound knowledge of physics are so great that we should remove all unneces

sary ones. We should set no task thoughtlessly; for example, we should try to make our *Questions* and *Problems* stimulating, and to load them with no superfluous, therefore exasperating, arithmetical drudgery.

In determining both the substance and the form of the laboratory notes the student should ask, not so much what kind of record college examiners or other examiners may require, but rather what kind of record will be most serviceable to himself in reviewing. He should keep the note-book in such a way that it will readily recall to his mind the purposes, methods, and results, of the several exercises performed. The record should be well-ordered, plainly legible, and concise. Simple drawings are the briefest and best descriptions of most apparatus. Long repetitions of directions or descriptions given elsewhere should be avoided.

Two[1] firms especially, the *L. E. Knott Apparatus Company*, of Boston, and the *Central Scientific Company*, of Chicago, undertake to furnish the apparatus called for in this book and described in two lists given in the teachers' edition or supplement. The author invites teachers who may purchase from either of them apparatus described in the lists above mentioned to write him freely, in commendation or in criticism as experience may dictate. He is more afraid that competition may lower the character of the goods than he is that too high prices may be asked for the right thing. Teachers should buy with care, insisting on good material and good workmanship, not expressed in fancy, and they should be willing to pay accordingly. Especial attention is called to the spring-balance, apparatus No. 3, made by *Chatillon & Sons*, of New York, in accordance with the author's directions.

E. H. H.

April 8, 1912.

[1] Probably also a third, the *Standard Scientific Company*, New York.

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ELEMENTS OF PHYSICS

CHAPTER I.

INTRODUCTION.

1. Definition of Physics.—Physics is the science of *mechanics, heat, sound, light, electricity,* and *magnetism.* Everybody knows something about these things before he begins to study them in a regular way, but he may not know them by the names which are given to them in books.

2. Use of Physics.—In flying kites or sailing boats, in walking or swimming, in almost any kind of bodily work or play, we have to do with physics, that part of physics which is called mechanics. We learn to do many mechanical acts very well by observation and experience, without thinking very much about them or knowing exactly how we do them; but when we have to do something that we have never done before and have never seen any one else do, something, perhaps, that nobody ever did before, we must study carefully the way in which common things behave and the way in which familiar acts are done, thus to discover what are called the laws of nature and the uses to which we can put these laws.

3. Qualitative Knowledge and Quantitative Knowledge.—Everybody knows that a piece of wood will float in water, and that a stone will sink. Everybody knows that, if a stick and a stone are tied together and put into water, the stone tends to sink the stick, and the stick tends to float the

stone. This kind of knowledge is called *qualitative*. It tells in a general way how the stick and the stone and the water act toward each other.

But we may know enough about the laws of flotation to calculate with accuracy how large a stick of a known kind of wood will be needed to float a stone of known size and weight. We then have what is called *quantitative* knowledge of the matter. We can tell *how much* the stone will pull down on the wood, and the wood pull up on the stone, when the two are together in water.

Everybody knows that a beam has a quality which we call strength—it can bear a load. This is qualitative knowledge. Everybody knows that a thick beam is stronger than a thin beam. This is quantitative knowledge of a kind, a rather indefinite kind. We may know how much stronger the thick beam is than the thin beam. We then have a more complete quantitative knowledge.

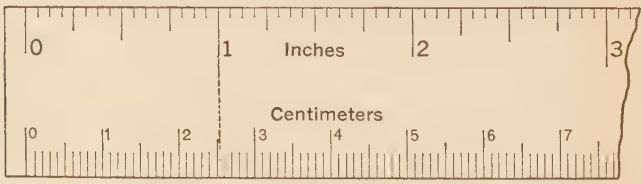


FIG. 1.—Comparison of inches and centimeters.

It is evident that quantitative knowledge is more useful than mere qualitative knowledge. The former includes the latter.

4. Object of this Course in Physics.—The knowledge of physics which children and older people get by merely knocking about in the world is mostly qualitative. The course of work laid out in this book is intended to add greatly to the user's stock of this kind of knowledge, and to do something more. It aims to make him familiar with

quantitative work, and to give him a considerable amount of quantitative knowledge.

5. Units and Standards of Measurement.—The importance of having definite units of length, of weight, etc., so that any man in dealing with his neighbor may know just how much is meant by the words *foot*, *pound*, and the like, is so great that in all highly civilized countries the exact meaning of such words is fixed by law, and very great care is taken to make and preserve government *standards*, as they are called, standard yard-sticks, standard pound-weights, for example, with which as patterns the measuring instruments used in business are compared and tested.

Measuring-rods for school use are in many cases marked off in inches on one side and in *centimeters* on the other. From such rods it will be found that 1 inch = 2.54 centimeters, very nearly.

A *centi*meter, often written 1 cm., is the hundredth part of a meter, as one *cent* is the hundredth part of a dollar.

Spring balances for school use in weighing light objects are commonly marked off in ounces on one side and in *grams* on the other, as in Fig. 2. From such balances it will be found that 1 oz. = 28.3 gm., very nearly, 1 lb. being, accordingly, about 454 gm. One *kilo*-gram = 1000 gm., the prefix *kilo* meaning, in the so-called *metric system* of weights and measures, *one thousand*.

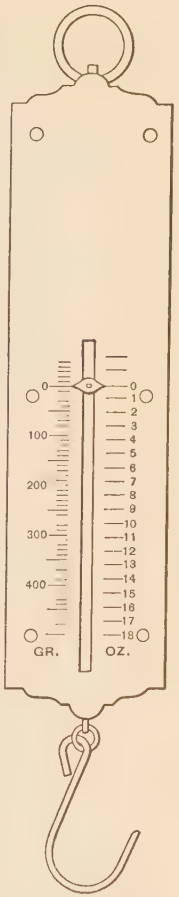


FIG. 2.—Spring balance with double scale.

The gram and the centimeter are connected by the fact

that the *definition* of the gram is *the weight of one cubic centimeter of water.*[1]

The simple ratios, 10, 100, 1000, etc., which hold between units of the same kind in the metric system, and the relation between volume-units and weight-units, which is shown by the definition of the gram, make this system much more convenient than others, and much use will be made of metric units, especially the centimeter and the gram, in this book.

[1] To be exact we must add *at its temperature of greatest density* (see Exercise 1), which is near 4° of the centigrade scale of temperature. For the purpose of this book such exactness is unnecessary.

CHAPTER II.

DENSITY AND SPECIFIC GRAVITY.

6. Definition of Density.—The weight of a unit volume, 1 cu. cm. or 1 cu. ft., for example, of a substance is called the *density* of the substance. If we know the density, we can calculate the weight of any volume of that substance. Engineers and others often have to find by this method the weight of objects which it would be inconvenient to weigh, buildings and bridges for instance. The density of a substance may be expressed as the weight in grams of 1 cu. cm., or as the weight in pounds of 1 cu. ft., or in any one of many other ways. For brevity, we call the first method of expression just given the *density in grams per cubic centimeter*, and the second, the *density in pounds per cubic foot.*

If the body is one which we can easily weigh and is of such shape that we can easily measure its volume, we can readily find its density by experiment. In Exercise 1 the density of a rectangular block of wood is found.

7. Density of Water. Ratio of the Two Densities.—The density of water in grams per cubic centimeter is 1; that is, 1 cu. cm. of water weighs 1 gm.

The density of water in pounds per cubic foot is very nearly 62.4; that is, 1 cu. ft. of water weighs about 62.4 lbs.

These numbers for water should be committed to memory.

Let us call *density in pounds per cubic foot* $D(p.\ f.)$ and *density in grams per cubic centimeter* $D(g.\ c.)$. For water $D(p.\ f.) \div D(g.\ c.) = 62.4$. Does the same ratio hold with other substances?

For any substance we have by definition,

$$D(p.\ f.) = (Wt.\ in\ lbs.) \div (Vol.\ in\ cu.\ ft.),$$
$$D(\ g.\ c.) = (Wt.\ in\ gm.) \div (Vol.\ in\ cu.\ cm.).$$

Hence

$$\frac{D(p.\ f.)}{D(g.\ c.)} = \frac{Wt.\ in\ lbs.}{Wt.\ in\ gm.} \times \frac{Vol.\ in\ cu.\ cm.}{Vol.\ in\ cu.\ ft.}$$
$$= \frac{1}{453.6} \times 28300 = 62.4.$$

Problems.

(1) If a piece of iron 10 cm. long, 8 cm. wide, and 7 cm. thick weighs 4200 gm., what is its density in gm. and cu. cm.? What is its density in lbs. and cu. ft.?

(2) The density of mercury in gm. and cu. cm. is about 13.6. How many lbs. would 1 cu. ft. of it weigh?

(3) The density of gold being 19.3 gm. per cu. cm., what would be the volume, in cu. ft., of 1 ton of this metal?

(4) The density of brick being 2.1 gm. per. cu. cm., how much, in lbs., is the weight of a "thousand of brick," averaging 8 in. long, 4 in. wide and 2 in. thick?

(5) How great, approximately, is the weight, in tons, of a brick wall 30 ft. long, 20 ft. high and 8 in. thick?

(6) Estimate approximately the number of bricks, and the total weight of the same in tons, in the outer walls of some brick building well known to you.

8. Finding the Density of an Irregularly Shaped Body.—Evidently it is easy to do this, if we can find the volume of the body. If the body is one which is not injured by water, we can, in some cases, find its volume readily by putting it into a space already full of water, letting it drive out just so much water as it can take the place of, and then finding the volume of this displaced water, either by measuring it in a glass vessel suitably marked or by weighing it and counting 1 cu. cm. for each gram.

This method is well illustrated by Figs. 3 and 4. In Fig. 3, C is a vessel from which the water has just ceased to overflow through the tube t, and p is a vessel for catching the water which, in Fig. 4, has been displaced from C by the body sunk in it. To test the accuracy of the method, which requires that the water shall always stop overflowing at the same level, it is well to try it first with a rectangular body, the volume of which can be found correctly by measurement. If it works well, as it will if the apparatus is properly constructed and handled, it can be used to find

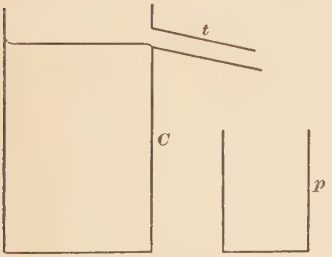


FIG. 3.—Use of overflow can: first stage.

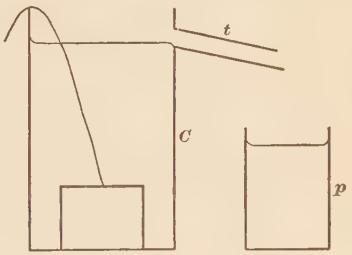


FIG. 4.—Use of overflow can. second stage.

the volume of such things as potatoes, lumps of coal, etc., the density of which it may be desirable to know.

9. Weight.—Before going farther we need to think carefully about the meaning of the word *weight*, which we have already used a number of times, and shall have to use very often. The word has two meanings.

Sometimes when we speak of the weight of a body we mean the *amount* of the body, as when we say 10 lbs. of butter or 100 lbs. of iron.

At other times we mean by the weight of a body the amount of the earth's downward pull upon that body, as shown by the spring balance, for instance.

It is somewhat hard to remember this distinction, because

the *units* in which we tell the *amount* of a body have the same name as the units in which we tell the *pull* which the earth exerts upon the body. For example, we say that the earth exerts a pull, or *force*, of 5 lbs. upon 5 lbs. of wood, or 5 lbs. of coal, or anything which consists of, or *is*, 5 lbs. of substance.

Often when we use the word *weight* it makes no difference which of its two meanings we have in mind, but sometimes it does make a difference. Thus, when we put a body into water, and say that it appears to lose *weight* in going from air to water, we do not mean that there appears to be any less of the *body* in water than there was in air. We mean that it requires a smaller pull of the spring balance to keep the body from sinking in water than it does to keep it from sinking in air.

10. Mass.—In strict scientific language the word *mass* is commonly used in speaking of the *amount* of a substance, and the word *weight* in speaking of the earth's pull upon that substance. For example, a piece of iron the *mass* of which is 50 lbs. is subject to a *weight*[1] or pull of 50 lbs. exerted by the earth.

SPECIFIC GRAVITY.

11. Discussion.—It is often convenient to know the *ratio which the weight of a body bears to the weight of an equal bulk of water*. This ratio is called the *specific gravity of* the body. For example, if the weight of a certain piece of iron is 7.8 times as great as the weight of an equally large body of water, we say that the specific gravity of the iron is 7.8.

Gravity comes from a Latin word *gravis*, meaning *heavy*.

[1] Such distinctions, which use words in a *scientific* sense different from the popular every-day sense, are often necessary in science, but it would be rather absurd to try to make the popular use of the words agree with the scientific use in all cases.

Specific here means *distinctive*, or *particular*. The *specific gravity* of a body is its *particular heaviness*—the degree of heaviness which distinguishes this body from other bodies of the same size but different weight.

Problems.

(1) If the weight of 5 cu. cm. of iron is 39 gm., what is the specific gravity of the iron? What is the weight in lbs. of 1 cu. ft. of it?

(2) If the sp. gr. of lead is 11.3, what is the weight in grams of 1 cu. cm. of lead? What, then, is the density of lead in grams per cu. cm.? In lbs. per cu. ft.?

(3) If the density of a certain kind of wood is 40 lbs. per cu. ft., what is the specific gravity of this wood?

Sinking and Floating.—In finding the specific gravity of solid bodies which are not injured by wetting it is a common practice to put them into water. We may divide bodies into two classes: first, those which, when they have no hollow spaces from which the water is kept out, will naturally sink in water; second, those which, when they have no hollow spaces from which the water is kept out, will naturally float in water.

We shall deal first, experimentally, with Class 1.

12. Principle of Archimedes.—If we suspend a body from some form of balance by means of a thread and weigh it first in air and then in water, its weight, as indicated by the balance, will be found smaller in the second case, the difference being equal to the weight of the water which has been pushed out of place by the body.

More briefly, *the apparent loss of weight of a body immersed in water is equal to the weight of the water displaced by the body.*

In place of the word *water* we can here use the general word *liquid*, as further experiments would show, if it seemed necessary to make them.

This important relation is called the law, or principle, of Archimedes.

A little reflection on this law, in connection with the definition of specific gravity, will indicate a simple method of finding, by experiment, the specific gravity of such bodies as can be conveniently weighed first in air and then in water. This method is used in Exercise 2.

Problems.

(1) A certain body weighs 100 gm. out of water and 50 gm. in water. How great is the volume of the body?

(2) A certain body 5 cm. long, 3 cm. wide, and 2 cm. thick, weighs 200 gm. in water. How much does it weigh out of water?

(3) If a piece of marble weighs 50 gm. in air and 31.5 gm. in water, what is its specific gravity?

(4) If a piece of copper, of sp. gr. 8.8, weighs 176 gm. in air, how much will it weigh in water?

(5) If a piece of zinc, of sp. gr. 7.0, weighs 300 gm. in water, how much will it weigh out of water?

13. Relation of Specific Gravity to Density.—Some of the problems already given illustrate the fact that the *specific gravity* of a body is numerically the same[1] as its *density in grams per cubic centimeter*. Of course, then, the specific gravity is not equal to the *density in pounds per cubic foot*, but is only equal to this density divided by 62.4. It is plain that the use of the metric units, grams and cubic centimeters, is more convenient in this respect than the use of the old-fashioned English units, pounds and cubic feet. Indeed, when the metric units are employed, it is rarely necessary to use the term *specific gravity*, as the shorter term *density* means practically the same thing in most cases. But we shall continue to speak of specific gravity in the next few exercises.

[1] This statement is not strictly correct, owing to the fact that 1 cu. cm. of water does not weigh exactly 1 gm. except at one particular temperature. This inaccuracy may well be disregarded in ordinary cases.

Problems.

(1) A brick-shaped piece of rock-salt 20 cm. long, 10 cm. wide, and 5 cm. thick weighs 2200 gm. What is its density in gm. per cu. cm.? What is its sp. gr.?

(2) A piece of sulphate of copper having a volume of 700 cu. cm. and a density of 2.27 weighs how many lbs.? What is its sp. gr.?

(3) A block of granite 20 ft. long, 10 ft. wide, and 5 ft. thick, weighs about 169,000 lbs. What is its density in lb. and ft. units? What is its sp. gr.?

(4) A cake of ice having a volume of 700 cu. ft. and a density of 57.4 lbs. per cu. ft. weighs how many kgm. (§ 5)? What is its sp. gr.?

14.—Various Expressions for Specific Gravity.—By definition we have

$$Sp.\ grav.\ of\ a\ body = \frac{Wt.\ of\ the\ body}{Wt.\ of\ an\ equal\ volume\ of\ water}.$$

It is evident that the quantity written below the line in this definition may be expressed in other ways. We may write,

$$Sp.\ grav.\ of\ a\ body = \frac{Wt.\ of\ the\ body}{Wt.\ of\ water\ displaced\ by\ the\ body\ when\ immersed},$$

or

$$Sp.\ grav. = \frac{Wt.\ of\ the\ body}{Loss\ of\ weight\ of\ the\ body\ when\ immersed},$$

or

$$Sp.\ grav. = \frac{Wt.\ of\ the\ body}{Lifting\ effect\ of\ water\ upon\ the\ body\ when\ immersed}.$$

These expressions all mean the same thing, but sometimes one of them is more convenient than the others. In the Exercise 3 the last form is used with advantage.

15. Forced Immersion.—We come now to the methods of finding the density of bodies that would naturally float in water. These methods are two, in one of which the body is forcibly immersed by means of some other body, called a

sinker, placed on it or in some way attached to it, while in the other the body is allowed to float. The first method is used in Exercise 3.

Problems.

(1) A block that weighs 4 oz. in air is fastened to a sinker that weighs 6 oz. in water, and the two together weigh 3 oz. in water. What is the sp. gr. of the block?

(2) A block the sp. gr. of which is 0.5, and which weighs 100 gm. alone in air, is fastened to a sinker that weighs 150 gm. alone in water. How much will both together weigh in water?

(3) If a man floats in water with 0.01 cu. ft. of his bulk above the surface,

(a) How much would the upward push of the water upon his body be increased if it were wholly submerged?

(b) How large a piece of lead, of sp. gr. 11.3, attached to his feet would be just sufficient to keep his body entirely submerged?

16. Flotation.—Thus far we have been considering the buoyant action of water on bodies wholly immersed in it. We shall now have to do with floating bodies and may well begin by trying through experiment to find how much, or what weight of, water is displaced by a body that floats in it.

Place the overflow can on one pan of a balance, fill the can and permit the overflow, then balance it with its contents by means of weights placed on the other pan. Place a floating cylinder (No. IV) gently in the can and permit the overflow to occur again, catching and removing the displaced water. When the flow ceases, the pans should again balance each other without further change of the load on either of them.

The principle, or law, here brought out, that *a floating body displaces its own weight of the liquid in which it floats*, is closely akin to that principle of Archimedes, relating to immersed bodies, which was given in § 12. Indeed, we can combine the two in the one statement, which we may call the general principle of Archimedes. *A body placed wholly or partly in a liquid is acted on by the liquid with an*

upward force equal to the weight of the liquid displaced by the body.[1]

If the greatest bulk of liquid the body can displace, which equals of course the bulk of the body, weighs less than the body itself, the body will sink; otherwise it will float, and the lighter the body is compared with the liquid the higher it will float. This fact leads to a method of finding the specific gravity of floating bodies which is used in Exercise 4

Problems and Questions.

(1) A block, of sp. gr. 0.6, floats in water. How much of it is below the surface?

(2) A block, of volume 1000 cu. cm. and sp. gr. 0.4, floats in water. How many cu. cm. of the block are about the surface?

(3) Is the sp. gr. of the human body much greater or much less than 1?

(4) Why does filling the lungs with air help one to *float* in water?

(5) What is the use of the "air-bladder" in fishes?

(6) B in Fig. 5 is a bottle filled with water standing at the bottom of a jar of water. The two tubes running through the stopper are open at each end. What will happen to the bottle if one blows long and hard into the rubber tube ? (Try the experiment.)

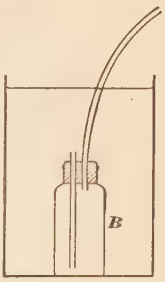


Fig. 5.

(7) (a) Does (6) suggest a method of raising the sunken hull of a ship built with closable compartments?

(b) Does it suggest any device for raising the utterly ruined frame of a ship, like that of *The Maine*, for example?

(8) What is the principle of a "pontoon bridge"?

17. Finding the Specific Gravity of a Liquid.—There are several convenient methods of doing this. Two of them

[1] The liquid must be free to get *under* the body, in order to have this statement hold true. A cylinder protruding vertically through the bottom of a vessel would not be buoyed up by the liquid in the vessel.

are used in Exercise 5, while a third, somewhat less simple, is given in Exercise 6.

Knowledge of the specific gravities of liquids may be useful in several ways. For example, if we know the specific gravity of a solution of sulphate of copper in water, we can tell the *strength* of the solution, that is, how much sulphate of copper a given amount of the solution contains.

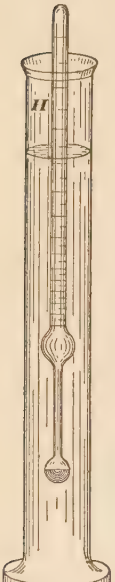


Fig. 6. Hydrometer.

Hydrometers.—A glass instrument like *H* in Fig. 6, shaped and weighted so as to float upright and carrying suitable marks and numbers on its stem, can be used for telling at once the specific gravity of the liquid in which it is placed, the particular mark which rests at the surface of the liquid indicating the value required. In order to mark, or *graduate*, such instruments properly it is necessary to try them in liquids the specific gravities of which are already known, but when once made they are very convenient.

Some hydrometers are suitable for use in water or liquids lighter than water, such as alcohol, gasoline, or oils; others are fitted for use in water or liquids heavier than water, such as acids and most solutions of things in water.

Questions and Problems.

(1) A glass sphere which weighs 100 gm. in air weighs 60 gm. in water and 40 gm. in sulphuric acid of a certain strength. What is the sp. gr. of the glass? What is the sp. gr. of the sulphuric acid?

(2) A vessel contains a layer of water and above this a layer of kerosene (sp. gr. 0.8). What is the weight of a cube, each edge of which is 10 cm. long, that, if placed in this vessel, will sink till one-half its volume is in the water and one-half is in the kerosene? What is its sp. gr.?

(3) A certain ship weighs with its cargo 10,000 tons.

(a) How many cubic feet of fresh water would it displace?

(b) How many cu. ft. of sea-water of sp. gr. 1.026 would it displace?

(4) If 1 cu. cm. of mercury weighs 13.6 gms. and 1 cu. cm. of cork weighs 0.25 gm., how deep will a cylinder of cork 20 cm. long sink when placed on end in mercury?[1]

(5) Which has the greater sp. gr., cream or skimmed milk?

(6) A piece of cloth thrown upon water will float at first and afterward sink. Why does it not sink at once?

(7) Can you tell from your own observation which of the following substances are more dense and which are less dense than water; kerosene oil, ordinary lubricating oil, butter, cheese, potatoes, eggs, meat, ice, india-rubber?

[1] In this question the effect of the surface-tension of the mercury (§39) is to be ignored.

CHAPTER III.

FLUID PRESSURE.

18. Fluids.—Water and air readily *flow* from one position or shape to another. They are examples of the class of substances called *fluids*. Fine sand and other like substances flow in a certain way, but examinations show them to consist of little hard or tough particles very different from equally small particles of water.

Fluids are divided into *liquids* and *gases*. Water is an example of the liquids, air an example of the gases.

19. Fluid-pressure.—Fluids settle snugly around solid, that is *non-fluid*, bodies placed in them and act upon these bodies, when at rest, with a peculiarly even pressure. We have had something to do with liquid-pressure in the preceding Chapter but have not studied it particularly. We must now give it especial attention and learn its principles, or laws, of action.

When we say that the pressure on a given area is a certain number of pounds or grams, for example, we may mean that the pressure on the whole area is equal to the load mentioned or we may mean that the pressure on unit extent of this area, a square inch or a square centimeter, for example, is equal to this load. When we need to distinguish between these two meanings we shall use the phrase *total pressure* to indicate the first. For the second, we shall have *intensity of pressure*, or *pressure per unit area*, or simply *pressure*.

Pressure at a point means the intensity of pressure at the point, that is, the total pressure which would exist on a

unit area, if the pressure were as intense all over this unit area as it is at the point in question. It is equal or nearly equal to the total pressure on a very small area, of which the point is the centre, divided by the extent of this area.

Familiar terms in the measurement of pressure are *pounds per square inch, grams per square centimeter, inches* or *centimeters of mercury*, the last indicating a pressure equal to that which would be exerted by the weight of a layer of mercury of depth equal to the number of inches or centimeters mentioned.

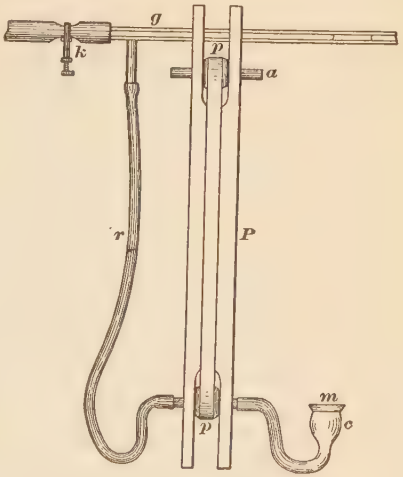


Fig. 7.—Pressure gauge.

By means of a properly contrived instrument, called a *pressure-gauge* (Fig. 7), the following laws or facts, holding for liquids *at rest*, can be illustrated:

1. Pressure increases with descent in the liquid and decreases with ascent.

2. Pressure at a given point in a liquid is equally great in all directions.

3. Pressure is equally great at all points on the same level in any one body of a single liquid.

Like facts or laws hold for gases *at rest*. The word *fluid* can be used instead of liquid in all three statements.

The laws of fluids at rest are called the laws of *hydrostatics*.

We shall make much use of these laws. Just here we can see that they explain, at least in a general way, why a body

immersed in water weighs, or appears to weigh, less than when in the air. For it is plain that there is an upward pressure of the water against the under side of the body, and that this upward pressure is greater than the downward pressure against the upper side of the body.

20. Slight Effect of Pressure upon the Density of Water.—Having seen that there is greater pressure on low levels than on high levels in water, we may well ask whether this greater pressure crowds the particles of water closer together on the low levels, thus making the water *denser* than on high levels. In fact there is an effect of this kind, but it is so slight that we need take no account of it in any ordinary case. At the bottom of a lake a mile deep the density of water is not 1 per cent. greater than at the surface, if the temperature is the same at all depths.

In some laboratory experiments, however, the volume of water has been reduced as much as 20 per cent. by a pressure of about 30,000 "atmospheres." See §§ 25 and 27.

21. Uniform Increase of Pressure with Depth.—We have not made, and cannot well make with the gauge shown in Fig. 7, any accurate measurement of the rate at which pressure changes with change of level in water. The fact is, however, that if we place a surface of 1 sq. cm. horizontal at any depth in water at rest, the column of water just above it is resting upon the given surface.[1] If we carry the given surface down 1 cm. farther, we now have resting upon it a load somewhat greater than before, greater by the weight of the additional 1 cu. cm. of water which is now above it. As 1 cu. cm. of water weighs 1 gm., the pressure upon a surface of 1 sq. cm. changes by 1 gm. for each 1 cm. change of level in the water.

[1] The pressure upon the given surface may be greater than the weight of the column of water resting upon it, for there may be, and usually is, a downward pressure of air or something else upon the top of the water-column.

If we were dealing with any other liquid than water, and if d were the density of this liquid, the pressure on a horizontal surface of 1 sq. cm. at a depth of h cm. in this liquid would be hd gm. greater than the pressure on 1 sq. cm. at the upper surface of the liquid.

22.—Balancing Liquid Columns.—It is a familiar fact that water stands at the same level in the spout and in the main part of a watering-pot, each being open at the top to the outside air. If either part is not thus open, if, for example, the spout is closed by a stopper, or if one blows into the spout, the water-level may be different in the two. It is likely to be different, too, if the water is warmer in one part than in the other, as the density of any liquid is usually changed by change of temperature. This temperature effect we shall leave out of account for the present, though it is of much practical importance in questions of heating, as we shall see later.

If the pressure on the upper surface of a liquid is p gm. per sq. cm. and if the density of the liquid is d gm. per cu. cm., the pressure in the liquid at any depth h cm. below this surface is $p+hd$. If, then, we have two liquid columns pressing against each other, one having a top-surface pressure p_1 gms. per sq. cm., a density d_1, and a height of h_1 cm. above the meeting place, while the other has a top-surface pressure p_2, a density d_2, and a height of h_2 cm. above the meeting place, we must, in order to keep equilibrium, have

$$p_1+h_1d_1=p_2+h_2d_2.$$

If the $p+hd$ of one column is greater than that of the other, the first column will descend and push the other up till, perhaps, the change of h's and of p's makes a balance between the two.

Nothing is here said about the size, or cross-section, of

the columns. The size is of no importance, unless it is small enough for the attraction which exists between the liquid and the containing wall to become worth considering. This effect may be large in tubes of small bore, called *capillary* (*hair-like*) tubes, and in narrow crevices. It is perceptible in tubes several millimeters in diameter, such as are used in Exercise 6.

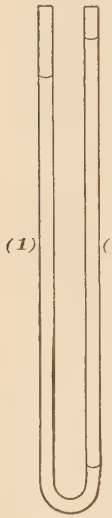


FIG. 8. Balancing columns.

The equation given above is useful in a certain method of finding the specific gravity of liquids. Let us suppose that our liquid 1 is water and our liquid 2 some oil, kerosene for example, the two columns meeting in a bent tube, as in Fig. 8. Here the top pressure, p, would be the same for each arm, that is, the atmospheric (§ 26 pressure), and, measuring h_1 and h_2 from the meeting-level up, we have, from equation (1),

$$h_1 d_1 = h_2 d_2, \text{ or } d_2 \div d_1 = h_1 \div h_2.$$

But $d_2 \div d_1$, and so $h_1 \div h_2$, = the specific gravity of the kerosene.

The simple arrangement of Fig. 8 is not suitable in the case of a liquid which naturally mixes with water. For such a case we can use the device shown in Exercise 6.

23. Average Pressure; Pressure on a Vertical Surface.—If we find the total pressure, P, on a given surface and divide this total by the extent, or area, A, of the surface, we get what is called the *average* pressure, $p = P \div A$, on the given surface. If the pressure is equally great at all parts of the surface, the average pressure is equal to the actual pressure per unit area at every part of the surface; otherwise, the average pressure is greater than the smallest pressure and less than the greatest pressure, per unit area, to be found on the surface in question.

The average pressure on certain surfaces, *e.g.*, a circle or a rectangle, placed vertically in a liquid, is easily seen to be equal to the pressure at the central point; for pressure increases regularly with the depth, and any given little part of the surface lying any distance below the centre is offset by some perfectly similar small part lying at an equal distance above the centre.

From the definition of average pressure it follows that the total pressure on a surface is equal to the area of the surface multiplied by the average pressure; $P = pA$.

Questions and Problems.

(The pressure of the atmosphere, due to the weight of air, is to be disregarded in all the questions here asked, not because it is small but because we are not yet ready to consider it.)

A cubical box, 10 cm. along each edge, has extending from its top, as in Fig. 9, a tube 15 cm. tall and 1 sq. cm. in cross-section (inside).

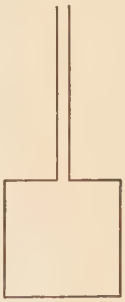


Fig. 9.

(1) If the box, but not the tube, is full of water, how great is the total water-pressure on the bottom?

(2) How great is the average pressure, and how great is the total pressure, on one of the vertical walls of the box?

(3) If the tube as well as the box is full of water, how great is the pressure upon that 1 sq. cm. of the bottom which lies just beneath the tube?

(4) Is the pressure equally great per sq. cm. at other parts of the bottom?

(5) How much is the total pressure now on the bottom of the box?

(6) How great is the pressure per sq. cm. at the top of the box just at the bottom of the tube?

(7) How great is the total *upward* pressure of the water against the top of the box?

(8) How great is the average pressure, and the total pressure, now on one of the vertical walls?

Suppose now that by means of a piston a pressure equal to the weight of 50 gm. is brought to bear on the top of the water in the tube:

(9) How great will now be the pressure on the sq. cm. which lies at the bottom of the box directly beneath the tube?

(10) How much will the total pressure on the bottom of the box be increased by the action of the piston?

(11) How much will the total pressure against the top of the box be increased by the piston?

(12) How great was the total pressure on 1 sq. cm. of the vertical wall of the box before the piston was applied, the centre of the square being at a depth of 5 cm. beneath the cover? at a depth of 8 cm.?

(13) How great is the pressure on each of these vertical squares, after the piston is applied?

(14) How much is the total pressure on one of the vertical walls of the box increased by the piston?

(15) What would be the answer to any one of these fourteen questions if sulphuric acid of sp. gr. 1.8 were used in the box and tube instead of water?

24. Pascal's Principle.—The question of the preceding Article lead naturally to the recognition of a principle which was announced by the French physicist and ecclesiastical writer Pascal about the middle of the seventeenth century.

"If a vessel full of water, closed in all parts, has two openings of which the one is a hundred times the other, placing in each a piston which fits it a man pushing the small piston will equal the force of a hundred men who push that which is a hundred times as large, and will surpass that of ninety-nine. Whatever proportion these openings have, and whatever direction the pistons have, if the forces that one applies on the pistons are as the openings, they will be in equilibrium."

The principle is usually stated nearly as follows:

Pressure exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force upon all equal surfaces in a direction at right angles to those surfaces.

This principle when properly applied is extremely useful, but, unless it is carefully illustrated and discussed, students

often get from it the false notion that pressure is equally great at all parts of the liquid.

An important application of Pascal's principle is made in the so-called *hydraulic press*, which will now be described.

25. The Hydraulic Press.—The *hydraulic press* usually depends for its action on the pressure applied to the liquid in a large cylinder from a plunger working in a smaller cylinder, often at a considerable distance. The larger plunger (A, Fig. 10) is so arranged as to work water-tight in

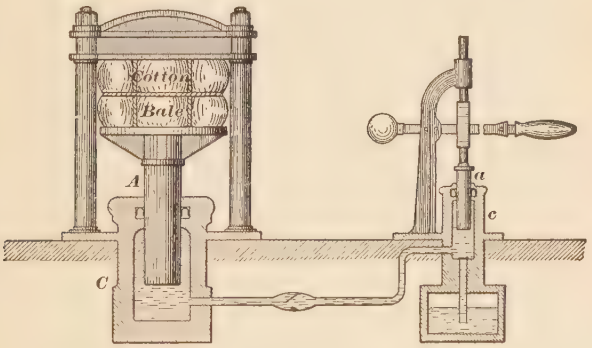


FIG. 10.—Hydraulic press.

the cylinder C, and the plunger a works similarly in the small cylinder c. Small quantities of water or some other liquid are successively forced from c into C by the movements of a, and A is raised very slowly, but with great force.

Since the pressure per square centimeter exerted by the piston a is transmitted undiminished to every part of the body of water in both cylinders, the total upward pressure, P, exerted through A, due to the total downward pressure, p, exerted through a, is such that

$P : p : :$ (cross-section, S, of A) : (cross-section, s, of a), or

$$P = p\ (S \div s).$$

Precisely the same principle holds if a gas, instead of a liquid, is used as the means of transmitting the pressure.

Rest one end of a board, about a meter long, on a table and the other end on the partly inflated "bladder" of a football. Then find how heavy a weight, placed on the board nearly over the bladder, can be lifted by forcing air into the latter by blowing from the lungs.

Of course, the small piston of an hydraulic press has to travel faster and farther than the large piston. This matter will be considered again in § 71.

A familiar use of the hydraulic press is the crowding of large piles of naturally bulky material, like hay or cotton fibre, into comparatively small bales, which, confined by strong bands, are convenient for transportation. The general principle, however, of the apparatus is used in many other applications, in hydraulic elevators, for example, the large piston, *A*, in this case being a long hollow shaft of steel which moves smoothly up and down in a deep enclosing tube, or well, in which it must fit closely enough to prevent the liquid below from leaking past it but not so closely as to be greatly resisted in its motion by friction.

By making the piston *A* end in or press against a small cylinder all the pressure passed through it can be concentrated upon a very small surface beyond, thus producing a very intense pressure. By such a device, and with a press worked by a hand pump, pressures of 30,000 "atmospheres" have been obtained, great enough to reduce water at 0° C. to 80 per cent. of its normal volume and to burst a steel tube having a half-inch bore surrounded by a wall 2 in. thick.

26. Atmospheric Pressure.—We will at the start repeat in a slightly varied form a famous experiment first made by Torricelli, an Italian, about the middle of the seventeenth century.

It is intended to show the weight-pressure of the air about us, which is called *atmospheric pressure.*

Take a strong glass tube about 100 cm. long and 0.8 cm. in diameter of bore, closed at one end. Holding this tube inclined, with the closed end downward, pour mercury slowly into it through a small funnel of glass or paper, tapping or shaking the tube occasionally to dislodge air-bubbles held in the mercury. When the tube is full, press one finger closely over the open end, invert the tube, plunge the end closed by the finger beneath the surface of mercury in a vessel, and then remove the finger. The mercury falls away from the top of the tube for a certain distance and then stops, leaving several centimeters of the tube apparently empty.[1] See Fig. 11.

Fig. 11. Torricelli's experiment.

Explanation.—It was known before the time of Torricelli that, if air were drawn from the upper part of a tube the lower end of which rested in water, the water would rise in the tube, but the true reason for this was not known. Torricelli maintained, and Pascal showed by experimenting at different heights in the air, that the pressure of the atmosphere, due to its weight, accounted for the rise of liquids in a vacuum. We have only to think of the fact that the air, although its density is very small compared with that of water, has, because of its great quantity, a great weight, and we see that the air, pressing upon the mercury surface in the open vessel, balances the column of mercury in the tube.

It has been stated (§ 19) that air pressure, like liquid pressure, is at any given point equal in all directions, if the air is at rest. This fact can be illustrated by covering the mouth of a glass "thistle tube" (Fig. 12) with an air-tight cap of strong sheet india-rubber, pumping out the air beneath the cap, so that the outside air pressure will force the cap into a deep cup shape, and then turning the mouth of the tube in all directions, the depth of the depression remaining constant.

[1] Really this space contains a very little air, from the bubbles that were in the mercury column before it was inverted, but so little that we may at present disregard it and consider the space above the mercury as empty. Such a space is called a *vacuum*, from a Latin word meaning *empty*.

27. Further Study of Atmospheric Pressure; the Barometer.—By measuring the difference in height of the two mercury surfaces in Fig. 11 we can get a measure of the atmospheric pressure. A "standard atmospheric pressure," often called simply "an atmosphere," is one equivalent to the weight-pressure at sea level[1] of a layer of mercury 76 cm. deep at 0° C. This is nearly the same as the average actual atmospheric pressure at sea level. It is about 1033 gm. per sq. cm., or 14.7 lbs. per sq. in. Large pressures are often reckoned in "atmospheres."

If we fasten the apparatus that is shown in Fig. 11 to a suitable support, it will serve permanently as a rude *barometer*, indicating the variations of the atmospheric pressure.

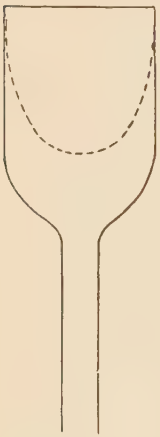


Fig. 12.

A better form of barometer is shown in Fig. 13. The mercury reservoir, or cistern, is covered to prevent loss of mercury when the instrument is carried about or jostled, and the glass tube, which may be 1.5 cm. or more in diameter, is placed for protection and support inside a tube of brass, on which is a scale divided either into inches and tenths of inches or into centimeters and millimeters.

The scale is marked as if it began at a certain level in the mercury cistern, at the point of a pin reaching down from the top of the cistern, and the indications of the barometer are correct only when

Fig. 13. Mercury barometer.

[1] To be exact we should have to add *at a certain latitude;* for example, the latitude of Washington or of Paris.

the surface of the mercury in the cistern is at this fixed level. In the use of a barometer of this class the level of the mercury in the cistern should first be adjusted, by means of a screw which presses against the flexible leathern bottom, and after this the height of the column is to be read on the scale alongside the tube.

The top of the mercury column in a glass tube is usually convex, and "capillary action" (see § 44) tends to push down such a column. The effect thus produced in a tube of considerable size, 1 cm. diameter for example, is very small. Sometimes a column that has recently been sinking has a flat or concave top. In such a case it is well to tap the side of the barometer with the fingers a number of times before taking a reading. The height of the very top of the convex surface of the mercury in the tube should be taken as the height of the column.

28. The Compressibility and Expansibility of Gases.—We have seen a number of resemblances between gases and liquids. We must now take account of one very marked difference between them. It is, as we know, very difficult to compress water much, but it is very easy to compress air.

On the other hand, no matter how much or how long a gas may be compressed, provided it remains a gas and is not turned into a liquid or a solid, as soon as the pressure is lessened the gas expands, and it will continue to expand as long as more room is given to it. Thus the gas which fills a few cubic centimeters of space behind the bullet in a rifle barrel and exerts a very great pressure on the bullet, driving it forward, would fill, in a sense, the largest gas holder, exerting a pressure, though a very slight one, on every square centimeter of the inner surface.

We have already seen that the weight of the atmosphere exerts a very considerable pressure per square centimeter, but we must now give attention to the fact that a very

small quantity of air or any other gas, the mere weight of which is too small to be worth reckoning, may by its expansive, or elastic, tendency exert a great pressure. In Exercise 7 the relation between the volume and the elastic pressure of such a body of air is studied.

29. Discussion of Boyle's Law.—The law in question was announced by Robert Boyle, an Englishman, in 1662, and by Mariotte, a Frenchman, about sixteen years later. It is often called Mariotte's law. It may be stated thus: *The volume of a given body of air, at a fixed temperature, is inversely proportional to the pressure to which it is subjected.* Or thus:

$$V \propto \frac{1}{P}; \text{ that is, } PV = a \textit{ constant quantity.}$$

Strictly speaking, Exercise 7 can establish this law only for a given amount of air at some one temperature and through a very small range of pressure. At very low temperatures and very high pressures it does not hold exactly.

Boyle, in announcing the law, spoke of the "*Spring of the Air*," and this phrase is very apt; for no matter how much or how long a gas has been compressed, it will always return to its original volume as soon as the original pressure and temperature are restored. In other words, gases have perfect *elasticity of volume* (§ 97).

30. Various Illustrations and Applications of Fluid Pressure.—There are many interesting cases in which liquid pressure and gas pressure are combined, and which the laws of hydrostatics help to explain. Of course, when we have *motion* of the fluids dealt with, new features of the situation appear, friction and inertia (§ 118), and the simple laws of fluids at *rest* are not enough for a complete understanding of all that occurs and of the exact way in which it occurs. But even here these laws are of very

great help in showing us how and why the operations in question *begin*. See (7) and (8) below.

In seeking to explain hydrostatic puzzles it is best to begin at a free surface, a surface where the liquid meets the outside air, if there is one; for we know how great the pressure is there. It is simply the barometric pressure. If from this surface we reckon down through the liquid, we find an increase of pressure; if we reckon up through the liquid, we find a decrease of pressure.

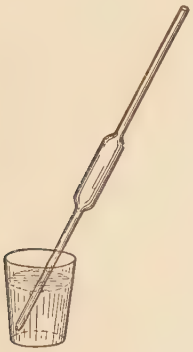


Fig. 14.—Pipette.

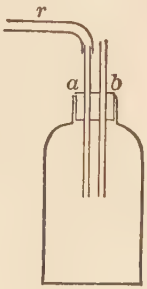


Fig. 15.

(1) Take a "pipette" (Fig. 14), or any long narrow glass tube open at both ends, and dip one end into a vessel of water. Apply the lips to the other end and draw water up into the tube.

In what sense is the water *drawn* up?

Try to draw in the breath without allowing the chest to expand.

(2) After nearly filling the tube quickly close the top with a finger and then lift the lower end from the water. Uncover the top of the tube for an instant, then cover it again.

Explain the behavior of the water during these operations.

(3) Fig. 15 shows a bottle closed with a rubber stopper through which two glass tubes, *a* and *b*, open at both ends, extend. To one of the tubes, *a*, is attached a rubber tube, *r*. The bottle and the two glass tubes are full of water.

By applying the lips to the outer end of the tube r water can be "drawn" into the mouth. Can this be done when the tube b is closed by a finger at the top?

If the bottle is inverted while b is closed will water run out?

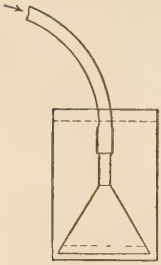


Fig. 16. Principle of "diving-bell."

Why will water not run evenly from a nearly full bottle?

Why is it necessary to loose the cap or stopper at the top of a kerosene can or the "bung" of a barrel in order that the liquid may flow freely from the spout?

(4) Fill or nearly fill a tumbler or broad-mouthed bottle with water and then cover it with a sheet of thick paper. Hold the paper firmly in place with the hand and invert the tumbler; then take away the hand that holds the paper. (As accidents may happen, the tumbler should be held over some large dish.)

In this experiment it should be noticed that the paper does not press close against the rim of the tumbler after the inversion. It hangs rather loose, having dropped down or sagged a little, thus allowing the air above the water to expand a trifle, decreasing in pressure.

(5) *Diving-Bell.*—The principle of this useful device, which enables men to keep their heads and bodies in air while laying masonry or doing other work at a considerable depth under water, is shown in Fig. 16. Air blown into the tube drives the water down to any level in the funnel.

Similarly, air under a high pressure is used during the construction of tunnels under river beds to keep the water and mud from entering the excavation through small holes and crevices.

Fig. 17. "Cartesian diver."

(6) *Cartesian Diver.*—In Fig. 17, J is a glass jar containing water, d is a small glass vial, mouth down, containing just enough air so that it will float pressing lightly against the well-fitting stopper S. When pressure is applied to S, d sinks to the bottom of the jar, but it rises again when the pressure on S is lessened. Explain.

This is one form of the *Cartesian diver*, invented as an instructive toy by René Descartes, a famous French philosopher and mathematician (1596–1650).

(7) *Mariotte's Bottle.*—The apparatus shown in Fig. 18 was invented by the same philosopher who shares with Boyle the honor of discovering the law of compression of gases. The bottle and the tube, open at both ends, which goes through the stopper, are filled with water. The side tubes, *a*, *b*, and *c*, are at first closed.

What will happen if *a* only is opened? *b* only? *c* only? *a* and *b* together?

(8) *The Siphon.*—This apparatus is illustrated in Fig. 19. As here shown it consists of a rubber tube connecting two glass tubes, but it is used in a great variety of forms.

The closed end of the tube is below the free surface of water in the jar.

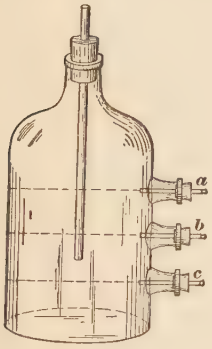


Fig. 18.—Mariotte's bottle.

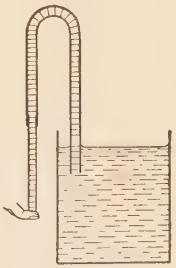


Fig. 19.—Siphon.

Is the water pressure against the finger that closes the tube now greater or less than the atmospheric pressure upon an equally large surface?

What will happen then, when the finger is removed? Try.

Repeat the experiment, but now hold the outer end of the tube, before opening it, higher than the level of the water in the jar.

Is there any natural limit to the height which there may be in a working siphon between the water level in the reservoir and the topmost part of the siphon tube?

31. Pumps.—Many contrivances for making fluids run from one place to another are called *pumps.* A flow may

be caused by decreasing the pressure at the place where the fluid is to be delivered or by increasing the pressure at the place from which it is to be removed, provided there is some channel filled with fluid connecting the two places.

Most pumps have parts called *valves* which open to permit flow of liquid or gas in one direction, but do not open to permit flow in the opposite direction. Valves are of many forms and kinds.

FIG. 20.—"Lifting-pump."

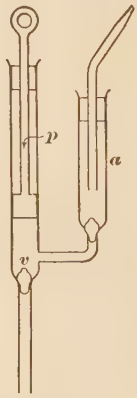


FIG. 21.—"Lifting and force" pump.

The forms of glass apparatus shown in Figs. 20 and 21 illustrate the principles of pump action. Fig. 20 is a "lifting-pump," Fig. 21 a "lifting and force-pump." In each, *p* is the "piston rod".

If such pumps are used to draw water from a reservoir which is open to the atmosphere, is there any limit to the height at which the lower valve, *v*, may be placed above the water in the reservoir? If so, about how great is this limit under usual conditions?

Notice the behavior and use of the air in the air-chamber, *a*, of the force-pump. A steam "fire-engine" is a force pump and the pear-shaped dome on its top is an air-chamber.

Air-Pumps.—The principles of construction of the air-pump are essentially the same as those of the water-

pumps just described; and, indeed, the great blowing-engines, that supply the intense blast of air to many kinds of furnaces for smelting metals, work force-pumps which drive air through tubes, much as an ordinary force-pump drives water. The ordinary air-pump of the lecture-room, however, is more like the lifting-pump of Fig. 20.

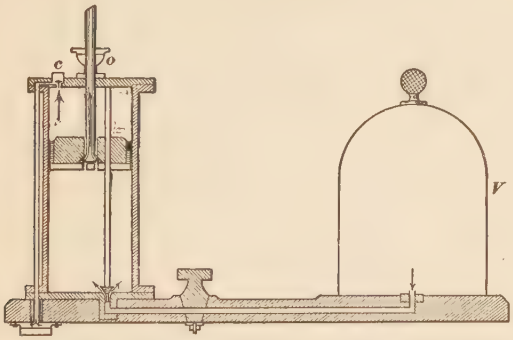


Fig. 22.—Air-pump, with exit valve at top of cylinder.

If the valves of a pump used to exhaust the air from some vessel are worked merely by the difference of air pressure on their opposite faces, the effective action of the pump ceases when the air pressure in the vessel becomes too small to open the first valve. This often happens. Fig. 22 shows a pump with an *automatic* lower valve at the bottom of the cylinder. This valve is opened by a rod which is lifted a short distance by the ascending piston.

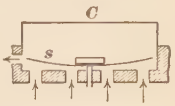


Fig. 23.—Details of exit valve.

A valve under *c* (Figs. 22 and 23) at the top of the cylinder allows air to escape during the upward stroke, but prevents air from entering from above during the downward stroke of the piston. Without this valve there would be a heavy pressure on the top of the piston all the time, which would make the labor of working the pump very great and would be likely to cause leakage downward past the piston during the upward stroke. This upper valve is often a mere disk of oiled silk, fastened down at its center but free to rise at the edge, which covers a number of small holes. The cup *o* holds

oil which gradually finds its way down the piston-rod, thus lubricating the cylinder.

However carefully made the air-pump cannot give a perfect vacuum. Even if it is so constructed that the valves will be opened and shut automatically, the continued action of the pump can only reduce the fraction of an atmosphere remaining in the exhausted vessel to smaller and smaller values without ever diminishing it to zero. For each upward stroke of the piston can at most remove only the air in the cylinder, leaving the air in the vessel (V, Fig. 22) and connecting-pipe to expand and fill the entire space contained by the cylinder (below the piston), the connecting-pipe, and the vessel.

To make the case as simple as possible, suppose the cylinder and the vessel to be of equal capacity, and the pipe to be so small that its contents may be neglected. If the volume of air before the first up-stroke were 2 liters, the volume after that stroke, in vessel and cylinder would evidently be 4 liters, and the pressure would have fallen to one-half what it was at first. The down-stroke would make no change in the pressure within the vessel. After the second up-stroke the pressure would be $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$, and so on.

32. Pumps for "High Vacua."—To produce the most perfect attainable, or "highest," vacua (*empty spaces*), mercury-pumps are commonly used, that is, pumps in which a quantity of mercury acts as a piston, driving the air before it. Sometimes the mercury is used in a large slow-moving mass, sometimes as a small stream, or rapid succession of small columns through a tube.

It is obvious that a liquid piston fits better and moves more readily than a solid piston. The special qualities which recommend mercury above other liquids for use in pumps are its density, its peculiarly slight tendency to evaporation, its strong self-attraction, which prevents it from wetting the glass tubes or vessels in which it flows, and its small power of absorbing gases.

Fig. 24 shows in very simple form the upper part of a so-called Sprengel air-pump. Mercury descends from a reservoir at the top, falls in drops from the bottom of the small glass tube within the bulb-

shaped space, and as these drops descend through a narrow tube below they carry between them small quantities of air which have flowed in from the vessel which is being exhausted. The vertical tube through which the drops descend from the bulb must be taller than the barometric column.

Formerly mercury-pumps, in spite of their slow action, were considered necessary for exhausting the bulbs of incandescent lamps, which must, for effective working, be almost completely emptied of air. Now pumps with solid pistons, similar in their general construction and action to that shown in Fig. 22, are used for getting out the greater part of the air, two pumps being used in *tandem*, one to take air directly from the bulb, the other to take it from the cylinder of the first pump. After these have done their work, much of the small amount of air remaining is burned out by flashing some combustible material in the bulb.

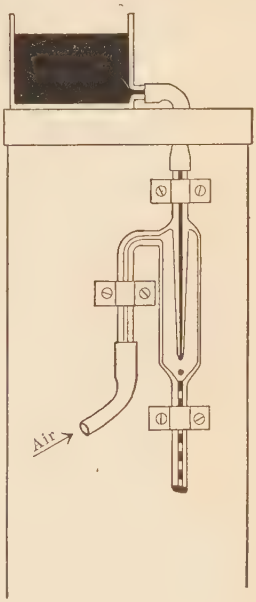


Fig. 24.—Mercury-drop air-pump.

Questions and Problems.

(1) A water-tank 10 ft. deep is to be emptied by means of a tube used as a siphon. What is the least length the tube can have?

(2) Do you understand the operation of the common household "trap," which allows water to flow from a sink to a sewer, but does not allow gas to come from the sewer to the sink?

(3) Do you know what serves as a valve in a common bicycle-pump?

(4) Make sketch of an air-pump which would take in air at one orifice and force it out at another orifice, both at the bottom of the cylinder. Make the valves very simple and show how they would work.

33. Density of Gases.—The density of air, or any other gas, depends on its temperature and the pressure to which it is subjected, but it is customary to give that value of the

density which holds when the temperature is that of freezing water and the pressure is equal to that of 76 cm. of mercury. These are the so-called *standard conditions*, but in Exercise 8 the density of the air is to be found at the temperature and pressure which hold in the laboratory at the time of the experiment.

34. Weight of the Atmosphere; Magdeburg Hemispheres.—After the air has been pumped out from the glass "receiver," V, in Fig. 22, the receiver and the plate of the pump will act as if they were fastened together, and if the receiver is lifted the plate goes with it.

A similar experiment is frequently performed with two hollow hemispheres from which most of the air has been pumped out. "The experiment with the hemispheres was made for the first time by Otto von Guericke, burgomaster of Magdeburg, who repeated it, in 1654, at Ratisbon, before Ferdinand III. and the assembly of the College of the Empire. The hemispheres were 24 *pouces* 8 *lignes* [about 26 inches] in diameter. Although much air remained within them, it required 12 horses to pull them apart. The gas (air) rushing into the vacuum, at the moment of separation, produced a violent explosion. This celebrated experiment helped to spread the doctrine of the weight of the air, and to popularize the air-pump, which Otto von Guericke had invented."[1]

Questions.

(1) Why does the lower hemisphere, with the plate of the air-pump, rise when the upper hemisphere is lifted? Are the two *stuck* together as if by glue?

(2) If the hemispheres are 10 cm. in diameter, inside, how great is the force required to lift the upper one away from the lower one? (Neglect the weight of the hemisphere and observe that, as a solid hemisphere surrounded by air is not driven in either direction by the

[1] Daguin's *Traité Élémentaire de Physique.*

air pressure, the total downward air pressure on the upper hemisphere must be just as great as would be the total air pressure on a disk of the same diameter as the sphere.)

35. Variations of Atmospheric pressure; Barometric Measurements of Height; Weather Indications.—Pascal showed in the early days of the barometer (see §26) that atmospheric pressure in general decreases with increase of height from the earth's surface; in fact, this demonstration may be said to have convinced the world that the atmosphere does exert pressure because of its weight.

The rate at which atmospheric pressure decreases with increase of height being well known, it is a common practice to estimate the height of mountains by noting the difference of atmospheric pressure at the summit and base. "Aneroid" barometers are frequently used for such work. *Aneroid* means *without liquid.* An aneroid barometer contains no mercury or other liquid. It is an air-tight metal box with a flexible metal cover. The middle of the cover moves in or out slightly with changes of pressure, and its slight motions are magnified to the eye by various mechanical contrivances. Some aneroid barometers are about as large as ordinary watches and look much like them.

For small changes of elevation, near the sea-level, a fall of 0.1 in. of the barometer corresponds to an elevation of 87 ft., 0.1 cm. to 11 meters, nearly, but this simple rule does not hold for great changes of height, such as occur in balloon ascensions or in mountain climbing.

Fig. 25 shows how, in still air at the temperature of freezing water, the pressure diminishes with increase of height above sea level. The height H, in miles, is represented by vertical distance; the pressure P, in fractions of "an atmosphere," by horizontal distance.

The daily and hourly changes in the height of the barom-

eter at one place may be considerable. These barometric changes must be noted when any careful experiments upon pressure of gases are undertaken, in order that the experimenter may know to just what pressure the gas under examination is really exposed. Their chief interest, however, lies in the fact that they give useful indications of

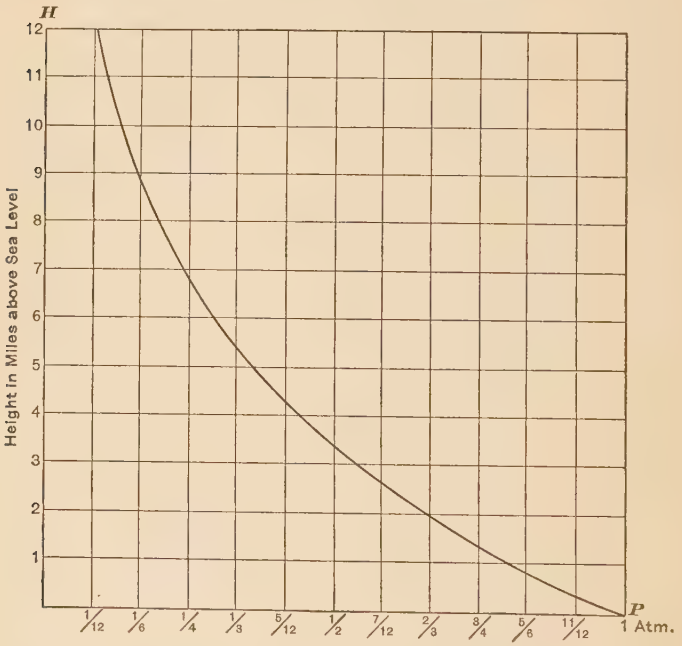


Fig. 25.—Relation of height and atmospheric pressure.

coming changes of weather. As a rule, the coming of a storm is preceded by a decrease in atmospheric pressure, a "fall of the barometer," though sometimes the barometer will stand high, above its usual fair-weather height, during a storm. "Weather maps," from which weather predictions are made, are thickly strewn with lines and figures showing

the state of atmospheric pressure in various parts of the country.

36. Height of the Atmosphere; Composition of the Atmosphere.—The "*height of a homogeneous atmosphere*" is a common phrase which means the height that would be necessary to produce the "standard" atmospheric pressure, if the air had at all heights the same density which it has at the sea-level under "standard conditions," that is, standard barometric pressure and temperature of freezing water. In fact the density of the air must diminish with the lessening pressure as the height increases, and it is impossible to tell how high our atmosphere extends. It has probably no definite limit.

The atmosphere consists of several gases. Nitrogen furnishes about three-fourths of its weight and oxygen about one-fourth. Argon, a gas quite unknown as distinct from nitrogen before 1895, when it was discovered by Lord Rayleigh, makes about 1 per cent.; water vapor, not fog or cloud but the true invisible vapor, usually a fraction of 1 per cent.; carbonic acid, perhaps 1 part in 2000 of its weight. All these gases mix freely with each other so that the composition of the air is about the same on the top of a high mountain as at the base.

Questions and Problems.

(1) If the density of mercury is 13.6 and that of standard air is .00129, each in gm. per cu. cm., how great is the height of the "homogeneous atmosphere?" *Ans.* 8012 m. nearly.

(2) How much, approximately, is the difference between the readings of two accurate mercury barometers, placed one at the bottom and the other at the top of a building having 20 stories averaging 9.5 ft. high?

(3) If water were used in a barometer instead of mercury, how many cm. tall would the column be under ordinary atmospheric pressure?

(4) How tall would the mercury barometric column be if its base

were at a depth of 20 m. in water, the pressure at the surface of the water being 76 cm.?

(5) What is the weight, in kgm., of the air contained under standard conditions (see Problem 1) in a room 20 m. long, 15 m. wide, and 8 m. high?

37. Buoyancy, Floating Power, of Air.—It is a mistake to suppose that any form of matter rises naturally, that is, without the help of any force. The principle of Archimedes, for the buoyant effect on an immersed body, holds in a gas just as it does in a liquid. Soap bubbles filled with air will sink, though slowly, through air; for on account of the enclosing liquid film they are slightly heavier than the air they displace. If filled with ordinary illuminating gas, which is lighter than air, they will rise in air. We have here the principle of the balloon, the old-fashioned balloon, which has no driving or lifting machinery. Large balloons are usually filled with illuminating gas; so-called fire-balloons contain mainly air, made light by expansion due to the heat of the fire carried below.

Balloons which are to reach a great height must be open at the bottom, or capable of being readily opened, in order that the pressure within may not be very different from that outside; otherwise they would burst in rarefied upper air, or having too great internal weight, would not ascend there. Of course, the buoyancy of the surrounding air grows less as the upper regions of lower pressure and lower density are reached.

To provide for emergencies balloons are equipped with bags of sand, which can be spilled out to help an ascent, and with means for letting out the interior gas suddenly, thus allowing the floating bulk to diminish without corresponding loss of weight, in order to hasten a descent at the right time.

Often bodies which appear to be floating or even rising in air are really sinking slowly through the air immediately

about them, which may be moving in an upward current. This is true of small liquid particles, fog or cloud, and small solid particles, dust or smoke.

Questions and Problems.

(The specific gravity of mercury when needed in the following problems is to be taken as 13.6. In dealing with most problems the student will be greatly assisted by making a simple sketch of the apparatus or conditions to be considered.)

(1) The upper surface of the water in a reservoir is 40 m. above a city street. What is the pressure, in kgm. per sq. cm., in the water-main at the surface of the street, in excess of the atmospheric pressure? in lbs. per sq. in.?

(2) A man uses just his whole weight, of 70 kgm., to force a smoothly fitting plug into a hole in the water-main of Problem (1). What is the cross-section of the plug?

(3) A vessel is filled with water to a depth of 40 cm. A cylinder of wood 30 cm. long and 100 sq. cm. in area of cross-section, the specific gravity of which is 0.5, extends upward through a hole in the bottom of the vessel, the top of the cylinder being 20 cm. beneath the surface of the water. Show whether the cylinder tends to rise or to fall, and how great a force is required to hold it in its present position.

(4) How great is the weight of the water in the box and the tube, both full, described in §23?

(5) (a) If this box with its contents, the tube being full of water but without the piston mentioned in § 23, were placed on the platform of a balance, how great would its downward pressure on the platform be, the weight of the box itself being disregarded?

(b) Would this pressure be just equal to the weight of the water as found in (4)? If not, explain the difference.

(c) Would it be just equal to the total downward pressure of the water on the bottom of the box? If not, explain the difference.

(6) A pressure-gauge is arranged so as to register the highest pressure to which it is subjected. The barometer-reading is 76 cm. The gauge is lowered into a lake until it registers a total pressure (from atmosphere and water together) of 5000 gm. per sq. cm. To what depth was it lowered?

(7) The bore of an air-gun, or pop-gun, 50 cm. long and 1 sq. cm. in cross-section, is fitted with a frictionless piston which slides in it

air-tight. The tube is filled with air at 76 cm. barometer, and the piston is then forced in to a point 5 cm. from the closed end. What pressure in grams is applied?

(8) A bubble of air liberated at a depth of 20 meters under water has a volume of 5 cu. cm. What will be its volume at the moment of reaching the surface, if the barometer stands at 76 cm.?

(9) In an air-pump the capacity of the cylinder is 0.5 liter, and that of the vessel to be exhausted 4 liters. The capacity of the connecting tube may be neglected. How much of the air originally present will remain in the vessel after five double (complete) strokes?

(10) Water, in a pipe leading from an open reservoir to the cylinder of a pump, stands at a height of 8 meters above the level in the reservoir. The barometer reads 75 cm. How great, reckoned in centimeters of mercury, is the pressure of the air just above the water in the pipe?

(11) The water surface in a diving-bell is 60 ft. below the free surface of the sea. The specific gravity of the water is 1.03. How much, in pounds per square inch, is the pressure of air in the bell above the pressure of the atmosphere?

(12) If the water has all run down to its natural level out of the cylinder and suction-pipe of a lift-pump, it may usually be pumped up again by filling the cylinder with water and working the pump rapidly. Can you explain this?

CHAPTER IV.

MOLECULAR ATTRACTIONS: SURFACE TENSION, ABSORPTION, SOLUTION.

38. Adhesion and Cohesion.—The surface of water close to a vertical wall of glass is not horizontal, but is turned upward, so as to appear concave as seen from above. The surface of mercury, on the other hand, is turned downward near a vertical surface of glass, so as to appear convex when seen from above. These facts are not accounted for by the laws given on page 17. The glass appears to hold the water against itself by some force of attraction, and it does do this. A piece of glass dipped into water comes out wet; that is, it pulls some of the water away from the rest and against the force of gravity. On the other hand, glass may appear to repel mercury; but it does not do so.

FIG. 26.—Adhesion of glass and mercury.

If a clean and dry glass disk, 5 or 6 cm. in diameter, suspended from the hook of a sensitive spring balance, as in Fig. 26, is allowed to rest for a moment on a flat mercury surface, a considerable force, in addition to the weight of the plate, will be shown by the balance when the disk is lifted. If water is used instead of mercury, the force in question is smaller.

Attraction between neighboring layers of different substances is called *adhesion*. Attraction between very small neighboring particles, or *molecules*, of the same substance is called *cohesion*. Cohesion in solid bodies is very evident (see § 93), and even in a liquid it may be large. We shall

see (Exercise 25) that the amount of energy (§ 139) required to separate the molecules of a kilogram of water and turn it into a vapor is very great. If a column of some liquid, mercury for instance, is inclosed in a tube to the sides of which it adheres, so that it cannot *dwindle* before pulling apart, it shows a very considerable tenacity. Newton showed[1] that a column of mercury much taller than the barometric column can be sustained vertically in a glass tube, provided the air is excluded and the mercury is in contact with the closed top of the tube. The mercury is, in fact, suspended by its top, and must be in a somewhat *stretched* condition.

When two layers of molecules are very close together the cohesion-attraction between them is probably very great, in comparison with their weight, but if they are not very close together this attraction is imperceptible. Quincke, a German, found that the attraction of water or of mercury for glass acted through a very thin layer of some third substance placed between them, but if the thickness of the layer was made as much as 0.00001 cm., or even somewhat less, the action apparently ceased.

39. Surface-Tension.—At points well within a body of liquid the attraction of molecule for molecule presses them hard together, but is equally great in all directions and does not tend to change the *shape* of the liquid body; for no change in the shape of the body would allow the interior molecules to come nearer together. But the molecules at the surface, not being surrounded on *all* sides by attracting molecules, may be drawn inward, thus coming into close relations with more of their fellow molecules. This operation is necessarily accompanied by a contraction of the surface. The surface of a body of liquid therefore tends to become smaller and smaller, acting *somewhat* like a film of stretched

[1] See Maxwell's article on *Capillarity* in the Encyclopædia Britannica.

india-rubber. This fact suggests the name *surface-tension*, used in connection with many curious phenomena, of which all are interesting to the student of physics and some are of great practical importance.

When a body of liquid is left free to follow its own tendencies, unaffected by forces from without, surface-tension brings it to the form having the least surface, that is, the spherical form. Drops of melted lead, falling from a great height in a "shot-tower," take the spherical form as they descend, and cool sufficiently to retain that shape on reaching the bottom, where they fall into water. Thus shot are made. A very small drop of mercury on a table-top assumes a form which is very nearly spherical, in spite of its weight which tends to flatten it out. If the amount of mercury is larger, the drop is perceptibly flattened, though it is still convex at its edge. Water, on surfaces which are not *wetted* by it, on leaves and feathers, for example, under certain conditions, acts like mercury on wood. If a liquid wets the body on which it rests, the attraction between the two acts, with the weight, to prevent the liquid from taking the spherical form.

40. Liquid Films.—A liquid film attached to a solid boundary takes that shape which makes the surface the smallest possible under the circumstances.

(All the wire frames and the pipe described in the following experiments[1] are grouped in the list of apparatus as No. L.)

(1) Blow a small soap-bubble with a common pipe, holding the mouth of the pipe downward. Watch the bubble for a little time, and see whether it contracts, taking care not to close the mouth-end of the stem.

(2) Take a wire frame about 4 cm. wide, like that shown in Fig. 27, the light cross-wire *s* being free to slide on the guides *g* and *g*. After dipping the parts *s* and *c* into a soap solution, pull *s* some distance away from *c*, hold *g* and *g* horizontal, and then release *s*.

[1] See Plateau's "Statique des Liquides."

Experiments properly carried out would show that the pull of the film on the wire s is equally great, whatever the *length* of the film, in the direction g, or the thickness of the film, provided it is not made very thin indeed. In this respect the action of the film is unlike that of a stretched film of india rubber, to which in some particulars it may be compared. This accords with the statement, made in § 39, that the shape-changing tendency of the liquid is to be found very near the surface, and justifies the term *surface* tension.

Fig. 27.

Fig. 28.

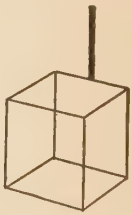


Fig. 29.

In the quantitative sense the *surface tension* of a liquid is equal to *one-half* the contractive force exerted by a double-surfaced film of the liquid 1 cm. wide, the width of the film being measured at right angles with the direction in which the force is measured. Measurement of surface tension is undertaken in Exercise 9.

(3) Take a wire frame of the shape shown in Fig. 28, the ring being about 4 cm. in diameter. Dip this ring into the soap solution, and then blow gently against the film stretched across it.

(4) Use two rings like Fig. 28, dip both into the solution, and then, after putting them flat together, draw them two or three cm. apart, observing the shape of the film formed by this operation. Touch the central diaphragm of this film with the heated end of a wire.

(5) Take a cubical frame of wire, like Fig. 29. Dip the whole of the cube into the soap solution, withdraw it, and observe the behavior of the film upon it.

Touch various parts of this film with the hot wire.

41. Surface Action on Larger Bodies of Liquid.—However great the body of a liquid may be, we must think of its surface as tending to contract. The exposed surface of the water in a large vessel is in a condition somewhat like that of a film of india-rubber stretched across the vessel. We see no evidence of this fact under common conditions, but if we lessen the tension at any one spot the result is very striking, the adjacent parts of the surface film drawing quickly away from the affected spot.

(1) Place a small cork stopper upon the surface of water in a large vessel, and then with a small tube let fall upon the water near the cork a drop or two of alcohol.

(2) Drop some alcohol upon a thin layer of water on a flat plate.

42. Oil Films on Water.—The experiment just described, with the cork stopper on water, may be repeated with success many times with the same body of water, if only a small quantity of alcohol is used each time. If a drop of oil be used instead of a drop of alcohol, the effect is about the same, *the first time,* provided the water and the cork are at first free of oil or grease. A second drop of oil produces little or no effect.

The fact is that the alcohol quickly mixes with the main body of the water, and so produces very little permanent effect upon the surface. But the single drop of oil covers the whole exposed surface of the water in an instant with a very thin layer, and the second drop makes little or no change in the surface condition. The film of oil upon the water may be detected by the iridescence, play of rainbow colors, which it shows. The visible formation of this film from the first drop is very interesting.

43. Use of Oil in Stilling Waves.—Not long ago people who thought themselves well informed used to ridicule the notion that a gallon or two of oil poured upon the surface of a stormy sea could serve as an efficient protection

to ships in distress; but it is now admitted that under certain conditions this can be. A ship driving before a gale, and in danger of being overwhelmed by the pursuing waves, trails out astern canvas bags filled with oil, which gradually finds its way into the water and, spreading over the surface, visibly abates the violence of the ocean. The theory is that the film of oil, being more viscid than the water surface, is less easily shaken up into the microscopic ripples which give the wind a hold upon the surface. *Practically* there is less friction of wind upon an oil surface than upon a water surface.

44. "Capillary" Action.—It is well known that, if one end of a small open tube is dipped into water or any one of many other liquids, the liquid rises in the narrow bore of the tube somewhat above the general level of its surface outside the tube. See Fig. 30.

FIG. 30.—Capillary elevation.

If the tube is very slender indeed the bore may look like a hair, and this fact suggests the name *capillary* tube, which is commonly used for any very narrow tube. Accordingly, the elevation of liquids in such tubes is called *capillary* action. It is explained as follows: The wall of the tube attracts strongly the adjacent layer of water, and so tends to spread this layer out over a larger part of the wall surface, in opposition to the surface-tension of the liquid itself, which resists such an extension. In most cases the spreading tendency of the wall's attraction prevails, and the liquid close to the wall creeps up a little, both inside and outside the tube, thus taking a concave shape.

Inside the tube the curvature reaches completely across, and the top of the liquid is brought into the shape of a

shallow cup. If we now think of this surface as held and supported at its edge by the glass, and remember that it is all the time tending to contract, we shall see that it tends to lift the liquid column beneath it, either directly by cohesion (§ 38), or indirectly by relieving the column from some portion of the atmospheric pressure upon its top, somewhat as the piston of a lifting-pump (§ 31) works. The indirect action is the ordinary one; but the direct action may occur, if the experiment is tried in a vacuum.

When mercury, instead of water, is used in a glass tube, the attraction of the glass for the mercury, although it is greater than the attraction of glass for water, does not succeed in *spreading out* the adjacent layer of mercury so as to make it cover more of the glass surface. On the contrary, the surface tension of mercury, which is very great, prevails, and the edges are drawn down, as in Fig. 31. The contraction of the *convex* surface thus formed within the tube forces the liquid column down. That is, mercury stands lower in the tube than in the vessel outside the tube.

Fig. 31.—Capillary depression.

So-called capillary action is perceptible in tubes of considerable size, as we have seen in a number of laboratory experiments. It occurs also in narrow spaces which are not tubes; in the meshes of a lamp-wick, for instance, thus raising the oil to the flame, or in crevices between solid bodies.

Water and oil, for example, tend to work into such crevices, while mercury often successfully resists the extension of its own surface which such a movement would cause.

Experiment has shown that water may work its way by

capillary action into the air spaces of a fine powder, zinc oxide, until the imprisoned air beyond it has a pressure of five atmospheres above the external pressure.

(1) Note the difference in level of the two water surfaces in the apparatus of Fig. 32, in which the connecting tube is of rubber. Then raise the cup end and note the difference of level of the two surfaces just before the water begins to run from the small opening.

Fig. 32.

Try a like experiment with mercury instead of water.

(2) Dip one face of a cube of sugar into water, and note how quickly the whole lump becomes wet through.

(3) Place two rectangular pieces of glass vertical in water, and let them meet at a very acute angle. Note the shape of the water surface near the angle.

(4) Push a small piece of paper down close against the flat bottom of a glass vessel containing mercury, then leave it to itself. Raise the vessel and look at it from below.

(5) Place a smooth disk of brass, 5 or 6 cm. in diameter, on a smooth, dry, brass plate (Fig. 33) and with a spring balance find the force required to move the disk slowly and steadily along the surface of the plate.

Put four or five very small drops of water between the disk and the plate and repeat the pulling experiment.

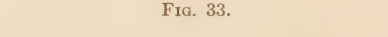


Fig. 33.

Put enough water on the plate so that it will wet the whole under face of the disk and show around its edge and repeat the pulling experiment.

In which case is the required pull the greatest? In which case the least? Can you suggest an explanation? In which of the three conditions do you find it hardest to lift the disk from the plate?

Apparent Attractions and Repulsions of Small Floating Bodies.—In Fig. 34 let M and N be two floating bodies, ww the general liquid-level, bb the edge-level to which the

liquid rises on the outer sides of the bodies, *aa* the edge-level to which it rises between them. Above the level *aa* there is equal pressure, the atmospheric pressure, on both sides of each body; below the level *bb* there is equal pressure, from the water, on both sides of each body; between the levels *aa* and *bb* there is full atmospheric pressure on the outer side of each body and a less pressure, of the raised column of water, on the inner side of each; so that the two bodies are driven toward each other by the excess of the outer-side pressure over the inner-side pressure between the levels *aa* and *bb*.

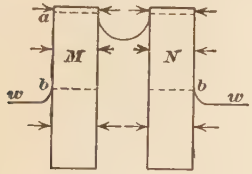


FIG. 34.—Apparent attraction: first case.

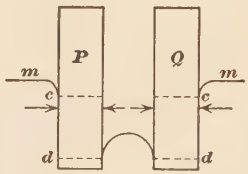


FIG. 35.—Apparent attraction: second case.

In Fig. 35 we have, let us say, two pieces of iron, *P* and *Q*, floating in mercury, which does not wet them and stands lower between them than on their outer faces. Above the level *cc*, and below the level *dd*, the opposing pressures on the inner and outer face of each body will balance each other. Between the levels *cc* and *dd* the pressure is greater on the outer side than on the inner side of each body, and accordingly the two approach each other.

In Fig. 36 we have one body, *M*, which is wetted by the liquid and another, *P*, which is not wetted by it. They may be blocks of wood and wax, respectively, in water. Between the levels *a* and *b* the inner-side pressure on *M* is greater than the outer-side pressure, and so *M* goes to the left. Between the levels *c* and *d* the inner-side pressure

on P is greater than the outer-side pressure, and so P goes to the right. M and P appear to repel each other.

It is very easy to show the case of Fig. 34 by means of two corks floating in water. It is easy to show the case of Fig. 35 by floating a clean piece of iron in clean mercury and bringing near it in the mercury the end of a rod of glass or wood.

It is not so easy to get the conditions of Fig. 36. But if one undertakes to make a floating cork, C in Fig. 37, fall over the edge E, of a vessel, the water-surface at E becomes convex before it overflows, the liquid edge falls lower on the nearer side of the cork than it stands on the farther side, and the cork floats away as if repelled by the side of the vessel.

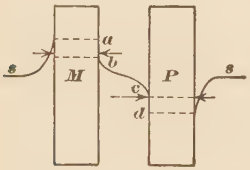


Fig. 36.—Apparent repulsion: discussion.

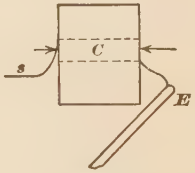


Fig. 37 Apparent repulsion: experiment.

45. Capillary Action in Soils.—On the tops of great sand hills near the ocean, where the surface looks as dry and incapable of vegetation as powdered glass, one sees occasional tufts and patches of grass, and by digging a few inches will find evidence of moisture in the darker color of the sand grains. This is the remnant of rain water which has fallen upon the hills and has for the most part found its way to greater depths, perhaps out into the ocean. By digging far enough we might find the sand actually wet, the spaces between the grains being filled with water, and evidence of capillary action, the water probably standing somewhat higher in the crevices between the sand grains than in the ocean at the base of the hill.

The clinging of water to the mere surface of the particles,

enough to darken their color, which is seen above this capillary level, is not strictly capillary action, as it does not depend on the narrowness of the spaces between the grains, and it might be found at any height, however great, to which the sand could be piled above the ocean level. The dryness of the particles at or very near the upper surface is due to evaporation into the air. Water is doubtless creeping back to these dry surface grains, by way of points of contact with the damp grains a few inches below, or coming back as vapor, but not rapidly enough to keep them visibly damp in spite of the evaporation.

Rain water escapes from the soil, and is therefore lost for immediate use in promoting the growth of plants, in three ways: first, by simply running off without really penetrating the soil; second, by draining off through the lower strata; third, by evaporation from the surface. The methods used in so called "dry-farming," that is, successful farming without artificial irrigation in regions where the annual amount of rain is small, undertake to lessen all of these wastes, first, by breaking up the surface of the soil to a depth of some inches, so that the rain can readily soak in but cannot readily come back to be evaporated, second, by working the soil below these few inches into a compact condition, so that it will not readily let the water run down through, but will hold it by capillary action within reach of the rootlets of plants.

46. Solution; Absorption; Adsorption.—When a liquid takes into itself another liquid, a solid, or a gas, so that the thing taken in, the *solute*, disappears in the original liquid, the *solvent*, giving a clear, transparent, liquid result, we call the action *solution*, and we call the resulting liquid *a solution*.

The name *absorption* is also used for this action, when the solute is a gas.

The taking in of a liquid or a gas by a solid is also called absorption.

Quickness and amount of absorption depend very much on the nature of the two kinds of matter engaged in it, and it is probable that ready absorption of one substance by another depends on some attraction between the minute particles, or molecules, of the two.

The name *ad*sorption is sometimes used, in cases where it is supposed that a solid attracts a gas to its external surface merely or to the surface of its *pores*, without taking it into its substance. The distinction between absorption and adsorption is not perfectly sharp, and the general term absorption will here be used to cover adsorption.

When we begin to speak of special attractions between the molecules, or smallest particles, of different kinds of matter, we are near, if not strictly upon, the ground of chemistry; but there is a region of science which physics and chemistry hold more or less in common, and in this field lie the phenomena which we now have to look at. We shall not here study those in which chemical actions, strictly so-called, occur, actions which involve the making and unmaking of molecules.

47. Solution of Solids in Liquids.—We are all familiar with cases of this kind; for example, the solution of sugar or of salt in water. The most notable general facts of such cases are the following:

Solution is limited: A given amount of a given liquid will dissolve, at a given temperature,[1] a given amount, not more, of a given solid. When a liquid has dissolved all it can of a solid at the existing temperature, the solution is said to be *saturated*. Stirring liquid and solid together

[1] To be exact we must say *and pressure;* but the effect of pressure is here very slight.

hastens the process of solution; but it does not increase the amount which can at last be dissolved.

Solution is selective: Thus 100 gm. of water at 10° C. will dissolve about 36 gm. of sodium chloride, common salt, and about 20 gm. of potassium nitrate, saltpeter.

Water probably dissolves more kinds of material than any other liquid; but some things dissolve more freely in other liquids; resinous substances and oils, for example, in alcohol; sulphur and india-rubber, in carbon disulphide.

Solution, as a rule, is increased by rise of temperature: For example, sulphate of copper dissolves much more readily in hot water than in cold water, and, if a hot saturated solution of this substance is allowed to cool, a considerable part of the solute comes out of solution in the cooling, appearing in crystals which grow from the bottom of the vessel and adhere very strongly to it.

Common salt is not much more soluble in hot water than in cold water, about 40 gm. at 100° C. and about 36 gm. at 0° C., dissolving in 100 gm. of the liquid.

There is usually a slight shrinkage of total volume when a solid is dissolved in a liquid.

48. Solution of Liquids in Liquids.—Solubility, or *miscibility*, of liquids in liquids is limited in some cases and unlimited in others, a fact which may be illustrated by experiments like the following:

(1) Into a test-tube nearly full of water put a few drops of ether and shake the whole thoroughly. Is there any visible separation of the two liquids after the shaking?

Put a few drops of water into a test-tube nearly full of ether, and proceed as before.

Mix equal quantities of water and ether in a test-tube and proceed as before.

(2) Mix equal quantities of water and alcohol in a test-tube and proceed as in (1).

Familiar cases of unlimited solubility are the following: Sulphuric acid and water, acetic acid and water, glycerine and water.

Evidently, from the experiments described above, solubility of liquids in liquids is *selective.*

As a rule, rise of temperature increases the solubility of liquids in liquids. In some cases pairs of liquids which have limited solubility for each other below a certain temperature will mix in all proportions above that temperature.

Usually solution lessens total volume.

Pour water into a test-tube till it is nearly half full. Put a thin slice of cork, fitting well enough in the test-tube to keep its position therein, on the top of this water. Pour alcohol gently in above the cork till the tube is nearly filled. Mark with a rubber band the position of the top of the alcohol column. With a thin wire upset the cork partition and then, closing the tube carefully with the thumb, shake the whole thoroughly. Compare the final position of the top of the column with the first position.

49. Solution of Gases in Liquids.—Selective solubility holds here, as in the cases of solution previously discussed. For example, water dissolves ammonia gas much more freely than it dissolves carbon dioxide ("carbonic acid"), and it dissolves carbon dioxide much more freely than it does oxygen or hydrogen.

In other respects we come on quite new laws here:

1. *The Pressure Law.*—The amount of any particular gas which a given liquid can dissolve, or absorb, at a given temperature is proportional to the pressure with which the gas is held against the surface of the liquid.

This law, called the law of Henry, is not exactly true, especially in the case of the very soluble gases; but it is near enough to the truth to be very useful. It implies that, if we bring a gas into contact with a liquid and keep the pressure of the gas constant, the liquid will absorb the

same *bulk* of the gas, whether the pressure of the gas is great or small. The liquid will absorb a certain fraction of its own volume or a certain number of times its own volume, whatever the external pressure of the gas. Accordingly, the *solubility* of a given gas in a given liquid is given as so many "volumes," so many times the volume of the liquid. A table of solubilities in water of some familiar gases is here given. The numbers are not strictly accurate:

T	Oxygen	Nitrogen	Hydrogen	Carbon dioxide	Ammonia
0° C.	0.049	0.024	0.021	1.71	1299[1]
20°	0.033	0.017	0.019	0.94	760
40°	0.026	0.014	0.018	0.61	.....
60°	0.024	0.012	0.018	0.41	.....
80°	0.023	0.013	0.020	.....	.....
100°	0.023	0.014	0.022	.....	.....

(1) Half fill a small bottle with water and fit it with a rubber stopper through which a glass tube extends externally. Exhaust the air from the space over the water and look for the freeing of small bubbles of previously dissolved air.

(2) Put ammonia water in place of the water in (1), and repeat the experiment, looking now for the appearance of bubbles of ammonia gas. (Only a cheap air-pump should be used with ammonia.)

The escape of carbon dioxide gas from "soda-water" when it is drawn from the "fountain," or from beer when the cork of a bottle is loosened, is due to the lessening of external pressure.

2. *The Temperature Law.*—As a rule, increase of temperature reduces the solubility of gases in liquids (see the table above) and accordingly the heating of a liquid tends to drive out previously dissolved gases.

[1] Water may increase greatly in bulk by absorption of ammonia gas.

(3) Heat ammonia water (*A*, Fig. 38), in a flask and let the released ammonia gas enter the bottle *B*, driving out the air through the thistle-tube *T*. When strong fumes are coming from *T*, stop the heating, close the pinch-cock *P*, pour a little water into *T*, then close the stop-cock *S*. Note the behavior of the mercury in *G*, occasionally shaking *B* to hasten absorption of ammonia gas by the water.

Liquids absorb certain gases or vapors which are commonly detected only by the sense of smell or of taste. Thus water or milk left in a refrigerator with certain vegetables, especially if the latter are warm, is apt to acquire an unpleasant flavor from them. Beverages should not be left uncovered in a sick room.

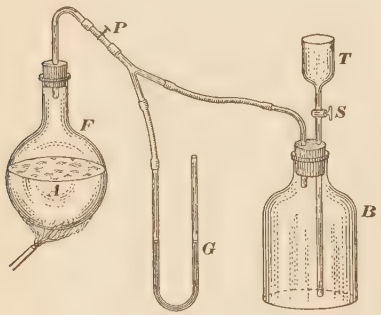


Fig. 38.—Absorption: ammonia vapor and water.

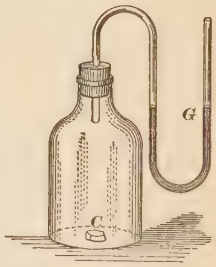


Fig. 39.—Absorption: ammonia vapor and charcoal.

50. Absorption of Liquids or Gases by Solids.—It is well known that air and water may enter the pores of wood, and one would expect a very porous body, a piece of charcoal for example, to absorb a relatively large amount of a fluid. The following experiment, however, will show that there must be some attraction between charcoal and ammonia; for the solid will be found to absorb more than its own bulk of the gas. The use of charcoal for the absorption of foul-smelling, noxious, vapors or gases is common.

Fill the bottle of Fig. 39 with ammonia vapor, produced as in Fig. 39, drop into it a lump of charcoal, put in the rubber stopper, and then watch the behavior of the connected mercury pressure-gauge *G*. Estimate roughly the number of volumes of vapor absorbed by the charcoal.

The power of solids to absorb gases is by no means confined to those which have visible pores. There are curious cases of selective absorption in which certain gases enter and even pass through certain metals. Thus hydrogen passes through white-hot platinum, and carbon monoxide, a product of the imperfect burning of coal, passes through hot cast-iron. Perhaps the most remarkable example of this kind is found in the power of the metal palladium to absorb, when cold, under favorable conditions, several hundred times its own bulk of hydrogen.

51. Gases in Gases.—Gases which have no chemical action on each other mix in all proportions and are sometimes said to form solutions with each other, but the word solution is rather misleading here. In fact, when two or more gases which are chemically indifferent, inactive, to each other are mixed, each one, if time is allowed without disturbance, will distribute itself through the mixture space as if it were alone there.

52. Emulsions.—Turbid or opaque fluid mixtures are not strictly solutions but *emulsions*, that is, liquids in which are suspended small particles of other liquids or of solids.

For example, milk, common ink, paints mixed ready for use, are emulsions.

If the suspended particles are very small indeed and not very different in density from the liquid in which they lie, or if the liquid is a very thick and sticky one, the particles may be a long time in sinking to the bottom, as the solids finally do in paint mixtures, or in rising to the top, as the cream particles finally do in milk. Indeed, if the particles

are sufficiently small, they in time distribute themselves through the liquid as the particles of one gas distribute themselves through another gas (see §51) in spite of difference in density.[1]

53. Osmosis.—The passage of liquids through membranes which have no perforations, in the ordinary sense of the word, is called osmosis.[2] In many cases a liquid thus passing carries with it the substances which are dissolved in it. In some cases, however, the solvent of a solution, water for example, will go through a membrane which will not allow the solute, sugar for example, to pass. If a drop of some strong solution enclosed in such a membrane is placed in clear water or in a weaker solution, water will find its way into the strong solution from the weak one and the drop will grow in bulk, stretching the containing sac.

Place a drop of saturated solution of ferrocyanide of potassium in a weak solution of sulphate of copper, attaching the drop to some floating solid or to the wall of the containing vessel, so that it will not sink. Instantly upon the contact of the two liquids a sac of ferrocyanide of copper is formed around the drop, which very soon begins to swell visibly.

A membrane which permits a solvent to pass while arresting the solute is called a *semipermeable* membrane. By forming such a membrane with much care on the inner surface of a porous earthen cup containing a 6 per cent.[3] solution of sugar and then placing the cup in clear water, a German physiologist, Pfeffer, found that water continued to enter the cup by osmosis until the pressure within it was about four atmospheres greater than the pressure on the water outside the cup. The excess of pressure produced in this way is called *osmotic pressure.*

[1] See Perrin's *Brownian Movement and Molecular Reality* (London).
[2] Greek *ὠσμός, impulse, pushing.*
[3] That is, 6 gm. of sugar in 94 gm. of water.

54. Physiological Applications of Solution, Osmosis, etc.— The solids of our foods must, as a rule, be dissolved by the various liquids of our bodies before they can be absorbed by the walls of our mouths, stomachs and intestines. Some parts are conveyed in the form of an emulsion to the places where they can be finally acted on and made ready for absorption into the blood and tissues of the body.

Oxygen passes through the lung membranes into solution in the blood, while carbon dioxide passes in the opposite direction.

Plants can take in their solid food from the soil only when this food is dissolved in water. This passes readily through most plant tissues. The skin of fruits, however, apple skin for example, is not easily penetrated by water, a fact which prevents the fruit from speedy drying up.

One-sided osmosis, water passing through a partition to dilute a solution on the other side, and thus producing a high osmotic pressure there, plays a great part in the life processes of plants. The swelling and bursting of seeds when placed in water is due to this action, and so, probably, is the very great pressure which is exerted by the sap in the roots and limbs of trees in the spring. In one case[1] a pressure equal to that of a column of water nearly 86 ft. tall was required to prevent the "bleeding" of sap from the severed end of a birch-tree root, which had been cut free from the trunk, though left planted in the soil.

[1] Goodale's *Vegetable Physiology*, Vol. 2, p. 267.

CHAPTER V.

EQUILIBRIUM OF FORCES: THE LEVER.

55. Force and Equilibrium: Newton's First Law of Motion.—The word *force*, as commonly used in physics, means a *push* or a *pull*. In the phrases "forces of nature," "natural forces," etc., the word has a different and often a more vague meaning.

The effect of a single force applied to any body is to set the body in motion, or to change in some way the motion that the body already has. But two or more forces, if applied to the body at once, may neutralize each other in such a way that the body will act as if no force were applied to it, provided we may neglect any change of shape or size that the body undergoes under the action of the forces. Forces so neutralizing each other are said to be in *equilibrium* with each other.

For a very long time it was supposed that a body under the influence of no force or a self-neutralizing set of forces must come to rest and remain at rest. It was supposed that motion could be maintained only by a continual application of force. It was supposed, for instance, that the planets must be *driven* or *carried* around the sun. All this was an error.

It is true that the moving bodies with which we are most familiar on the earth do come to rest. The error lay in supposing that no force is acting upon them while they are coming to rest. Experiment shows that, when better and better means are taken for lessening friction, resistance of air, etc., moving bodies show less and less tendency to

come to rest, and as a result of all experience and reasoning it is now believed that no body ever comes to rest except because of some *resistance*, some application of force which stops it. In fact, *the natural behavior of any body, the behavior of a body not acted upon by forces outside itself, is to remain at rest if at rest, to move with unchanging motion if in motion.* This is the substance of Newton's *First Law of Motion.*

The subject of setting bodies in motion or changing their motion will be taken up in Chapter XI. We shall in the present chapter and the next be dealing with cases of *equilibrium;* that is, *cases in which bodies are at rest or, as nearly as may be, in uniform, unchanging motion.* We shall in studying these cases have in view several objects, one of which is to determine what relations must exist in a set of forces in order that they may balance each other in their action upon the body to which they are applied. These relations are called the *conditions necessary for equilibrium,* or simply the *conditions of equilibrium.*

56. Illustrations of Equilibrium.—Floating bodies owe their stability to the joint action of gravity and the upward thrust or buoyancy of the liquid. A picture hung against the wall is supported in position by the simultaneous action of gravity, the pull of the suspending cord, and the push of the wall against the lower part of the picture-frame. A car may run with uniform velocity down an incline, the force of gravity which tends to hasten its motion being offset by friction.

57. Description and Representation of Forces.—In order to describe a force completely or to calculate what effect will be produced by it, we must know the *amount,* the *direction,* and the *position,* or *line of action,* of the force.

The directions of forces with reference to each other may be represented by lines, and the lengths of the lines

may represent the relative magnitudes of the forces. One end of a line may represent the point of application of the force, and an arrow-head upon the line may indicate whether the force is directed this way or that along it.

THE LEVER.

58. Definition and Illustration.—Civilized men do most of their work with tools or machines. Many tools and many parts of machines consist of a piece of iron or wood or other material movable to a certain extent upon a support called a pivot, or axis, or fulcrum, by means of which a force applied in one direction at a certain spot may produce another force different in direction or in magnitude, or in both, at another spot. *Such a tool or part of a machine is called a lever.*

One of the most familiar examples of the lever is a crowbar. A hammer, as used to *draw out* a nail from a board, is another example. Each half of a pair of scissors is a lever. We shall study first, in Exercise 10, a very simple form of the lever, to find out what relations hold between the forces exerted at different points.

Questions (after Exercise 10).

(1) If a mass of 6 oz. is suspended from a point 4 cm. to the left of the centre of the lever in Exercise 10, how great a load placed at a distance of 10 cm. to the right of the centre will make equilibrium?

(2) A mass of 8 oz. is suspended from a point 5 cm. from the centre of the lever and is balanced by a mass of 10 oz. How far from the centre is the latter placed?

(3) Two masses, 4 oz. and 12 oz. respectively, are to be suspended from a lever. Describe three possible arrangements of the masses, any one of which will cause them to balance.

(4) A boy pushing down at one end of a lever 6 ft. long pries up a stone weighing 100 lbs. at the other end. The fulcrum is 2 ft. from the stone. The weight of the lever itself is neglected. How great is the force exerted by the boy?

(5) A plank, extending east and west, is balanced in a horizontal position on a fulcrum. A boy weighing 100 lbs. is placed on the plank 6 ft. west of the fulcrum; another weighing 60 lbs. is placed 6 ft. east from the fulcrum. Where shall a third, weighing 80 lbs., be placed in order to balance the whole?

(6) If the third boy were to be placed 4 ft. from the fulcrum toward the east, what should his weight be?

(7) How great is the total force, in addition to the weight of the plank itself, exerted upward by the fulcrum against the plank in problem 5? In Problem 6?

59. Point of Application and Line of Application of a Force.—In the experiments of Exercise 10 the weights have been suspended from the top of the lever on a level with that part of the pivot upon which the lever rests. We have now to find out what would be the effect of hanging one or more of the weights from points higher or lower than the point of support of the lever. For this purpose we shall use the piece of apparatus shown in Fig. 40, in which the straight lever thus far used is replaced by a circle of wood about 20 cm. in diameter, supported by a screw passing horizontally through the centre. Such a circle, or disk, of wood comes under the general definition of a lever.

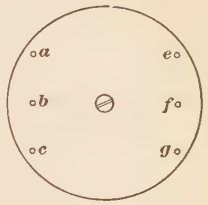


FIG. 40.—Lever in disk form.

Hang at b and f such weights as will balance each other, leaving the disk in equilibrium, and then turn the disk so as to make b somewhat higher and f somewhat lower than the pivot. Will the equilibrium continue after this change?

Move one of the weights from b to a or c, leaving the other at f, and make the line bf horizontal. Will the weights now balance each other? If they do, turn the disk a little in either direction, so that the line bf will no longer be horizontal, and find whether the whole will now be in equilibrium.

60. Lever-Arm of a Force, Moment of a Force, etc.—We see from such experiments as those of § 59 that the

point of application of a force can be changed to any new position lying in the line of action of the force without changing the power of the force to produce rotation of the body on which it acts.

Accordingly we see that the distances which are really important in the application of forces to a lever are not in all cases the distances between the points of application of these forces, which points, like *a*, *b*, *c*, in Fig. 41, may not lie on one straight line, but are the distances, the *shortest* distances, between the *lines of action* of the forces.

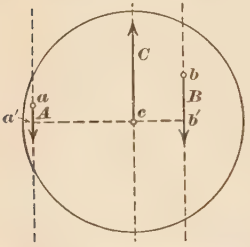


FIG. 41.—Illustrating the definition of *lever-arm*, etc.

Lever-Arm.—The shortest distance from the line of action of a force to the point regarded as the pivot[1] of the body to which the force is applied is called *the lever arm of the force* with respect to the given pivot.

Thus in Fig. 41 the lever arm of the force A with respect to the pivot c is the distance $a'c$. The lever arm of B with respect to c is the distance $b'c$.

Moment of a Force.—The product of any force by the lever arm of that force with respect to any given pivot is called the *moment of the force*, or turning-power of the force, with respect to this pivot.

Thus in Fig. 41 the moment of A with respect to the pivot c is $A \times a'c$; the moment of B with respect to c is $B \times b'c$. Evidently the moment of C with respect to c is zero. If a is taken as the pivot, the moment of A with respect to this pivot is 0, the moment of C is $C \times ca'$, the moment of B is $B \times b'a'$. If b is the pivot, the moment of

[1] The word *pivot* is here used as equivalent to *point at which an axis, around which the body to which the force is applied is supposed movable, passes at right angles through a plane containing the line of the force.*

B is 0, the moment of C is $C \times cb'$, the moment of A is $A \times a'b'$.

We shall call the moment of a force *positive*, or give it the plus[1] sign, when it tends to produce rotation clockwise[2] around the chosen pivot, and we shall call it negative, or give it the minus[1] sign, when it tends to produce rotation counter clockwise.

61. Laws of Equilibrium of Three Parallel Forces in One Plane.—We have studied a number of cases in which three parallel forces in one plane balance, or make equilibrium with, each other. Let us now try to state the conditions necessary for equilibrium in all such cases, so that when a diagram showing the forces and distances in any particular case is put before us we can readily tell whether this is or is not a case of equilibrium.

To be in equilibrium, a set of forces must, as a whole, have (a) no tendency to *slide* the body to which they are applied, (b) no tendency to *rotate* the body around any pivot whatever.

The first condition was indicated in Exercise 10 by the fact that the one upward pull appeared to be equal in all cases to the sum of the downward pulls. If we do not confine ourselves to upward and downward directions, but merely agree to call a force in one direction positive and a force in the opposite direction negative, we can state our first requirement for equilibrium thus:

(a) *The algebraic sum of the forces must be zero.*

The second condition, which we began to learn in Exercise 10, may with the help of the definitions given in §60 be stated as follows:

(b) *The algebraic sum of the moments of the forces must be zero with respect to any and every pivot.*

[1] Many writers use these signs in the opposite way.

[2] That is, in the direction of rotation of clock hands.

It may seem at first as if it would be difficult to make sure of this second condition. The fact is, however, that, if a set of forces satisfies condition (a) and has zero moment with respect to some one pivot such as we have been considering (see foot note p. 66), the moment of this set of forces will be zero with respect to any other point whatever taken as a pivot.

For example, if we take the case shown in Fig. 42 and neglect the weight of the body to which the forces are applied, as we shall do for the present, we see that we have a set of forces in equilibrium, *if Q is the fulcrum.* But evidently, if the body is at rest, we could put a peg through it anywhere into any fixed body beyond without disturbing the equilibrium. The mere putting in of a peg would not tend to make the body turn around that peg. So in such a case we may think of the fulcrum, or pivot, as anywhere we please to have it, if we have only equilibrium to consider.

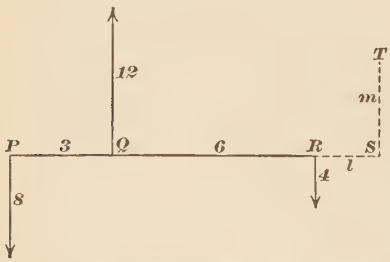


FIG. 42.—Illustrating conditions of equilibrium.

But shall we have

$$(\textit{moment of } 8) + (\textit{moment of } 12) + (\textit{moment of } 4) = 0,$$

with respect to every point we may choose for the pivot? Yes. Let us see how this can be proved.

For Fig. 42 we have, as the moments with respect to P,

$$\left.\begin{aligned} 8\times 0 &= 0 \\ -(12\times 3) &= -36 \\ +(4\times 9) &= +36 \end{aligned}\right\} = 0.$$

With respect to Q,

$$\left.\begin{aligned} -(8\times 3) &= -24 \\ 12\times 0 &= \quad 0 \\ +(4+6) &= +24 \end{aligned}\right\} = 0.$$

With respect to R,

$$\left.\begin{aligned} -(\ 8\times 9) &= -72 \\ +(12\times 6) &= +72 \\ 4\times 0 &= \quad 0 \end{aligned}\right\} = 0.$$

Let us next imagine a pivot at a point lying in the extension of PR, at a point S at a distance l from R. The moments with respect to this pivot are:

$$\left.\begin{aligned} -[\ 8\times(9+l)] &= -72-\ 8l \\ +[12\times(6+l)] &= +72+12l \\ -[\ 4\times l] \qquad &= \qquad -\ 4l \end{aligned}\right\} = 0.$$

Now as l can have any magnitude, and can be either negative or positive, without destroying this equation, it is shown that, with the given system of forces, the algebraic sum of the moments is zero for any pivot on the line PR or its extension.

But imagine a pivot at a point T at a distance m from S, on a line drawn from S parallel to the forces. The moments with respect to this pivot are, as a little consideration of the definition of moments will show, precisely the same as the moments with respect to the pivot at S. Now as m, like l, may have any magnitude and be either positive or negative, we see that the proposition which we are studying is true in the case represented by Fig. 42. Further trial would prove it to be true for any set of three parallel forces in equilibrium. Accordingly we can put in place of the statements (a) and (b) the following single statement:

A set of three parallel forces lying in one plane is in equilib-

rium, if the algebraic sum of the forces is zero and if the algebraic sum of the moments, taken with respect to some pivot lying in the plane of the forces, is zero.

62. **Laws of Equilibrium of Any Number of Parallel Forces in One Plane.**—The statement given at the end of the preceding Article has been illustrated in our discussion by cases of three forces, but, if *three* be omitted, it holds for the equilibrium of any number of parallel forces in one plane. The Second Part of Exercise 10 gives a case of four such forces.

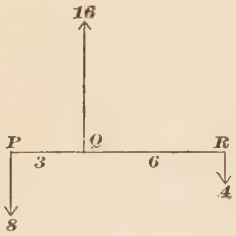


FIG. 43.—Illiustratng non-equilibrium.

63. **Cases of Non-equilibrium; Couples.**—The conditions given at the end of § 61 are not only true for every case of equilibrium of parallel forces in one plane; but one or both of them must be untrue for any case of non-equilibrium of such forces. They are the "necessary and sufficient" conditions of equilibrium of such forces.

For example, let us consider the set of forces shown in Fig. 43, which we know is *not* a case of equilibrium.

It meets the second requirement, for the algebraic sum of the moments with respect to Q is zero; but it does not meet the first one. It would not meet the second requirement if any point not on the line of the 16 force were taken for the pivot.

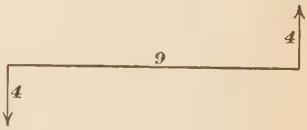


FIG. 44.—A "couple," with negative moment.

The cases shown in Figs. 44 and 45 meet the first requirement, but not the second. We should seek in vain for any pivot with respect to which the sum of the moments would here be zero.

Definition: The kind of combination shown in Fig. 44, two parallel equal forces in opposite directions, but not in the same line, is called a *couple*.

Moment of a Couple: Trial with respect to various pivots will show that the moment of the couple described in Fig. 44 is always *negative* and always equal to 36, that is, to the product of one of the forces by the *arm of the couple*, the distance between the lines of the forces.

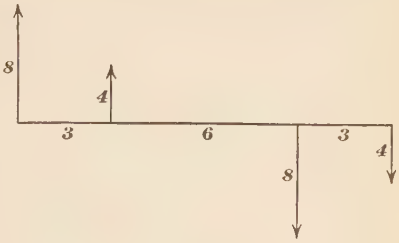


FIG. 45.—Two couples, each with positive moment.

In Fig. 45 we have two *positive* couples, one having the moment 8×9, the other the moment 4×9.

We shall do something more with couples later.

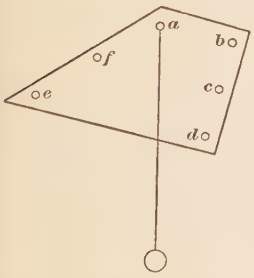


FIG. 46.—Illustrating centre of gravity.

64. Centre of Gravity; Weight of a Lever.—In the experiments upon the lever thus far, the lever itself, whether a bar or a disk, has *balanced*, when left to itself without load. We have, therefore, not had to consider the weight of the lever itself. But many levers are used in such a way that their own weight helps or hinders the operation to be performed with them. To understand such cases we must learn something about what is called the *centre of gravity* of a body.

If an irregularly shaped board (Fig. 46), is hung by the hole a on the horizontal rod r (Fig. 47), on which it can turn easily, and if a bullet is suspended by a thread from r, the thread will indicate a vertical

line from *a* down across the board. If this line is marked on the board with a pencil, and if then this operation is repeated with the board hung by the holes *b*, *c*, *d*, etc., in succession, all the pencil marks will cross at or very near one point on the face of the board.

If the board is placed horizontal with the point in question resting on some narrow flat support, the head of a lead pencil, for example, it will balance in this position.

Centre of Gravity.—We see that a certain point of the board or, rather, a certain line running from the located surface-point straight through the board, always hangs vertically beneath the support *r* or lies just over the supporting pencil top. We see that the board acts in these experiments as if the whole of its weight, the whole pull of gravity upon it, were concentrated at a point somewhere on this line, probably the point at the middle of the thickness of the board. This point, at which we may regard the whole weight of the board as centred, might be called the centre of weight or the centre of heaviness of the board, but it is commonly called the *centre of gravity.*

FIG. 47. Another aspect.

We now see how to take account of the weight of a lever, when we need to do so. All of its weight, wherever the fulcrum may be, may be regarded as concentrated at a certain point fixed in or in a certain relation to the body of the lever.[1] In the use of some levers, the crowbar for example, the weight of the bar itself may play an important part, when the fulcrum is a long way from the centre of gravity of the bar.

Centre of gravity will be taken up again, and the different kinds of *equilibrium*, stable, unstable, and neutral, will be considered in Chapter VI. An illustration of the use

[1] It is assumed that the lever is rigid enough not to change shape in use. Change of shape of a body usually changes the position of its centre of gravity.

of the term and of the idea it stands for is given in Exercise 11.

Problems.

(1) An oar, the centre of gravity of which is 3 ft. from the end of the handle, weighs 4 lbs. It rests in the rowlock at a point 2 ft. from the end of the handle. How great a force applied at the end of the handle will keep it balanced?

(2) A boy weighing 100 lbs. is seesawing alone on a plank 14 ft. long weighing 50 lbs. The boy's centre of gravity is 1 ft. from one end of the plank. How far from the same end of the plank is the fulcrum? (The centre of gravity of the plank is at its middle.)

(3) If the plank mentioned in the preceding problem is to balance on a fulcrum 8 ft. from one end, with a 100-lb. boy 1 ft. from this end and another boy 1 ft. from the other end, how much must the second boy weigh?

(4) How great is the upward pressure exerted by fulcrum in each the of these cases, (1), (2), and (3)?

65. Equilibrant, Resultant, etc.—A single force that will just balance, or make equilibrium with, two or more others is called their *equilibrant.*

In Fig. 42 the 12 is the equilibrant of the 8 and the 4;
" 8 " " " " " 12 " " 4;
" 4 " " " " " 12 " " 8.

A single force that can exactly replace two or more others, so as to produce the same effect upon the body acted on (if the body is rigid so that it does not bend), is called their *resultant.*

In Fig. 42 the forces 8 and 4 could be exactly replaced by a force 12 acting *downward* from the point *Q*, and therefore this force is their resultant, etc.

The resultant and equilibrant in any given case are equal in magnitude, but opposite in direction, so that the two would exactly balance each other. Often the easiest way to find the resultant is to find first the equilibrant.

Two or more forces which together can exactly replace a single force are called the *components* of that force. In Fig. 42 a force 8 upward from P and a force 4 upward from R might be the components of the 12.

Questions.

(1) (*a*) What would be the magnitude of the equilibrant of the three forces in Fig. 48? Its direction? Where would be its line of application?

(*b*) What is the magnitude, direction and line of application of the resultant of the three forces?

(2) What would be the magnitude, the direction and the line of application of the equilibrant in Fig. 49? Of the resultant?

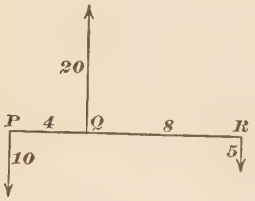


Fig. 48.

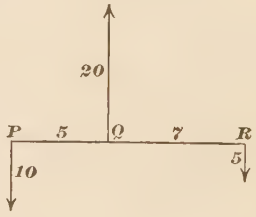


Fig. 49.

Fig. 50.—"Bent-lever": principle of claw-hammer.

66. Bent Levers.—This name is used for levers in which the forces applied are not parallel to each other. Thus in Fig. 50, which is in principle like a claw-hammer used in drawing out a nail, we have the load represented by a downward force, W, and the power by a horizontal force, P. The law of equilibrium, so far as it relates to the relative magnitudes of W and P, is merely, as in the case of the straight lever,

$$W \times (\textit{lever-arm of } W) = P \times (\textit{lever-arm of } P),$$

or

$$(\textit{moment of } W \textit{ with resp. to } F) + (\textit{moment of } P \textit{ with resp. to } F) = 0.$$

The force exerted at or *by* the fulcrum itself, F, in such a case is parallel neither to *W* nor to *P*, and we are not yet ready to deal with it fully. It may well be taken up in connection with the *Parallelogram of Forces* in Chapter VII.

67. Pulleys.—We have already learned to consider a disk pivoted at its centre as a kind of lever. When such a lever is worked by means of a cord or band lying upon its circumference, it is called a *pulley*.

It is hardly necessary for us to try any new experiments in order to see that, with the apparatus shown in Fig. 51, the force *P*, in any one of the positions indicated, will balance *W*, if the two are exactly equal.

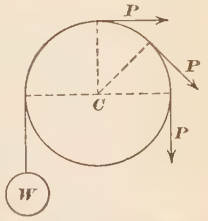


FIG. 51.—Circular lever, or pulley.

We see, then, that the object of a simple, single, pulley is not to enable us to lift a great weight by means of a small power, as we can do with some forms of the ordinary bar-lever. The advantage of the pulley is that it enables us to exert a pull in one direction by means of a pull in a different direction and to exert a longer pull, to lift a weight farther, for example, by one steady pull than we could lift it by means of a bar lever no longer than the diameter of the pulley. In fact, the distance through which we can lift a weight by means of a pulley depends merely on the length of the cord that runs over the pulley.

68. Windlass, Capstan, etc.—The *windlass* (see Fig. 52) is a familiar apparatus consisting of an elongated pulley, *d*, called the drum, or cylinder, turned by a power applied at the handle, *h*, and acting through the lever, or crank, *c*.

The crank and handle are sometimes called a *winch*. The same name is sometimes applied to the whole windlass.

If the power is applied at right angles with both *h* and *c*, and if the cord sustaining the weight is small, we have

very nearly, disregarding friction, which may be considerable,

$$P \times (\textit{length of } c) = W \times (\textit{radius of } d).$$

A *capstan* is the same in principle as a windlass, but has a vertical drum, so that the lever travels in a horizontal circle.

On shipboard capstans were formerly worked by means of a number of men walking about the drum and pushing against the levers, or *capstan-bars*, of which there may be several applied to one cylinder. This is an old fashioned way of raising anchor or, if there is much cable out, of

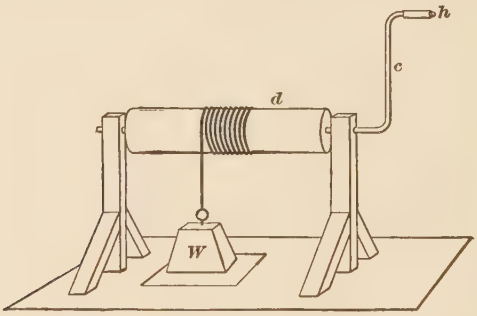


Fig. 52.—Windlass, or winch.

pulling the ship along to a position nearly over the anchor. Rafts of logs, great areas enclosed by a chain-boom, are, or used to be, moved along lake surfaces by the same device.

69. Movable Pulleys.—In the pulleys thus far mentioned the fulcrum has been fixed in its position with respect to the rest of the apparatus. But pulleys which move as a whole, up and down or back and forth, are used in various combinations, which make it possible to have P, the power applied, much smaller than W, the weight lifted or the resistance overcome.

Questions.

(1) In Fig. 53 the weight, W, is the sum of the weights of M, the suspended mass, and B, the movable pulley. What is the ratio of W to P[1] here?

(2) What is the ratio of $W(=B+M)$ to P in Fig. 54?

70. Another Rule for Relation of Power to Weight.—In each of the two cases just looked at there is a single, continuous, cord running through the system of levers, to one end of which cord the power, P, is applied. In any such case we can at once find the ratio of W to P by merely counting the numbers of lengths of this cord which lead up from, pull up on, the weight to be raised.

But in many cases, in Fig. 55 for example, the system and the calculation are not quite so simple, and, though it is always possible with any set of pulleys to find the ratio of W to P by means of the principles of the lever, as they have been given in the preceding pages, it is often more convenient to make use of a different form or statement of the lever law.

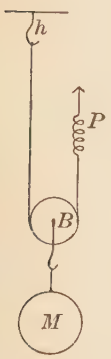


Fig. 53. Movable pulley.

This new form makes use

Fig. 54.—Combination of movable and fixed pulleys.

[1] In making an actual trial one must remember that friction is often large in pulleys, even when they are well oiled. When the load is being steadily raised the hand carrying the spring balance must lift harder than it would if there were no friction, but when the load is being steadily lowered, the hand, pulling just hard enough to prevent the load from *gaining* velocity, is assisted by the friction. The mean between the reading of the balance going up and the reading of the balance coming down will show, very nearly, what the pull required to sustain the load would be if there were no friction.

of the *relative distances moved through* by the power and the weight in any operation. If we study the various cases of lever, pulley, and combination of pulleys that have been described in the preceding pages, we shall see that whenever the weight is greater than the power the weight moves a less distance than the power does in any given *operation* of the apparatus; but whenever the weight is less than the power, friction being supposed zero, the weight moves a greater distance than the power does in the operation of the apparatus.

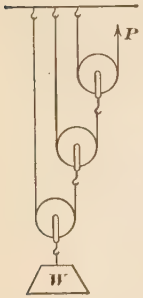


Fig. 55.

If we study the matter more closely, we shall find the following rule, or law, *suggested*, though we cannot say that it is *proved* by our preceding experiments:

$$P \times D_p = W \times D_w,$$

where P stands for the power applied;

W " " " weight lifted;
D_p " " " distance the power moves;
D_w " " " " " weight "

Applications of this Law.

(1) It is evident that this law can be readily applied to a case like that of Fig. 54. We can see at once that, if the pulley B were lifted 1 in. while P remained stationary, there would be *four* inches of loose string under B. To make the string taut again, P would have to rise 4 in. In actual use P and B rise at the same time, P moving four times as fast as B.

(2) Fig. 56 represents the rear wheel and the gear of a bicycle, with a cord running back over a pulley F, which does not move forward with the wheel.

If the weight of the rider, 150 lbs., rests entirely upon the pedal shown, in its present position, how great a weight, W, acting in opposition, as in the figure, will just neutralize the driving-power?

The correct answer could be found by means of a rather troublesome

calculation making use of the diameters of tire and sprocket wheels and the length of pedal radius; but it is not necessary to go through all this. If we make P descend a small measured distance, D_p, and find by measurement D_w, the distance the weight W is meanwhile lifted, which is equal to the distance the wheel moves forward, we have at once $W = 150\times (D_p \div D_w)$.

In a real case $(D_w \div D_p)$ might be 5, which would give 30 lbs. as the value of W.

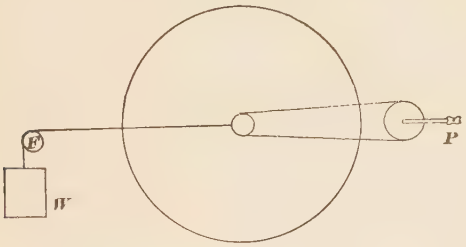


FIG. 56.—Leverage in a bicycle.

The higher the gear, that is, the greater the large sprocket-wheel compared with the small one, the greater will be the ratio $D_w \div D_p$, and the greater the distance traveled with one revolution of the pedal; but a high gear has a small pulling power and is therefore a disadvantage in climbing a steep grade.

71. Work; Principle of Equal Work.—The principle brought out and applied in the preceding Article is more conveniently stated and used if we have a name for such a product as $P\times D_p$ or $W\times D_w$.

Definition.—The product of a force by the distance the point of application of this force moves *in the direction of the force* is called the *work* done *by* the force. The product of a force by the distance its point of application moves in a direction *opposite* to that of the force is called the work done *against* the force.

The principle set forth in § 70 comes to this:

work done by the Power = work done upon the Weight.

This law holds strictly true only when there is no friction in the machinery between the power and the weight.

Weight is taken here in a general sense to include any resistance which the power has to overcome.

We shall find the application of this principle useful all through the field of mechanics; for example, in the hydraulic press (§ 25) and in the inclined plane (§ 88).

Among English-speaking people the most familiar unit for the measurement of work is the *foot-pound*, the work required to lift one pound one foot straight up. This unit is used, however, when there is no lifting. For example, if in drawing any body across a level table for a distance of 3 ft., we exert against friction a pull equal to the weight of 5 lbs., we do 15 ft. lbs. of work.

Another familiar unit of work is the *gram-centimeter*, the work required to lift one gram one centimeter straight up.

We can use any force-unit with any distance-unit to make a work-unit. Thus we may sometimes find it convenient for the moment to reckon work in *ounce-centimeters*.

Questions and Problems.

(1) A large pair of shears is used to cut a wire. One handle of the shears being held fixed, a power of 25 lbs. is applied to the other handle at a distance of 2 ft. from the pivot. The wire cut is placed 2 in. from the pivot. How great is the resistance offered by the wire; that is, how great a force applied just over the wire would drive the blade through it?

(2) A man lifts 10 lbs. of coal on a shovel. His left hand is at the end of the handle; his right hand is 18 in. distant from his left hand; the centre of gravity of the coal is 36 in. distant from the left hand. The shovel itself weighs 6 lbs., and its centre of gravity is 21 in. from the left hand.

(*a*) What is the direction and magnitude of the force exerted by the left hand? (Consider the right hand as the fulcrum.)

(*b*) What is the direction and magnitude of the force exerted by the right hand? (Answer this question as if the left hand were replaced by a weight.)

(3) A body weighing 160 lbs. is suspended from a pole resting on the shoulders of two men, A and B, of equal height. The point of suspension is 3 ft. from A's shoulder and 5 ft. from B's. How much of the weight does each man bear?

(4) Six men are working at a capstan, each exerting a force of 20 lbs., each at a distance of 6 ft. from the centre. The diameter of the coils in which the rope is being wound on the drum is 1 ft. How great is the strain on the rope, all friction being neglected?

(5) A man finds that by moving one point of a machine, consisting of levers and pulleys, forward 10 in. he can move another point of the machine 1 in. If a force of 5 lbs. is applied at the first point, how great a resistance applied at the second point will be required to neutralize it, if there is no friction in the machinery?

(6) Name tools or machines, not already mentioned in this Chapter in which levers or pulleys are used?

(7) To a rod extending east and west are applied two forces, one of 10 lbs., the other of 5 lbs., both due south, the points of application being 18 ft. apart. State the magnitude, direction, and point of application of their equilibrant; of their resultant.

(8) If the two forces applied to this rod were 6 lbs. due north and 15 lbs. due south, the points of application being 12 ft. apart, state what would be the magnitude, direction, and point of application of the equilibrant; of the resultant.

(9) Let two forces, one of 10 lbs. due north, the other of 10 lbs. due south, be applied to the rod, the points of application being 6 ft. apart. Find, if possible, the magnitude, direction, and point of application of the equilibrant; of the resultant.

(10) To the rod already mentioned let three forces be applied, one of 5 lbs. due north, one of 10 lbs. due south, one of 10 lbs. due north, the points of application being respectively 4, 8, and 12 ft. from the western end of the rod. State the magnitude, direction, and point of application of the equilibrant of these three forces; of their resultant.

(11) Let the forces be 6 lbs. north, 8 lbs. south, 10 lbs. north, and 15 lbs. south, the points of application being respectively 4, 8, 12, and 16 ft. distant from the western end of the rod. State the magnitude, direction, and point of application of their equilibrant; of their resultant.

(12) A force of 20 lbs. acts due north from a point 8 ft. from the western end of a rod. State the magnitudes, directions, and points of application of three pairs of forces, each one of which pairs would just neutralize this force.

(13) A board 1 ft. square is placed horizontal and is loaded at the northeast corner with 10 lbs., at the southeast corner with 10 lbs., at the southwest corner with 20 lbs. Draw a figure of the board and mark on it the position where a single supporting point must be placed and the load which this point must bear, the weight of the board being disregarded.

(14) Let a rower be pulling with a horizontal force of 60 lbs. at a point 2 ft. distant from his row-lock while the centre of the water resistance to the oar is 6 ft. from the row-lock, and suppose the foot-rest against which his feet are braced to be on a level with his hands, an unusual but possible position. Let the rower's body at this instant be neither gaining nor losing motion relative to the boat and suppose there is no friction of the seat.

In the system as thus described how great is the net force driving the boat forward? (Find the push of the oar against the row-lock and subtract from this the push of the feet against the foot-rest.)

CHAPTER VI.

GRAVITY AND THE CENTRE OF GRAVITY; STABILITY.

72. Direction in which Gravity Acts.—Frequent allusion has been made in the preceding pages to the force of gravity, and its familiar tendency to draw all objects toward the earth has been assumed to be known by the student. The direction of gravity may be found by means of a plumb-line (Fig. 57), and such a line will always be found to point nearly toward the centre of the earth. The line *OP*, or *OC*, passes through the centre of the plumb-bob, *P*, and shows the line of direction in which gravity acts upon the bob taken as a whole.

Fig. 57. Plumb line.

Since the earth is nearly spherical, no two plumb-lines on the same hemisphere could be precisely parallel, except in cases where local disturbances of the direction of gravity are caused by the neighborhood of mountains, etc. Plumb-lines far apart are in general very different in direction, as is shown in Fig. 58. On account of the magnitude of the earth, however, the lines of direction of gravity acting

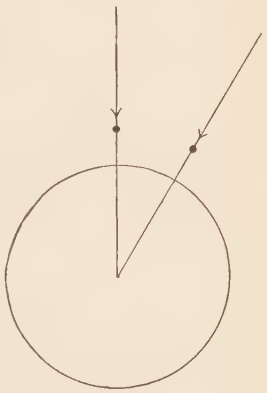


Fig. 58.—Various directions of plumb line.

upon the different parts of all ordinary objects are practically parallel. The action of gravity upon bodies, therefore, gives rise to a large and important class of problems dealing with parallel forces.

73. Centre of Gravity.—Since gravity must be supposed to act upon every particle of a body, it is clear that the total effect of gravity upon any object is the resultant of a very great number of parallel components. The point of application for this resultant, or the centre about which, as a point of support, a body will balance in any and all positions, is called the *centre of gravity.* (See § 64.)

It is, in many cases, an easy matter to find approximately by experiment the position of the centre of gravity. If a slender cylinder, such as a knitting needle, is, after a series of trials, made to balance across a knife-edge, it is plain that the centre of gravity must lie in the centre of that cross-section of the needle which is vertically above the knife-edge. If a flat piece of cardboard can be balanced on the point of a needle or a pin, the centre of gravity lies half-way between the two surfaces of the cardboard, and vertically above the point of the needle. In a body which is of uniform density throughout, the centre of gravity and centre of figure coincide; that is, the centre of gravity of a sphere is at its centre, that of a right cylinder is at the centre of its axis, that of a cube is at the intersection of its diagonals, and so on.

Such an object as a sheet of writing-paper, of cardboard, or of ordinary tinned sheet-iron may safely be assumed to have a nearly uniform thickness throughout. If the shape of such an object is a regular geometric figure, it is a simple matter to find its centre by geometry. If the shape is irregular, the centre of gravity may still be easily found by means of the plumb-line, used as in § 64.

The expressions "*centre of gravity of a line,*" "*centre of*

gravity of a surface," etc., when they are met with in books on mechanics, may be interpreted to mean centre of gravity of an extremely slender cylinder, of an extremely thin sheet, etc., of uniform thickness and density.

74. Stability.—When we speak of the stability of an object resting upon a supporting surface, we usually have in mind its freedom from danger of toppling over. In order that a body may *stand* at all, without being *fastened* to something, it is necessary that the pull of gravity, straight down through the centre of gravity, be directly opposed by an equal upward force acting along the same vertical line. This upward force may be exerted through a single supporting point vertically beneath the centre of gravity, or it may be the resultant of all the upward forces

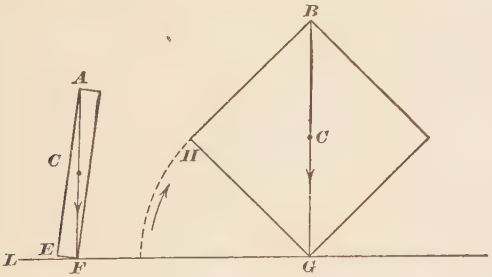


FIG. 59.—Stability: dependence on width of base.

exerted through the various supporting points on which the body rests. Such a resultant need not pass through any actual supporting point, but evidently it cannot pass outside the boundary of the body's base, which consists of the whole area included by the straight lines that connect the outermost supporting points all around. For instance, the base of a three-legged stool, or a tripod-support for a photographer's camera, is a triangle, of which the foot of each leg forms a vertex.

The degree of stability depends upon two factors, the area of the base on which the body rests and the height of the centre of gravity above this base.

In illustration of the influence of the area of the base, compare the angle traversed by the centre of gravity of a square prism, *A* (Fig. 59), 1 cm. square and 10 cm. long, placed on end and then overturned, with the angle traversed under similar conditions by the centre of gravity of a decimeter cube, *B*. The block *A* will not of itself overturn until after its diagonal *AF* has passed the vertical position which it has in the figure. In the same way *B* will of itself overturn only after *BG* has passed the vertical position. To be upset, therefore, the block *A* has to be tilted only a bit more than the very small angle *EFL*, while the block *B* has to be tilted a bit more than the angle *HGL*, of 45°.

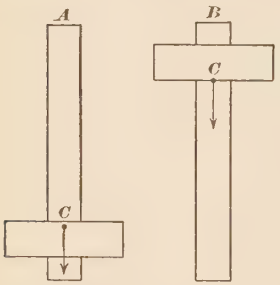


FIG. 60.—Stability: dependence on height of centre of gravity.

In order to show the influence of the height of the centre of gravity on stability, suppose a heavy iron nut to have a cylindrical piece of wood inserted in it, the centre of gravity of the whole being at *C* (Fig. 60). When the nut is near the supporting surface, as shown in *A*, the stability will be considerable; when the nut is near the upper end of the stick, as in *B*, the stability will be small.

The common term *top-heavy*, applied to objects like *B*, well expresses the actual reason of their instability.

75. Kinds of Equilibrium.—Bodies are said to be in *stable*, in *unstable*, or in *neutral*, equilibrium, according to their behavior upon being tilted or displaced through an extremely small angle. If they tend to right themselves,

that is to return to their first position, their equilibrium is stable. If they tend to become overturned, their equilibrium is unstable. If they remain indifferently wherever placed, their equilibrium is neutral. A single solid, the cone, in different positions illustrates all three kinds of equilibrium: resting on its base, on a level surface, it is stable; on its apex, unstable; on its convex surface, neutral.

It is evident that slight tilting, in the case of stable equilibrium, leaves the line of direction of the earth-pull, from the centre of gravity downward, well inside the boundary of the base. In the case of unstable equilibrium it throws the line of direction outside the boundary of the base, and in the case of neutral equilibrium leaves the line of direction unchanged in its relation to the base.

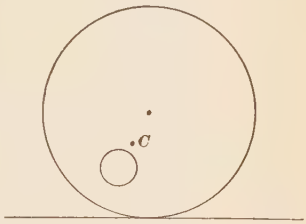


FIG. 61.—Cylinder with eccentric centre of gravity.

(1) Place a double cone on a track made of two rails placed somewhat higher and farther apart at one end than at the other. The roller moves toward the higher end of the rails because its two points of contact with the rails are not in the same vertical plane with the centre of gravity, being a little nearer the lower end of the rails, but its centre of gravity is really descending slightly all the time, for the greater distance between the rails near the upper end lets the roller farther down between them.

(2) Set rolling along a horizontal surface a cylinder like Fig. 61, loaded in such a way that its centre of gravity, *c*, does not lie at its centre of figure.

76. Stability of Suspended Bodies.—Since the centre of gravity, by its very definition (§ 64), has a constant tendency toward the earth's centre, it is evident that an object *W* hung from a point *O* (Fig. 62), about which it may swing freely, will be in stable equilibrium when the centre of

gravity of W falls in the line OG, which represents the direction of one of the earth's radii. W', then, is the position of rest for the weight when left free to settle. The plumb-line and the pendulum plainly illustrate this principle, but there are other contrivances in which it is less obvious.

Fig. 62.—Stability of a suspended body.

Fig. 63 represents two steel forks stuck into a cork, the whole balancing upon the head of a pin. In this case, as in many others, the centre of gravity, C, of the balancing body is not inside the substance of the body. Other similar toys are familiar.

77. Theory of a Simple Balance.—Fig. 64 represents a balance of a very simple kind, supported at the point O. The pans P_1 and P_2, hanging freely from the points K_1 and K_2, have the same effect as if their weights were concentrated at these points. The centre of gravity of the whole, consisting of the beam, the pans, regarded as at K_1, and K_2, and the pointer I, is, we will suppose, at C, a little below the point of support. When the balance is disturbed the pointer swings easily, like a pendulum, through its position of equilibrium in either direction.

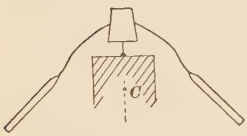


Fig. 63.—Centre of gravity, C, outside the body.

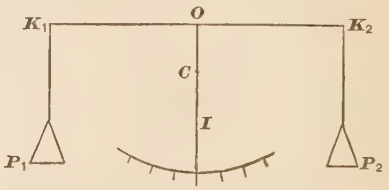


Fig. 64.—Principle of balance: without load.

If a small weight, p, is placed on P_2, the equilibrium is for the moment disturbed. The pointer swings and presently comes to rest in a new position. The condition of things is then illustrated by Fig. 65.

The point C, at which we may regard the weight of the beam, etc., concentrated, now hangs to the left of the vertical line through the point of support O. We have equilibrium when the moments calculated with respect to O balance each other (§ 61). The moment of the weight which we regard as applied at C is negative. The moment of p with respect to O is positive. The moment of C increases when the index swings farther from its first position; the moment of p decreases at the same time. So, at a certain position of the index, the two moments become equal and there is equilibrium.

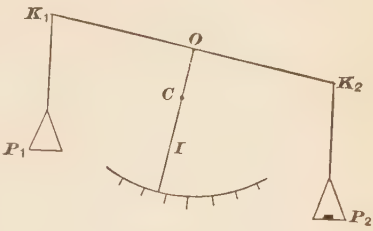


Fig. 65.—Principle of balance with load.

The nearer C is to O, other things being equal, the farther the index must move when the weight p is added. By attaching to the index a small rider than can be moved up and down upon it, the sensitiveness of the balance can be changed at will. Frequently an adjustable rider for this purpose is placed on a stem reaching upward from the middle of the beam.

It is evident from Fig. 65 that there are two other things that count in making the sensitiveness of a balance. The lighter the beam and the pans and the longer the beam, provided it is stiff, the smaller the weight, p, that will tip the beam through a given angle.

78. Centre of Buoyancy; Stability of Floating Bodies.—

In a body of liquid at rest any portion not at the bottom may be regarded at *floating* in the remainder.

The liquid pressure against any body wholly or partly submerged depends upon the shape and position of its submerged surface, but does not depend upon its kind of material. The resultant of the liquid pressure against the submerged part, whatever its density, is a force upward through what would be the centre of gravity of this submerged part if it were of the same density as the liquid. This point is called the *centre of gravity of the displaced liquid,* or the *centre of buoyancy.*

When a floating body, for instance the hull of a vessel, is in equilibrium, under the influence of gravity and buoyant pressure only, the centre of buoyancy lies in the same

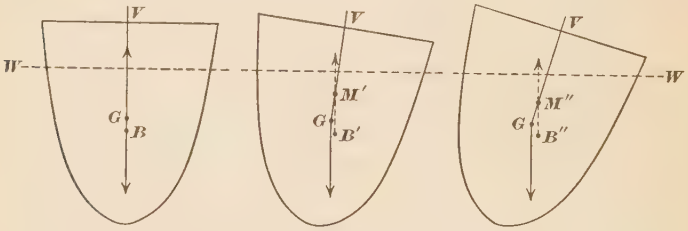


Fig. 66. Fig. 67. Fig. 68.

Figs 67 to 68.—Illustrating centre of buoyancy, b and b', metacentre, m', and metacentric height, mg.

vertical line with the centre of gravity of the body. When the vessel rolls, the upward push of the water and the pull of gravity make a *couple* (§ 63) which tends to right the vessel.

It is an interesting fact, which anyone can see for himself by floating a broad piece of cork, that a floating body may be in very stable equilibrium with the centre of gravity above the centre of buoyancy, and even above the surface of the liquid. The truth is in such cases that, when the

body is tipped, the centre of buoyancy, owing to the change of figure of the displaced body of water, moves in the direction of the tipping more rapidly than the centre of gravity does. Figs. 66, 67 and 68 show how this may occur, G being the centre of gravity, B the centre of buoyancy when the body is upright, and B' or B'' the centre of bouyancy when the body is inclined.

Metacentre.—The points M' and M'', where the vertical lines through B' and B'' respectively cross the line GV, which is vertical when the body is upright, are nearly the same, in ships, if the tipping is not many degrees, and this point is called the *metacentre.* The distance GM is called the *metacentric height.*

This height varies in ships from one or two feet to several feet. It is evident from Figs. 67 and 68 that, for a given angle of inclination, the *arm* of the couple which tends to restore the ship to the upright position from a moderate inclination, the *righting couple* we may call it, is proportional to the metacentric height.

Questions and Problems.

(1) Show that a brick laid on a table-top may have three positions, one of greatest, one of least, and one of medium stability. (Disregard the scarcely possible case of balancing it upon an edge or corner.)

(2) What kind of equilibrium has a straight bar balanced across a fulcrum beneath it and having a weight suspended by a string from each end?

(3) How would the equilibrium of the apparatus of Problem 2 be affected by replacing the strings by light, stiff wires rigidly attached?

(4) A telegraph-pole is made of three hollow iron cylinders joined end to end. Each cylinder is 3 m. long. The lowest weighs 200 kgm., the middle one 100 kgm., and the uppermost 50 kgm. Find the centre of gravity of the pole. (Imagine the pole in a horizontal position, calculate the *moment* of the weight of each section with re-

spect to one end of the pole, find the *force* which must be exerted by a single supporting point, then find the *position* of this point.)

(5) A cube of wood, 10 cm. on each edge and of specific gravity 0.5, is covered on one side by a plate of metal 10 cm. square and 1 cm. thick, of specific gravity 5.

(*a*) How far from the outer surface of the metal plate is the centre of gravity of the whole?

(*b*) If this object is floated in water, how far below the water surface will the centre of buoyancy be?

(6) A uniform lever, 6 ft. long and weighing 20 lbs., lies horizontally across a fulcrum 2 ft. from one end. A mass of 100 lbs. is suspended from the end of the short arm of the lever. How great must be the force applied at the end of the other arm in order that there may be equilibrium?

(7) A lever weighing 12 lbs. is placed horizontally, and carries at one end a weight of 8 lbs. The centre of gravity of the lever is 4 ft. from the same end. At what distance from this end must a supporting point be placed in order that the whole may be in equilibrium?

(8) A bar 6 ft. long has its centre of gravity 1 ft. from one end and balances upon a point 4 ft. from this end when a load of 10 lbs. is suspended from the other end. What is the weight of the bar?

(9) A bar 6 ft. long, uniform in cross-section and density and weighing 4 lbs., balances on a point 4 ft. from one end when a certain load is applied at the other end. How great is the load on the bar and how great the pressure on the fulcrum in this case?

(10) A uniform bar 10 ft. long and weighing 5 lbs. bears at a distance of 2 ft. from the end A a slider weighing 4 lbs., and at a distance of 6 ft. from the same end another slider weighing 7 lbs. How far from the end A must a point be placed in order that the rod with its loads may balance upon it in a horizontal position?

(11) Show what forces are at work to maintain equilibrium in the case of a vessel heeling (tilting) over somewhat under the action of a wind which blows at right angles to her course.

CHAPTER VII.

THREE FORCES DIRECTED THROUGH ONE POINT; THE PARALLELOGRAM OF FORCES.

79. Introductory.—In studying the lever we have usually, though not always, had parallel forces to deal with, forces acting straight up or straight down. But very often we have to do with bodies that are acted upon by forces not parallel to each other. Thus, when a ladder standing upon the ground leans against a house, we have at least three forces acting upon the ladder: 1st, the earth's attraction, or, as we call it often, the *weight* of the body, which acts as if the whole substance were at the centre of gravity; 2d, the push of the ground against the foot of the ladder, which push is not straight upward; 3d, the push of the wall against the top of the ladder.

Again, a flying kite is acted upon by the earth's pull, straight downward; by the force exerted by the air, which force, because of the wind, is not straight upward; by the pull of the string, which pull is not straight downward.

The way to begin the study of such cases is to take first the case of three forces all acting in the same plane, straight from or straight toward a single point. This is done in Exercise 12.

80. Parallelogram of Forces.—Experiment has led to the following rule for the equilibrium of such a group of three forces: From any point, P (Fig. 69), draw on a plane three lines, A, B, C, representing the directions and the magnitudes of the forces and, taking each pair of these lines in turn as two sides of a parallelogram, complete the

parallelogram. Then from the common point draw A', equal in length to A but opposite in direction, B', equal but opposite to B, C', equal but opposite to C. If A' proves to be the exact diagonal of the parallelogram BC, B' that of AC, and C' that of AB, the forces represented by A, B, and C will be in equilibrium; otherwise not.

This is called the law of the *Parallelogram of Forces.*

Inspection of Fig. 69 will show that, if the three lines A, B and C were put together, in any order, tail to tip, with-

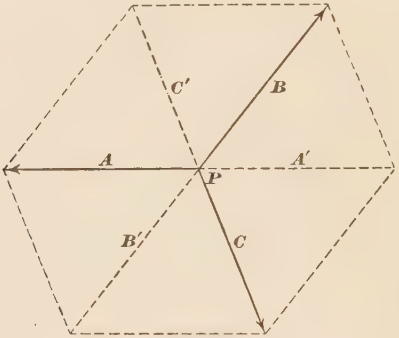


FIG. 69.—Illustrating the parallelogram of forces.

out change of *direction*, they would make a closed triangle. Accordingly, the term *Triangles of Forces* is equivalent to *Parallelogram of Forces.*

81. Composition and Resolution of Forces.—It is evident from the definitions of equilibrant and resultant (§ 65) that

C is the equilibrant and C'' the resultant of A and B,
A " " " " A' " " " B " C,
B " " " " B' " " " A " C.

We see, then, that the rule we are considering enables us to find at once by a graphical method, that is, by drawing and measuring on paper, the resultant or the equilibrant

of any two forces whose *lines of action pass through the same point;* for, as we found in § 59, we should find here by trial, if trial seemed necessary, that the actual point of application is unimportant, so long as it lies on the chosen line of action of the force.

After finding the resultant[1] of two forces we can find the resultant[1] of this resultant[1] and a third force, then the resultant[1] of this second resultant[1] and a fourth force, and so on till we have the resultant[1] of any number of forces the lines of action of which pass through any one point.

Finding the resultant or equilibrant of two or more forces is called the *composition of forces.*

Definition.—Any number of forces which, taken together, can exactly replace a given force are called the *components* of the force. Thus in Fig. 69 again.—

A and *B* are called the *components* of *C′*,
B " *C* " " " " " *A′*,
C " *A* " " " " " *B′*.

Evidently it is easy to find by a graphical method, if not otherwise, a pair of components which can replace any given force. This operation is called the *resolution* of forces. We *resolve*, or separate, the given force into its components.

There are, in fact, numberless pairs of forces, any one of which pairs may be regarded as the components of the given force, as there are numberless parallelograms which may have the same diagonal.

Combinations in which two of the three forces in equilibrium are at right angles with each other are especially easy to deal with; for in such cases the triangle (see § 80) into which the lines representing the forces can be combined is a right triangle, and therefore the lengths of the lines,

1 Or equilibrant.

and the magnitudes of the corresponding forces, have a well known simple relation to each other. See Figs. 71 and 72.

THE INCLINED PLANE.

82. Introductory.—The parallelogram of forces will enable us to understand a contrivance very often used for raising heavy weights. It is a common thing to see barrels of flour or other heavy objects loaded upon wagons by rolling them up a plank or a pair of rails, placed with one end on the ground and the other upon the wagon, so as to make the ascent gradual instead of straight up. The flat, slanting surface up which the body is rolled is called an *inclined plane.*

FIG. 70.—Illustrating the wedge.

Sometimes a body is lifted by forcing an inclined plane, the slanting face of a *wedge,* under it, as in Fig. 70.

Sometimes the force used with the inclined plane is parallel to the inclined surface; sometimes it is parallel to the base-line of the plane, the horizontal surface of a wedge, for example.

We will consider each case in order, seeking for the connection between the weight, steepness of incline, and force to be applied.

83. Force Applied Parallel to Incline.—This case is illustrated by Fig. 71, where

L represents the *length* of the incline,

B " " *base* " " "

H " " *height* " " "

W " " *weight* of the body on the incline.

W' is the equal and opposite of W;

N represents the force exerted upon the body by the

plane L, a force which is straight outward from the surface of the incline if there is no friction between the body and the incline;

P represents the pull, parallel to the plane L, which with the force N will just balance W.

By comparing the triangle $W'P$ with the triangle whose sides are L, B, and H, we see that

$$P:W' \text{ (or } W)::H:L, \text{ or } P \times L = W \times H, \text{ or}$$
$$P = W \times (H \div L).$$

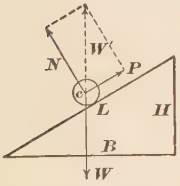


FIG. 71.—Force parallel to the incline.

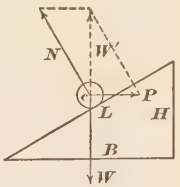


FIG. 72.—Force parallel to the base.

84. Force Applied Parallel to Base.—This case is illustrated by Fig. 72.

The line W' is not here, as it is in Fig. 71, the hypothenuse of the triangle $W'CP$; but it is evident that the triangle $W'CP$ is similar to the triangle LBH. P is the force applied parallel to the base, and just sufficient, with N, to balance W. We have, from a comparison of the triangles,

$$P:W' \text{ (or } W)::H:B, \text{ or } P \times B = W \times H, \text{ or}$$
$$P = W \times (H \div B).$$

85. The Wedge.—The *wedge*, as commonly used, is a case of the inclined plane with the applied force parallel to the base. It differs from the case shown in Fig. 72 in one respect. In Fig. 72 the body raised has a motion parallel to the base of the plane, while the plane itself has not. A wedge commonly has a motion parallel to its own base,

while the body raised or otherwise moved by it does not have a motion in this direction. The principle involved in the two cases is the same, and for a wedge used to lift a weight we have, leaving friction out of account as before,

$$P \times B = W \times H, \text{ or } P = W \times (H \div B),$$

where P stands for the driving force, W for the weight lifted, H for the thickness of the wedge, and B for its length.

86. The Screw.—The *screw* is essentially a form of the inclined plane, as the following experiment will show:

Cut out a long narrow triangle of paper (see Fig. 73) and then wind it upon a lead-pencil, beginning at the end H and keeping the line B all the time at right angles with the length of the pencil. The line L will make a regular spiral around the pencil, corresponding to the *thread* of a screw.

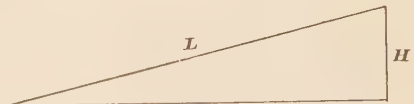


Fig. 73.

The distance from one turn of the thread to the next turn, measured parallel to the length of the screw, is called the *pitch* of the screw.

It is evident that in the ordinary use of a screw one revolution moves it forward or backward the length of its pitch.

87. Use of the Screw in Lifting.—A very large iron screw (Fig. 74), called a jack-screw, is frequently used for lifting very heavy bodies. The power is applied to such a screw by means of a long handle, or lever, which projects from the head of the screw at right angles with its axis, its central lengthwise line, a. Leaving friction out of ac-

count, we can find the relation between power applied and weight of the body lifted thus:

Let P = the power applied to the handle at d, at right angles with the handle and with the axis of the screw:

A = the distance, da, from the point of application of P to the axis of the screw;

r = the radius, ac, of the screw itself;

p = the pitch of the screw;

W = the weight of the body lifted.

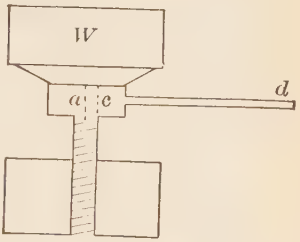


FIG. 74.—Jack-screw.

The force P produces, or is equivalent to, a force $P' = P \times (A \div r)$ applied at the thread of the screw.

This force at the thread is like the power used to drive a wedge. The circumference of the screw at the thread, which $= 2\pi r$, corresponds to the length of the base of the screw, while the pitch corresponds to the thickness of the wedge. We have, then (see § 85),

$$P' \times 2\pi r = W \times p, \text{ or } P' = W \times (p \div 2\pi r).$$

That is,

$$P \times (A \div r) = W \times (p \div 2\pi r);$$

whence

$$P \times 2\pi A = W \times p, \text{ or } P = (W \times p) \div 2\pi A.$$

88. The Work Law Again.—In § 71, while dealing with levers and pulleys, we came on the law,

power × distance it moves = weight × distance it is raised.

If we now take the *distance the power moves* as meaning the distance the point of application of the power moves *in the direction* of the power, we shall find that this same law holds in the case of the inclined plane and of that combination of inclined plane and lever which we have in

the screw. For example, in § 84, if the weight W is drawn the whole length of the incline, it is *raised* the distance H, and in order to produce this result the horizontal power P must move a *horizontal* distance B; so that the law $P \times B = W \times H$, which is found at the end of § 84, is merely a special case of the law given in § 71 and again here. The same general law, that the *work done by the power = the work done on the weight*, would in the case of the jack-screw (§ 87) bring us at once to the right conclusion, $P \times 2\pi A = W \times p$.

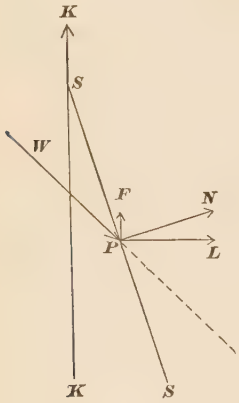


Fig. 75.—Action of wind on a sail.

This law holds strictly true only when there is no friction in the machinery between the power and the weight, and it must not be forgotten that friction is often a very important factor in the action or non-action of a machine.

89. Forces Acting upon a Sail Boat; upon an Aeroplane.—A boat sailing against the wind gives a familiar illustration of the combination of several forces.

Let KK (Fig. 75) represent the line of the keel, and SS that of the sail, which for simplicity is here assumed to be flat. Let WP represent the direction of the wind.

If we may disregard the *friction* of the wind against the canvas, the direction of the force exerted by the wind against the sail is at right angles with the sail and may be represented by PN. This force may be resolved (§ 81) into two components, one, PL, at right angles with the keel, the other, PF, parallel to the keel. PL tends to drive the boat sideways, and does so to a greater or less extent, in spite of the resistance offered by the broadside and by

the keel or centre-board that extends below the hull. *PF* tends to drive the boat forward in the direction of the keel, and the resulting motion carries the boat obliquely toward that quarter from which the wind is blowing.

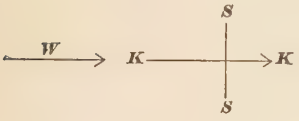


Fig. 76.—Sail before the wind.

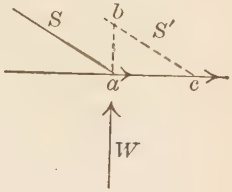


Fig. 77.—Wind at right-angles with keel.

Question.

Can a boat sail toward the wind when the sail is parallel to the keel?

It is said that ice-boats, that is, boats driven by wind across fields of smooth ice, sometimes sail faster than the wind itself is moving. This cannot occur, or, at least, can-

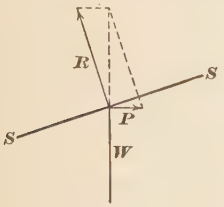


Fig. 78.—Aeroplane, nearly horizontal.

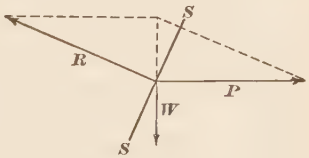


Fig. 79.—Aeroplane, nearly vertical.

not continue, when the boat is moving directly before the wind, as in Fig. 76, for the sail *SS*, gaining upon the air in front, would compress it, and, outstripping the air behind, would rarefy it, so that it would practically encounter a contrary wind.

But if the direction of the wind is, for instance, at right angles with the motion of the boat, as in Fig. 77, it is

possible, if friction is very slight indeed, for the boat to go faster than the wind, the sail moving the length of the line *ac*, from position *S* to position *S'*, while the wind moves the length of the line *ab*.

The main forces acting on an aeroplane are roughly indicated in Fig. 78, where *SS* is the plane, *P* the propelling power, *W* the weight, in excess of weight of displaced air, and *R* is the air-reaction on the under side of the plane, which is moving toward the right. Observe that, if the plane were more nearly vertical, as in Fig. 79, *P* would have to be larger in proportion to *W*.

Problems.

(1) A telegraph wire pulls north from a post with a force of 12 lbs.; another pulls west from the same post with a force of 16 lbs.

(*a*) How great is the resultant pull on the post?

(*b*) If a third wire is put in to neutralize the pull of the other two, should it pull more nearly south than east, or more nearly east than south?

FIG. 80.

(2) Two sticks of equal length, *OA* and *OB* in Fig. 80, each resting one end against the ground, meet at a right angle in a frictionless joint *O*. From this joint is suspended a mass of 5 lbs., the weight of which is represented by the line *OW*.

(*a*) How great is the total force exerted by each stick against the ground, the weight of the stick being left out of account?

(*b*) How great is the vertical push exerted by each stick against the ground?

(*c*) How great is the horizontal push exerted by each stick against the ground?

(If we think of *AOB* as one end of the roof of a house, the answer to (*c*) shows the tendency of the roof to push the walls apart. This tendency is met in actual roofs by beams or rods connecting *A* and *B*.)[1]

[1] A beam running from *A* to *B* would make a complete triangle, and a triangle made of rigid bars is a rigid figure, even if each joint, in itself, would, like *O* in Fig. 80, be capable of motion. Any figure made of more than three bars in one plane, without cross-bars or ties, is capable of distortion, if the joints are like *O*. See Fig. 86. Accordingly, triangular arrangements of beams are very common in framed structures.

(3) A wedge 1 ft. long on its base and 2 in. thick is used to lift a weight of 300 lbs., in a case where friction may be left out of account. How great is the force, parallel to the base, required to drive the wedge?

(4) A safe weighing 2000 lbs. is resting on an inclined plane 12 ft. long, one end of which is 2 ft. higher than the other. How great is the force, parallel to the incline, required to keep it from sliding down?

(5) A jack-screw having a pitch of $\frac{1}{4}$ in. and a handle 2 ft. 1 in. long is used to lift a mass of 5000 lbs. How great must be the power applied to the end of the handle?

(6) A loaded freight-car weighs 25 tons. It is held at rest on a grade which rises 1 ft. for every 50 ft. of track. What force parallel to the track is needed to hold the car in place?

(7) A boat is fastened to a pile driven into the bed of a river. The boat is pressed due southward by the river current with a force of 200 lbs. and due southeastward by the wind with a force of 80 lbs. Construct a diagram to show the amount and direction of the resultant pressure. Find its amount by measurement.

(8) A rope is attached to two hooks which are on the same level and 20 ft. apart. At the middle of the rope, which hangs 10 ft. below the level of the hooks, is suspended a weight of 100 lbs. How great is the pull upon each hook? (Ignore weight of rope.)

(9) *A* holds one end of a rope and *B* holds the other end. At a certain point between them a weight hangs from the rope, and the rope bends at a right angle at this point. *A* pulls with a force of 12 lbs., *B* with a force of 16 lbs. How great is the weight which hangs from the rope? (The weight of the rope itself is neglected.)

(10) Construct a diagram to show how equilibrium is secured in the case of a heavy sinker, attached to a fishing-line, dropped into a swift-flowing stream and allowed to move down stream until the line stops it. (Neglect the action of the water on the line, but consider the buoyant action and the friction of the water on the sinker.)

(11) A child is seated in a swing, 20 ft. long, which is drawn forward 5 ft., measured horizontally, from its position of rest and held motionless by a horizontal cord. Make a diagram to show the number and direction of the forces engaged, then assume that the child's weight is 50 lbs., and find by means of your diagram the magnitude of the other forces.

CHAPTER VIII.

COUPLES: TWO SETS OF FORCES AT RIGHT ANGLES.

90. Introductory; Moment of A Couple, etc.—It is important for the student not merely to learn how to do problems in physics, but to form the habit of looking for useful applications of his knowledge of elementary principles. In the study of mechanical structures the *couple*-idea, which is presented in §63 (Fig. 44), is often helpful.

We have already learned that *the moment of a given couple is always positive, or always negative, with respect to all pivots* (*p.* 66 *foot note*) *in the plane of the forces, and is numerically equal to one of the forces multiplied by the shortest distance between the lines of the forces.*

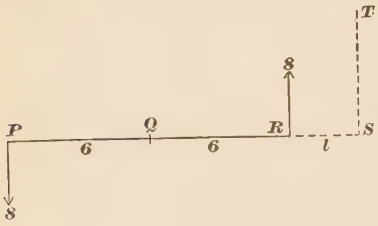


Fig. 81.

We may have noted also (see Problem 9, p. 81), that *there is no single-force equilibrant or resultant for a couple;* but it is well for us to look into this matter a little further.

Let Fig. 81 represent a couple applied to a body, and let us consider how the influence of the given couple could be neutralized. If there is a fixed pivot at the point Q, we have, as the moments with respect to this pivot of the two forces making the couple,

$$\left.\begin{aligned} -(8\times 6) &= -48 \\ -(8\times 6) &= -48 \end{aligned}\right\} = -96.$$

This moment of—96 can be neutralized in a great variety of ways. For instance, if we apply at P a force of 16 upward, its moment with respect to Q is $+96$, and we have equilibrium.

It may look as if we had here balanced a couple by means of a single force, but we have not. The rule (§61) that the algebraic sum of the *forces* must be zero for equilibrium shows that the pivot must now exert a force of 16 downward. The fact is that we now have *two* couples, an 8-lb. couple with a distance, or arm, of 12, and a 16-lb. couple with a distance, or arm, of 6, the former having a moment of —96, the latter a moment of $+96$. If we had applied a force of 32 upward at a point half-way between P and Q, the pivot would have exerted a force of 32 downward, and the moment of this 32-lb. couple would have been $+96$. There is no end to the combinations by means of which we can balance the original couple, but in every case we should find the additional forces equivalent to a couple of moment $+96$.

91. Two Sets of Forces at Right Angles.—In Exercise 13 we study experimentally the case of several forces all in one plane but not all parallel, being grouped in two sets which make right angles with each other. We might work out the laws for such a case from the experiments and discussions that precede, but it is well to reach them by more direct methods. The case is of great practical importance, as we shall see in the examples which are given in this Chapter and the next.

Illustration 1.—In Fig. 82, MN is a ladder resting upon the ground at M and leaning against a smooth vertical wall NO. D is the weight of the ladder acting downward through the centre of gravity C. The push of the wall against the top of the ladder is R, horizontal toward the right. The upward push, U, of the earth must equal the downward force D, making a negative couple with the *arm* PM. The left-hand push L must be equal to the right-hand push

R, making a positive couple, with the arm NO, and the moments of these two couples must be numerically equal. That is $D \times PM = R \times NO$, so that, if we know D, PM and NO, we can find R and its equal, L.

Illustration 2.—A door weighing 40 lbs. has its centre of gravity $1\frac{1}{2}$ ft. distant from a vertical line passing through the pivots of the two supporting hinges. The hinges are of the simplest character and 6 ft. apart from centre to centre. The load is divided equally between the two hinges, but the upper one is supposed to take all the horizontal pull, and the lower one all the horizontal push, caused by the weight of the door.

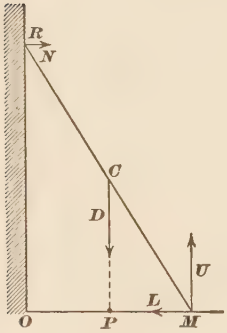


Fig. 82.—Forces acting on a leaning ladder.

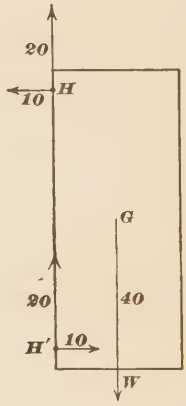


Fig. 83.—Forces acting on a door.

(*a*) How great is the horizontal pull upon the upper hinge?

(*b*) How great is the horizontal push upon the lower hinge?

Let H and H' in Fig. 83 represent the hinges, and G the centre of gravity of the door. The line W represents the weight of the door, 40 lbs., acting downward from G. Each hinge, bearing half the load, must exert upon the door an upward force of 20 lbs. We thus have acting on the door a positive *couple* of moment $(40 \times 1\frac{1}{2})$, or 60. This must be balanced by an equal negative couple acting at the hinges; and, as we have already taken account of the vertical forces at the hinges, this negative couple must consist of horizontal forces. As the arm of this couple is 6 ft., the distance between the hinges, each

horizontal force must be $60 \div 6 = 10$, a pull applied to the door by the upper hinge and a push applied by the lower hinge. It is evident that the upper hinge is much more likely to get loose than the lower one.

Illustration 3.—Fig. 84 represents a chair, the back of which is held firmly in a vertical position while the front feet are unsupported, and the seat is loaded. The conditions are pretty nearly the same as if a person were seated in the chair tilted back slightly.

If the joints B and C are perfectly firm or are cross-braced, so that the whole of the chair in front of its back may be regarded as one rigid body, we have a case very similar to that of the door discussed in

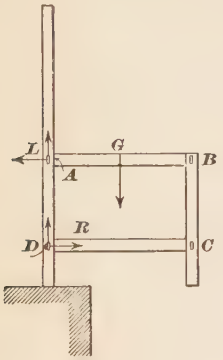


Fig. 84.—Forces in a chair frame: first case.

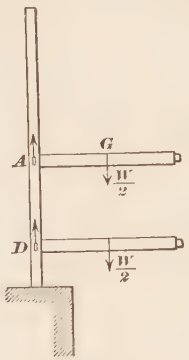


Fig. 85.—Forces in a chair frame; second case.

Illustration 2 above, and the horizontal forces L and R, at the joints A and D, may not be excessively large. But if the chair through misuse and lack of proper crossbracing has become loose at the joints B and C, the case is pretty nearly the same as if the front legs were gone and the load W were divided equally between the upper and lower horizontal parts, as in Fig. 85, and it is evident that, if the positive couple of the upper horizontal, with its moment $\frac{1}{2}W \times AG$, is to be neutralized by a negative couple of equal moment acting wholly within the *joint* A, the *forces* of this latter couple must be very much greater than $\frac{1}{2}W$. The like holds for D.

In short, there is, after the joints B and C get loose, a destructive action at A and D, and the chair soon goes to pieces, if it is tilted when loaded.

As a matter of fact, all the joints will probably work loose together, the rectangle $ABCD$ tending to the shape $A'B'C'D'$ of Fig. 86.

Evidently a *tie*, a connector capable of resisting a stretching force, running from A to C, or a *strut*, a stiff rod capable of resisting compression, running from B to D, would tend to prevent this racking.

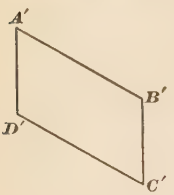


Fig. 86.—Distortion of chair frame.

Illustration 4.—In Fig. 87 the inclined line M represents a man supported by a scaling-ladder L, such as firemen use, which hangs upon a window-sill. A horizontal chain reaches from the man's belt to the hook of the ladder.

The forces applied to the man are his weight,[1] M_1, downward, and an equal force, M_2, upward, the two making a negative couple, M_3 horizontal toward the right and M_3 horizontal toward the left, making a positive couple. These two couples being equal and opposite, the man is in equilibrium.

The forces applied to the ladder are L_1 and L_2, a negative couple, L_3 and L_4, another negative couple, and L_5 and L_6, a positive couple.

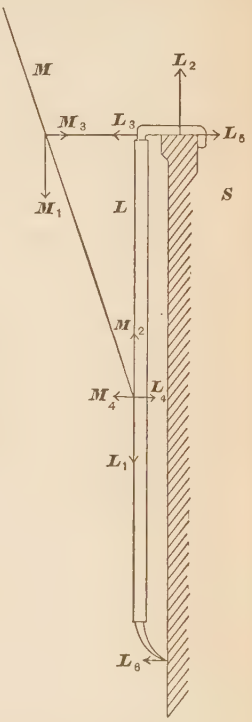


Fig. 87.—Forces acting on a scaling ladder.

Problems.

(1) Let the checkerboard of Exercise 13 be placed with the lines 1–7, 8–14, etc., east and west. Let a force of 10 lbs. act north from point 1, a force of 10 lbs. south from point 7, and a force of 15 lbs. west from point 8. Find the magnitude, direction, and point of application, of a single force that will make equilibrium with all the others.

(2) A ladder 20 ft. long and weighing 50 lbs., the centre of gravity being at the centre of its length, stands upon level ground and leans

[1] Fig. 87 assumes that the man's centre of gravity is on a level with his belt. This need not be the fact.

against a vertical wall, which is so smooth that the force between it and the ladder is wholly horizontal. The distance from the foot of the ladder to the wall is 8 ft. A boy weighing 100 lbs. is on the ladder. The ladder does not bend under his weight.

Find the magnitude of the horizontal push H and of the vertical push V, exerted by the ladder against the ground—

(*a*) When the boy is midway between the foot and the centre of the ladder;

(*b*) When the boy is midway between the centre and the top of the ladder.

Suggestion: Find first the magnitude and line of action of the resultant of the two given vertical forces, the weight of the ladder and the weight of the boy. Then treat the problem as a case of two equal and opposite couples, one horizontal and the other vertical.

(3) Let the keel of a boat 40 ft. long lie north and south, the bow south. Let one oarsman pull according to the conditions given in Problem 14, p. 82, with his oar out toward the west, and let another back water with equal force, his oar out toward the east. Let one row-lock be 2 ft. west of the keel, the other 2 ft. east.

Two forces, one acting east, the other acting west, are applied, one at the bow, the other at the stern, of the boat, so that the boat does not move. What is the magnitude and direction of the bow force? of the stern force?

CHAPTER IX.

PROPERTIES OF SOLID BODIES.

92. **Introductory.**—Very many bodies, such as pieces of wood, stone, and metal, have a shape of their own which they will keep under all ordinary conditions. These are called *solid* bodies, or, for brevity, *solids*.

Plainly distinguished from this class is another, made up of bodies such as air, water, oil, etc., which adapt their shape to the vessel they may happen to be contained in, showing no tendency to keep any form which can be called their own. Bodies of this second class (see § 18) are called *fluids*, a name which includes *gases*, as air, and *liquids*, as water.

The same *substance* may be at times solid and at times fluid. We are all familiar with at least one substance which occurs commonly in each of the three states here named—the solid state, ice; the liquid state, water; the gaseous state, steam.

The student has already, from common experience and from Chapters III. and IV. of this book, gained some knowledge of liquids and gases. Now we are to study some properties of solid bodies, the properties which distinguish them from fluids. In doing this we shall get an idea of the kind of knowledge of materials which engineers and other builders must have in putting together framed structures, like houses, bridges, and boats, upon the strength and stability of which we are dependent every day.

The properties which we shall study especially are *tenacity*, that is, resistance to breaking apart under a straight pull, and *elasticity* of various kinds.

TENACITY.

93. Illustration and Definition.—When an engineer wants to know how large a rod of a given material he must use to suspend a given load, he first inquires how heavy a load could be borne by a rod of the same material and quality 1 sq. in. or 1 sq. cm. in cross-section. We will suppose that he finds the load required to break, by a straight pull, a rod 1 sq. cm. in cross-section to be 4000 kgms. Then he sees that, if the load to be sustained is 20,000 kgm., he must use a rod at least 5 sq. cm. in cross-section. In fact, he probably would, to make perfectly sure, use a rod two or three times as large as that.

Definition: If F is the force required to break, by a straight, longitudinal, pull, a rod of cross-section S, the ratio $F \div S$ is called the *breaking-strength*, or *tenacity*, of the *material*. It is the breaking-force per unit area of cross-section.

The experiments may be made on fine wires, instead of rods, but even in such cases the tenacity found may be expressed as so many kilograms per square centimeter, or so many pounds per square inch, of cross-section.

In the preceding statements it seems to be taken for granted that the pull which a rod can bear is proportional, other things being equal, to its area of cross-section. This is, for ordinary purposes, taken to be the rule. It would be strictly true, or very nearly so, if the quality and condition of the material were exactly the same in all pieces of the same material and in all parts of the same piece, but this is not the case. The surface film of a rod is in a somewhat different internal condition from the interior parts. Flaws, due to the presence of impurities or to imperfections in the process of manufacture, occur in metals, knots and differences of grain in wood. Accordingly, though the engineer finds tables of the tenacity of various materials very

useful, he does not trust them as giving strictly accurate data for every case he may have in hand. If he is properly careful, he will probably put into the structure he is building parts of such dimensions that, if there were no flaws in his material and each piece were as good throughout as the best reported in the tables, the structure as a whole would be several times as strong, f times, let us say, as it need be for its uses. The quantity f is called the *factor of safety*.

APPROXIMATE VALUES[1] OF TENACITY, T., IN KGM. PER SQ. CM.

Material	T.	Material	T.
Brass	6000	Quartz	1630
Copper, hard-drawn	4000	Silver, h. d.	2900
Copper, annealed	3100	Silver, ann.	1600
Glass	300–900	Steel, h. d.	8000
Gold, h. d.	2700	Steel, ann.	6600
Gold ann.	1000	Steel, wrought	13000
Iron, h. d.	6100	Tin, h. d.	250
Iron, ann.	4700	Tin, ann.	170
Iron, wrought	4500	Zinc	1300
Iron, cast	2300	Wood, ash	1200
Lead, h. d.	210	Wood, beech	800
Lead, ann.	180	Wood, fir	850
Platinum, h. d.	3400	Wood, oak	700
Platinum, ann.	2400		

It is to be noted that annealing, softening by heating for a time, usually if not always decreases the strength of the material. Observe that brass, an alloy of copper and zinc, is stronger than either copper or zinc; there are many such cases among alloys.

[1] From Landolt and Börnstein's *Phyikalisch-Chemische Tabellen*, 3d edition.

94. Cohesion.—No one can at present *explain* tenacity. Why does it take force to break a wire? We may say, of course, that there is an *attraction* of neighboring particles of metal for each other, which we call *cohesion;* but that, after all, is merely saying that it requires force to pull them, or keep them, apart, when they are near each other. If cohesion explains tenacity, what explains cohesion?

No satisfactory answer to this question can now be made, in spite of all the attention that has been given to the subject, and perhaps none ever will be given, but that should not discourage us from trying to find out all we can about the properties of bodies; partly because the facts we discover may be of great practical use, and partly because so many things apparently unknowable have been found out, that it is rash to speak of any fact of nature as hopelessly out of our reach.

The immense importance of iron in its many forms has led to a corresponding amount of study of the changes made in its properties by various kinds of treatment and by various admixtures of other substances. Something of this is shown in the following article.

95. Iron and Steel.—Iron commonly contains small quantities of other substances, such as carbon, sulphur, and phosphorus, left in it by the process of manufacture from iron ore, or added to produce certain changes of quality. The first operation in the manufacture leaves considerable carbon, perhaps 2 or 3 per cent., in the iron. The metal in this condition can be melted with comparative ease, and when poured into moulds in the liquid state readily takes the form of the moulds. This operation is called *casting,* and iron thus treated is called *cast iron.* Many articles of household use —kitchen stoves, pots, and kettles, for instance—are of cast iron.

When cast iron is in the molten condition, much of its carbon can be burned out by exposing it to a stream of air. The decarbonized iron thus obtained is *worked* while hot by pressing, hammering, etc., and thus becomes *wrought* (that is, *worked*) *iron,* which is softer and very much tougher and stronger than cast iron. The iron used by

blacksmiths is wrought iron. It can be readily welded, which is not true of cast iron.

Steel, in the ordinary sense of the word, means iron containing less carbon than cast iron and more carbon than wrought iron. The amount of carbon in ordinary steel may vary from a small fraction of 1 per cent. to 1.5 per cent.

Sometimes steel is made by slowly burning out a part of the carbon from cast iron, occasional tests of the progress of the operation being made by taking out and cooling a small quantity of the molten metal. When the quality of the tested piece is satisfactory the operation is stopped. *Open-hearth* steel is made by this, the Siemens-Martin, process and sometimes contains so little carbon that it is very much like wrought iron.

For making the rails of car-tracks and for other rough purposes great quantities of *Bessemer* steel are used. This is made by burning out the carbon from melted cast iron very rapidly, streams of air being forced through the liquid metal in immense pots called *converters*. No attempt is made to stop the operation at the instant when the metal contains just the right proportion of carbon. It is easier to burn the carbon all out, or nearly so, and then add the desired amount in a certain mixture or compound of iron and carbon, called *spiegeleisen*.

Steel containing from 0.5 per cent. to 1.5 per cent. of carbon is stronger than wrought iron. One specimen of such steel in the form of wire is reported to have had a tenacity of nearly 345,000 lbs. per square inch. This steel is, in its softest state, harder than wrought iron, and when it has been heated to redness and cooled suddenly it becomes very hard indeed, some pieces being hard enough to cut or scratch glass. This process of hardening steel is often called *tempering;* but, strictly, tempering is the partial softening that the steel is subjected to after hardening, the degree of softening being adapted to the purpose for which the metal is to be used.

Other substances are used with carbon to give peculiar properties to steel or iron. *Mushet's steel*, which is exceedingly hard and brittle, contains *tungsten*. Steel containing 10 per cent. or 15 per cent. of *manganese* is strong and exceedingly tough. A mixture of steel and *nickel*, called *nickel-steel*, is so strong and tough and elastic that it has been used for the armor-plate of heavy war-ships.

Ordinary wrought iron cannot be cast in moulds with success, because of the "blow-holes," caused by gas-bubbles, which form in the cooling metal; but the addition of a small quantity of *aluminium*

(more commonly called *aluminum*) to the iron prevents this difficulty. Iron treated in this way is called *mitis* iron. It is used in place of wrought or forged iron in making the magnets of dynamos.

The demand for the very best kinds of iron and steel for the manufacture of armor-plate and large cannon has led to great improvements in the processes and products of manufacture during the last few decades.

The steel of which armor-plates for ships is made is not only strong, but it is so tough that a big solid shot fired against the armor may bed itself in the steel, as a stone beds itself in moist clay, without seriously cracking the plate.

Fig. 88.—Iron: microscopic condition after sudden cooling.

Fig. 89.—Iron: microscopic condition after slow cooling.

The distance that a piece of metal will stretch before breaking, divided by the length before stretching, is called its *elongation*. Some specimens of manganese steel are said to have an elongation as great as 50 per cent.

Microscopic study of the interior condition of samples of iron and steel has been very instructive. For example, it is known that if iron containing carbon is heated above what is called the critical temperature, about 700° or 750° C., and is then suddenly cooled, it shows under the microscope a fine-grained structure, as in Fig. 88; whereas, if it is cooled slowly from a like high temperature, it shows a coarse-grained structure, as in Fig. 89. The interpretation is, that at high temperatures the iron and the carbon are intimately blended, and that sudden cooling from this condition leaves them still closely

associated, while slow cooling allows the two elements to separate from each other to a considerable extent. Each of the two specimens here shown contains about 0.5 per cent. of carbon. Fig. 88 shows a harder and stronger, but at the same time more brittle, state than Fig. 89.

ELASTICITY.

96. Definitions: Stress and Strain.—We shall often find the scientific use of a word to be somewhat different from its popular meaning, and, unless this fact is recognized and remembered, much confusion of ideas, and much vain discussion, is likely to occur. Illustrations may be found in one or two words to be used in the next few paragraphs. Thus the word *strain* means, in scientific language, a change in the shape or size of a body; while the word *stress* is applied to the force which produces the strain. More strictly still, *strain* means the *ratio* of the change of dimension or size to the original dimension or size; and *stress* means the *ratio* which the whole force applied bears to the area of the cross-section through which it is applied. Thus, if a rod L cm. long is stretched until it is $(L+l)$ cm. long, or compressed until it is only $(L-l)$ cm. long, the strain, as to length, is $l \div L$. And if the area of cross-section of the rod is S sq. cm., and a force of F gms. is applied to lengthen or shorten it, the *stress* is $(F \div S)$; that is, the force per unit area of cross-section.

In popular usage *strain* has nearly the same meaning as *stress*. Again, in popular speech, a body is said to be highly *elastic* when it can suffer and recover from *great changes* of size or shape. India-rubber is the type of such elastic substances. But in the scientific sense the elasticity possessed by steel and glass is much greater than that of india-rubber.

97. Elasticity: of Volume, of Figure.—The power which a body has of recovering, more or less perfectly, its original

volume, after the force which has changed the volume is withdrawn, is called *elasticity of volume.* This property exists in perfection in liquids and gases, which recover completely from the effect of any compression, however long it has been continued, when the forces producing it have ceased to be applied.

The power which a body has of recovering, more or less perfectly, its original shape after the force which has changed the shape is withdrawn, is called *elasticity of shape or figure.* Liquids and gases are often regarded as entirely lacking in this property, although any body of liquid or gas left entirely to its own forces and at rest would finally assume a spherical shape. Elasticity of figure, then, is peculiarly a property of solids. It is called also *rigidity.*

So-called inelastic, or *plastic,* solids, are those which, like lead, wet clay, or putty, yield readily to a slight distorting force, showing little or no ability to resume their original shape when that force ceases to act upon them. Highly elastic solids, like tempered steel, spring-brass, and ivory, offer a constant resistance even to very long-continued distorting forces, and will immediately resume their original shape upon the removal of any such force, provided the strain does not exceed a certain value, called the *limit of elasticity,* beyond which the substance will act like a plastic solid.

98. "Modulus" of Elasticity; "Hooke's Law."—If several bars of various sizes and shapes, but made of the same material in the same state of purity, hardness, etc., are subjected to longitudinal forces, either stretching or compressing, that do not exceed the limit of elasticity, it is found that the quantity *stress ÷ strain* is the same for all of them. It is called the *modulus*[1] *of elasticity* for the given material under the kind of strain used. A different material

[1] Latin: a little measure.

has a different modulus; that is, a different value for *stress ÷ strain.*

Tables of such moduli, derived from experiments, are printed in many books used by physicists and engineers, but different grades of the same general substance will differ so much in their tenacity and elasticity that materials which are to be put to any severe and important use —steel for making heavy cannon, for instance—are often tried by special tests to prove their qualities.

The particular *modulus* which we are concerned with in tests of either stretching or shortening is called *Young's modulus*, from the English scholar, Dr. Thomas Young,[1] who suggested and used it.

We shall define the *stress* as the force, F, in grams, divided by the cross-section of the wire, S, in square centimeters. The *strain* will be the change, increase or decrease, of length, l, divided by the original length, L, both in centimeters.

If we use M to signify Young's Modulus we have

$$M = \frac{stress}{strain} = \frac{F}{S} \div \frac{l}{L} = \frac{F \times L}{S \times l}; \text{ and } l = \frac{F \times L}{S \times M}.$$

The equation last written, which, it must be remembered, is based upon experiment, shows that l, the change of length in any case, is proportional to the total force, proportional to the original length, inversely proportional to the area of cross-section, and inversely proportional to the modulus, M.

Hooke's Law.—The rule that the change produced by a force is proportional to this force, provided the change is small, holds not merely for change of length, but for any kind of elastic change. It is called *Hooke's law*, having

[1] Young was born in 1773 and died in 1829. He was a man of great learning and ability, and one of the founders of the wave-theory of light (§ 234); though Fresnel, a Frenchman, who took up the matter somewhat later than Young, was more active and efficient in establishing the theory.

been originally announced by Robert Hooke, an Englishman (1635-1703).

The same general laws hold for compression as for extension, and, moreover, the *modulus* for the compression of any substance is numerically the same as the modulus for its extension, provided the limit of elasticity is not exceeded. The limit of elasticity (§97) in compression is, however, not necessarily the same as the limit of elasticity in extension. Thus, cast iron will bear a much greater compression strain than extension strain before breaking, although the force required to produce a minute decrease of its length is practically the same as the force required to produce an equal increase. (See §104).

The value of M for steel bars of a certain grade is 2,500,000,000, or 25×10^8, and for copper wire of a certain grade, about 1,200,000,000, or 12×10^8.

Problems.

(1) How much will a steel rod 3 m. long and 5 sq. cm. in cross-section be stretched by a load of 5000 kgm.?

(2) How great a load, applied at the top of the rod in (1), would shorten it 0.2 cm.?

99. Bending.—Transverse forces, applied in such a way as to bend a body, stretch it in some places and compress it in others.

Take a long thick india-rubber "eraser" and measure carefully the length of the two opposite sides. Then bend the eraser sharply, in such a way that one of the measured sides will be on the inner arc and the other on the outer arc, taking care not to stretch or compress the rubber unnecessarily. Measure the length of the two arcs thus formed and determine for each side whether it is longer or shorter than before bending.

Draw a straight line along the middle of one of the other sides from end to end, and, when the rubber is bent exactly as before, measure the length of this line, which will be midway between the

concave and convex sides of the bent eraser, and see whether it has increased or decreased in length.

A strip of paper will be found convenient for making the measurements.

The general laws of bending, for rods of rectangular cross-section, that is, the ways in which the amount of bending depends on the load, and on the various dimensions of the rods, are studied in Exercise 14. As the grain of the wood may differ considerably in different rods, it is necessary, in deducing from this Exercise the laws relating to dimensions, to make use of the observations made with a considerable number of rods, all as much alike in grain as may be. The effect of length, of width, and of thickness, upon stiffness can in this way be made out with some approach to accuracy, though more extensive and precise observations than are here practicable would be required to make us sure of the laws in question.

100. Laws of Bending; Stiffness.—Comparing the mean deflection per gram of the First Case of Exercise 14 with the mean deflection per gram in the Second Case, we see the relation between amount of deflection and length of rod, other things being equal. Comparison of the First Case with the Third Case indicates the relation between deflection and width, other things being equal. Comparison of the First Case with the Fourth Case indicates the relation between deflection and thickness, other things being equal.

The relations in question are combined in the formula,

$$D \propto \frac{P \times L^3}{W \times T^3},$$

in which D stands for the total deflection,
P " " " load,
L " " " length,
W " " " width,
T " " " thickness.

The meaning of this formula is that in going from any case 1 to any case 2, provided the rods used in the two cases are of the same material and quality, we have

$$D_1 : \frac{P_1 \times {L_1}^3}{W_1 \times {T_1}^3} :: D_2 : \frac{P_2 \times {L_2}^3}{W_2 \times {T_2}^3}.$$

Problem.

If a load of 100 gm. bends 0.01 cm. a rod 40 cm. long, 1 cm. wide and 1 cm. thick, how much would a load of 1000 gm. bend a rod 80 cm. long, 2 cm. wide and 3 cm. thick?

Stiffness.—The greater the stiffness of a rod the less it will bend under a given load. That is,

$$\textit{stiffness} \propto \frac{P}{D}\left(\text{or } \frac{W\ T^3}{L^3}\right).$$

101. Breaking-Strength in Bending.—It is important to notice that the formula for stiffness suggested in § 100 will not express the dependence of *breaking-strength* upon dimensions in the case of rods submitted to the action of transverse forces. For instance, adding to the thickness of a rod increases its strength less than it increases its stiffness. A thick rod is hard to bend, but a little bending will break it. The following relations hold: $\textit{strength} \propto \textit{width}$; $\textit{strength} \propto \textit{thickness}^2$; $\textit{strength} \propto 1 \div \textit{length}$. Combining these three relations we get the formula,

$$\textit{strength} \propto \frac{\textit{width} \times \textit{thickness}^2}{\textit{length}}\left(\text{or} \frac{W \times T^2}{L}\right).$$

The relations in question are shown to be true or nearly true by experiment; but they can be profitably discussed from the point of view of couples, as various other practical matters were discussed in the preceding Chapter.

Let Fig. 90 represent a beam of length L, the weight of which may be disregarded, supported at the ends and bearing at the middle a load W, one-half of which may be regarded as carried by each half of the beam. Accordingly we have, as the moment of the couple

applied to the right-hand half of the beam by the weight and the support, $-(W/2 \times L/2)$, or $-WL/4$. This must be balanced by a positive couple or a combination of positive couples exerted on the cross-section *ab*, through the fibres connecting the right-hand half with the left-hand half of the bar, a push toward the right in the upper half and a pull toward the left in the lower half, as the horizontal arrows indicate. The arrows at the top and bottom are made longer than

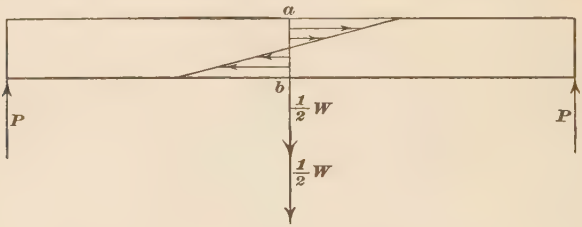


Fig. 90.—Fiber couples in a beam loaded at the middle.

the intermediate ones; for evidently the compression of the fibres is greatest at the top of the beam and the stretching greatest at the bottom. Evidently, too, if the beam is to break, it will, if the fibres are everywhere of the same natural quality, give way first at the top or at the bottom, that is, at one of the places of greatest *strain* (§ 96).

The parts of the bar at the section *ab* can bear a certain amount of

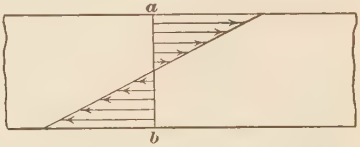


Fig. 91.—Fibre couples: effect of thickness.

stretching or compression, and no more. The moment of the couple they can exert has a certain limit. If the moment $WL/4$ becomes greater than this limit, the bar breaks.

Influence of Length.—Evidently the greatest load, W, which the beam can bear without breaking is inversely proportional to L.

Influence of Thickness.—If the bar is made twice as thick, as in Fig. 91, we may draw twice as many arrows to represent the molecular forces, but the outside arrows have the same limit of length as

before; for the limit of stretching or compression of the substance is the same as in the smaller bar. The average length of the arrows is the same, at the breaking condition, in the thick bar as in the thin one.

The right-hand and the left-hand resultants are twice as great as in the thin bar, and the points of application of these resultants are twice as far apart as in the thin bar. Hence the moment of the molecular couple just before breaking is four times as great in the thick bar as in the other. Doubling the thickness multiplies the strength by *four*.

Influence of Width.—Doubling the width of the bar would double the resultants of the molecular forces, but would leave the lever-arm of the couple unchanged in length. Hence it would merely double the strength of the bar.

102. Torsion, or Twisting.—We are, in the affairs of every-day life, less familiar with twisting strains than with stretching or bending, but when we reflect that all of the revolving rods, or "shafts," used to transmit power through machinery are subject to twisting, we see that engineers, at least, must make a study of the laws of torsion. The frequency with which mainshafts of great steamers break in mid-ocean is evidence that we do not yet know all that we should in regard to these matters, though such accidents are due rather to failure of quality in the materials used than to any lack of knowledge as to the general laws involved. Defining the amount of torsion, or twisting, as the difference in direction, produced by twisting, between two diameters originally parallel, one at one end, the other at the other end, of the object twisted, we have for straight rods or bars or wires the following laws:

(1) Torsion is proportional to the moment, M, of the twisting couple.

(2) Torsion is proportional to the length, L, of the piece twisted.

(3) Torsion is inversely proportional to the fourth power of the diameter, D, of the piece.

Putting all these laws together we have

$$\textit{torsion} \propto \frac{ML}{D^4}$$

103. Breaking-Strength in Twisting.—As in bending, so in twisting, we must distinguish carefully between stiffness and strength. The moment of the couple required to break a rod by twisting is independent of the length of the rod, but is proportional to the third power of the diameter.

104. Best Use of Material in Beams.—We have now seen that increase of thickness is very effective in adding to the stiffness and the strength of rods, whether subject to bending or to twisting. The same amount of material is more effective when it is placed some distance away from the

Fig. 92.—Iron beams: forms of cross-section.

centre of cross-section than when it is very near this centre. Accordingly, beams that need to be made as stiff and strong as possible in proportion to their weight, to resist bending or twisting, are not made of compact form, but rather of a cross-section like some one of those in Fig. 92, two of which are hollow.

This fact sometimes leads people to the conclusion that a hollow beam is stronger than a solid one of the same external dimensions, but this is a mistake. If saving of weight and material were no object, beams would always be made solid, but where, as in bicycles, flying-machines, etc., it is necessary to combine lightness with great stiffness and strength, and where *crushing in* is not likely to occur, the tubular form is almost always used, if the material used is one that can readily be obtained in this form. Witness the lightness and stiffness of quills.

It has already been noted (see § 99) that the outer arc of a bent rod is stretched and the inner arc compressed. It has also been stated (§ 98) that in a given substance the limit of elasticity in stretching is not necessarily the same as its limit in compression. Cast-iron, for example, will bear much more compression than stretching before giving way. Accordingly, cast-iron beams or rails are made much wider on the side which is to be stretched than on the side which is to be compressed, a cross-section like that shown in Fig. 93 being sometimes used. When such a beam, placed horizontal, is loaded in the middle, the broad under side is stretched less than the narrow upper side is compressed.

Fig. 93.
Cross-section of cast iron beam.

105. **"Fatigue" of Metals.**—The strain produced in a body by any application of force may depend upon the length of time, or the number of times, the force is applied. In Germany experiments have been made in which bars of metal have been loaded and relieved alternately by machinery day after day and week after week, until the number of repetitions has reached into millions. We are told, for instance, of a rod of iron that "was torn apart . . . after 10,100,000 tensions," and of another rod that was sound after 48,000,000 bendings. The general result of such experiments is to show that a bar of metal may be broken by a very long application, or by very many applications, of a force that applied once for a short time would do no apparent harm. This fact is recognized in the phrase the *Fatigue of Metals*.

Even crystals and pebbles are sometimes found which have become permanently distorted, without breaking, under great pressure long continued.

It is well known that cannon may be gradually weakened and finally burst by repeated discharges.

106. Characteristics of Solids.—The more closely we examine the characteristics of so-called *solids*, the less distinct appears the division between solids and liquids. Many substances ordinarily classed as solids will flow slowly under a slight pressure.[1] For instance, a stick of sealing-wax supported at its ends in a moderately warm room will gradually sag in the middle. On the other hand, all known liquids offer considerable resistance to rapid changes of form. Clerk Maxwell[2] showed that a certain thick liquid, Canada balsam, when it is inwardly disturbed by the movement of some body through it, exhibits properties resembling those of a strained solid. Much importance was attached to such experiments by Maxwell and by Lord Kelvin[3] in their discussions of the nature of the luminiferous ether (§ 300).

Questions and Problems on Chapter IX.

(1) Define elasticity and limit of elasticity. Illustrate your definitions.

(2) If No. 27[4] brass wire breaks with 15 lbs. pull, calculate the breaking-strength of No. 25 brass wire.

(3) If No. 27 spring-brass wire breaks with 15 lbs. pull, and No. 30 annealed iron wire with 5 lbs., find the relative tenacity of spring-brass and annealed iron.

(4) What pull would a wire of each kind, mentioned in Problem (3), 1 sq. mm. in area of cross-section sustain?

[1] The following story was related by one of the sons of the late Alvan Clark, Sr., the great telescope-maker: A file was carelessly laid on the top of a large mass of pitch. After a time, the weather being warm, it was noticed that the file was nearly swallowed up by the pitch, and to pull it out by main force seemed impracticable. The elder Clark, upon being appealed to, remarked, *It went in slowly, and it will have to come out slowly.* So a contrivance was rigged to exert a steady, moderate, pull upon the file, and in the course of a week it came out.

[2] A British physicist (1831–1879) best known for his development of the electromagnetic theory of light, though he did much other important work.

[3] Originally William Thomson, later Sir William Thomson, finally Lord Kelvin. Probably the greatest British physicist of his time (1824–1907).

[4] See table of wire-gauge numbers and diameters, Appendix IV. In these calculations it is to be assumed that wires of the same metal are of exactly the same quality and condition. In practice this is not always the case. A thin wire is likely to be stronger, relatively to its size, than a thick wire.

(5) If a force of 1 kgm. stretches 1 mm. a wire which is 1 m. long and 0.1 sq. mm. in cross-section, how great a force is required to stretch 5 mm. a wire of like material 10 m. long and 1 sq. mm. in cross-section?

In Problems (6–10) it is to be understood that the beams or rods are horizontal, supported at the ends and loaded at the middle.

(6) What is the ratio of the stiffness of a rod 50 cm. long to that of another rod 100 cm. long, but similar in all other respects?

(7) What is the ratio of the breaking-strength of the two rods in (7) under simple bending?

(8) If a beam 10 ft. long, 4 in. wide, and 6 in. thick, is bent 1 in. by a load of 500 lbs., how much would a beam 5 ft. long, 2 in. wide, and 3 in. thick, be bent by a load of 250 lbs.?

(9) If a certain beam 10 ft. long, 4 in. wide, and 6 in. thick, is bent 1 in. by a load of 500 lbs., how great a load would bend 1 in. a beam 20 ft. long, 8 in. wide, and 12 in. thick?

(10) Is the stiffness of two similar rods, laid one on the other, as great as if the two were really one?

(1) (*a*) If one end of a rod is firmly fastened, and certain forces applied to a cross-piece at the other end twist the rod 10°, how many degrees would it have been twisted by forces twice as great and similarly applied?

b) How many degrees would these latter forces twist a rod twice as long as this one, but similar in other respects?

(12) If a certain rod is twisted 80° by the application of a certain couple, how much could another rod of the same material and length but twice as thick and twice as wide be twisted by the same couple?

CHAPTER X.

FRICTION.

107. Introductory.—When we push a heavy block along on the top of a table we feel a certain resistance. We know from experience that by making the surface of the table and the surface of the block very smooth we can lessen the resistance. This resistance, the amount of which depends upon the condition of the rubbing surfaces, is called *friction.*

Friction always opposes motion, whatever may be the direction of the motion; that is, it merely tends to stop the motion. It never helps to push the block back to the position where it started.

Friction is one of the most important conditions in the construction and operation of very many mechanical appliances. It enters largely into the list of resistances to be overcome, as in the rolling friction of the car-wheels upon the track or of wagon-wheels upon common roads. Every axle revolves in its bearings with a measurable amount of friction, which can be diminished but not overcome by oiling the surfaces in contact. On the other hand, many machines and mechanical appliances would be valueless without friction. Upon this the efficiency of belting, of brakes, of nails and screws of every description, is dependent. The driving-wheels of engines or of electric street-cars, the feet of men or of horses, would be unable to produce or maintain locomotion without the aid of friction. If its operation were suspended, every river would become a cataract, soon running itself out.

We shall consider first the laws of sliding friction between solid bodies.

108. "Laws of Friction."—The so-called laws of sliding friction between solids are:

1*st. Friction is independent of the velocity*[1] *of one surface across the other, other things being equal.*

2*d. Friction is independent of the area of the rubbing surfaces, other things being equal.*

3*d. Friction is proportional to the total pressure of one surface against the other, other things being equal.*

The experimenter in Exercise 15 need not be surprised or disappointed if his observations do not agree exactly with these statements. In fact, the "laws" are not strictly true; but they are near enough to the truth to be of very great use.

109. Cofficient of Friction.—*If the total pressure between two surfaces, at right angles with each of them, is called P, and if the friction between the two surfaces is called F, the ratio* $F \div P$ *is called the coefficient of friction.*

There is a method of finding this coefficient without measuring either P or F. It makes use of an inclined plane and the parallelogram of forces.

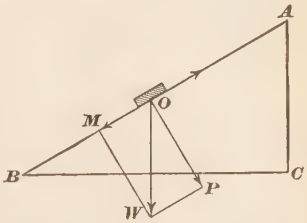


FIG. 94.—Illustrating coefficient of friction.

In Fig. 94, O is supposed to be a body resting upon the incline AB, which is just steep enough to keep O moving with uniform velocity, in spite of friction, if it is once started down the incline.

The line OW represents the weight of the body. This is

[1] It is, however, a matter of common knowledge that "static" friction, the friction which must be overcome in starting a body which has come to rest on another, is greater than the friction of continued motion.

equivalent (§81) to a force OP at right angles with the incline and a force OM down the incline. It is the force OP that causes the pressure to which the friction is supposed to be proportional. It is the force OM that maintains motion in spite of the friction.

If the body moves with *uniform* velocity down the incline, as we have supposed, the force OM must be exactly equal and opposite to the resistance of friction. For, if OM were greater than the friction, the body would move faster and faster down the incline; if OM were less than the friction, the body would move more and more slowly down the incline.[1] While the body is moving downward friction is represented by the arrow pointing from O toward A, equal and opposite to OM.

Therefore, in accordance with the definition given at the beginning of this article, we have:

$$\textit{coefficient of friction} = OM \div OP.$$

A comparison of the triangle OWM, in which $WM = OP$, with the triangle ABC shows that they are similar, and hence

$$OM : OP :: AC : BC.$$

That is, the

$$\textit{coefficient of friction} = AC \div BC.$$

The inclined-plane method of studying friction leads directly to the question whether there is a natural limit to the steepness of permanent slope of sand banks or hills of loose material. There is such a limit, depending of course upon the nature and condition of the material and the amount of jarring to which it is subject, and this limit is

[1] It requires force to set any body in motion, and it requires force to stop any body that is in motion. If a body is moving along in a straight line with uniform velocity we know that the various forces acting on it balance each other. See sections 55 and 118.

much lower than most people suppose it to be. A rise of 1 in 2 on a permanent slope that is not of rock is unusual.

The greatest angle of slope at which one body can rest upon another without slipping under the pull of gravity is called the *angle of repose.*

In the following Table[1] certain important coefficients of friction, f, are given approximately, together with the angles of repose, ϕ:

Materials	f	ϕ
Wood on wood, dry	0.25 to 0.50	14.0° to 26.5°
Metals " oak, "	0.50 to 0.60	26.5° to 31.0°
Leather " metals, "	0.56	29.5°
" " " wet	0.36	20.0°
" " " greasy	0.23	13.0°
Metals " " dry	0.15 to 0.20	8.5° to 11.5°
Earth " earth	0.25 to 1.00	14.0° to 45.0°

110. Rolling Friction.—The friction encountered by a moving body is usually much less when it is on wheels or rollers than when it slides, though it is true that, on snow, *runners* are better than wheels. The wheels of ordinary carriages do not get rid of sliding friction altogether, for the surfaces of the axle and the hub slide over each other. The "ball bearings" of bicycles do away with this sliding friction almost entirely.

The coefficient of friction of iron wheels rolling on iron rails may be as small as 0.002, so that a pull of 4 lbs. may keep in motion, on a level track, a carriage weighing 2000 lbs. The coefficient of sliding friction of smooth dry iron upon iron is perhaps 75 or 100 times as great as this.

[1] *Smithsonian Physical Tables*, 1903.

The question naturally arises whether, for carriages, large wheels offer more or less resistance than small wheels. The answer depends on circumstances. What is called rolling friction is mainly due to sliding friction at the axle and unevenness or softness of track. Axle friction, with an axle of given size, is less important with wheels of large radius; for, in going a given distance, a large wheel makes fewer revolutions on its axle than a small wheel. Moreover, a large wheel more easily surmounts an obstacle, as one can see by considering Figs. 95 and 96, and noting the greater

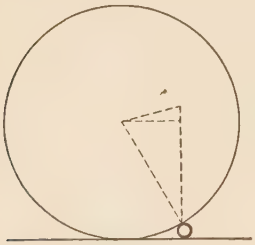


Fig. 95.—Obstacle under a large wheel.

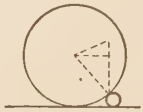


Fig. 96.—Obstacle under a small wheel.

steepness of the curve followed by the centre of the small wheel in getting over the obstruction.

On the other hand, large wheels are of course heavier, and, if the velocity is great, the greater air-resistance which they meet may be a serious argument against them.

The conclusion is, then, that, where roads are rough and the pace is slow and there are no special devices for reducing axle-friction, which devices would be rather out of place on a rough road, large wheels are preferable; but that, where the opposite conditions prevail, as in racing carriages on smooth tracks, small wheels are likely to be better. Witness the large wheels of ox-carts, where such vehicles are still to be seen, and the small wheels of racing sulkies, bicycles, and automobiles.

111. **Slipping of Wheels.**—In the very early days of the use of steam for drawing railway carriages the driving-wheels of the locomotive were made with cogs fitted to a cogged rail along the track. This device was found to be unnecessary for ordinary work, but it is used on very steep inclines running up the sides of mountains.

Even upon ordinary railroads, when there is a heavy train to be set in motion on an up grade, the driving-wheels sometimes slip and revolve, while the train refuses to start. Now, the frequency of the puffs from a locomotive depends upon the speed of revolution of the driving-wheels, and when an engine that has been puffing very slowly in starting a train suddenly gives three or four puffs in very quick succession, we may conclude that the driving-wheels are slipping on the rails. Engines are provided with sand-boxes, from which sand can be sprinkled upon the rails in front of the driving-wheels when slipping occurs.

112. **Friction Between Solids and Fluids.**—The laws of friction between solids and fluids are very different from those which hold between solids. Friction between solids and fluids changes comparatively little with change of pressure, but it changes a good deal with change of speed, being for many cases pretty nearly proportional to the square of the velocity. The resistance of the air is an important obstacle to very rapid motion, as in the case of a railroad train or an aeroplane at great speed, and the frictional resistance of the water to the hull and propeller of a steamer demands most of the steam-power required to propel the vessel. The air-resistance to a bullet-shaped projectile 12 inches in diameter moving 2800 ft. per second is estimated at 4000 lbs. Railroad trains are appreciably delayed by headwinds.

On the other hand, there is no static friction (§108) with fluids. A pull of 1 lb. would keep the greatest ocean liner

moving, very slowly of course, if there were nothing but the friction of still water and still air to oppose.

The friction which we are now considering depends also on the area of the surface of friction. When bodies like feathers or very fine particles of dust, which have a very large surface for their weight, are allowed to fall in still air, they gain no great velocity before the resistance of friction becomes practically equal to the pull of gravity, and thereafter they fall with practically unchanging velocity. Dust sent up by great volcanic eruptions is sometimes months in falling back to the earth. Liquid particles

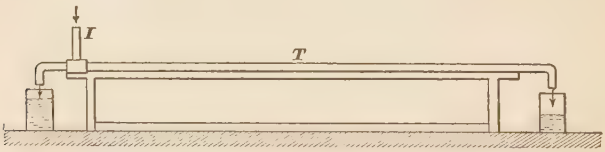


FIG. 97.—Friction of water in a tube.

in air act much like solid particles. In some famous experiments at the University of Cambridge the size of very small particles of water was estimated from the velocity of fall of the cloud which they made.

113. Friction in Tubes and Narrow Crevices.—The friction of liquids or gases flowing through long narrow tubes or crevices is very considerable.

(1) If water is run into the tube *T* (Fig. 97), at the inlet *I*, which is much nearer one end than the other, and if vessels of equal size are placed beneath the two outlets at the same instant, the vessel nearer *I* will be much more rapidly filled than the other.

(2) If air is forced into the tube *T* (Fig. 98), through the inlet *I*, which is much nearer one end than the other, a small wind-mill, *W* (Figs. 98 and 99), may be driven by the current of air from the short arm of the tube against the current from the other, and nearer, outlet.

(3) If a piece of ordinary window-glass is laid flat on a somewhat smaller piece 3 or 4 cm. square and is then suddenly lifted, the smal-

ler piece is raised also, but soon drops. The sudden lifting produces a partial vacuum between the two plates, and the air cannot work into the narrow crevice quickly enough to restore full pressure there before the lower plate is pushed up by the excess of air pressure beneath it.

Problems.

(1) A body weighing 20 lbs. rests upon a horizontal surface upon which its coefficient of friction is 0.2. How great is the force required to keep the body moving along the surface?

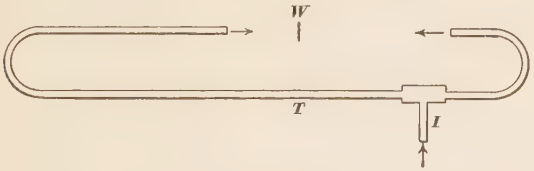


Fig. 98.—Friction of air in a tube.

(2) A sledge weighing 10 lbs. can be drawn along a certain level surface by a force of 0.25 lb. How great may we expect the force to be which will just maintain motion when a load of 50 lbs. is placed on the sledge?

(3) A sledge weighing 50 lbs., having runners 1 in. wide, is drawn along a level surface by a force of 15 lbs.

(*a*) How great is the coefficient of friction?

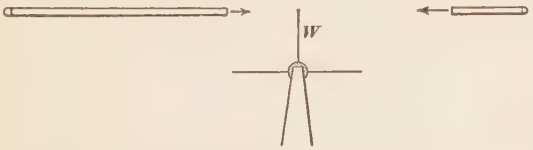


Fig. 99.—Another aspect of the apparatus of Fig. 98.

(*b*) How great would the force required to maintain motion be if the runners were 2 in. wide, the weight being unchanged?

(4) According to Raukine's *Civil Engineering* the coefficient of sliding friction of loose earth on earth may be as great as 1, though it is generally less. What angle must a slope of such earth make with the horizontal in order that the surface particles, if started, will continue to slide down with uniform velocity?

(5) A body weighing 5 lbs. rests upon an inclined plane 10 ft. long, with the height 6 ft. and the base 8 ft.

(*a*) How great is the pressure which the body's weight exerts normal to (perpendicular to) the inclined plane?

(*b*) How great a force parallel to the incline is required to prevent the body from sliding down when there is no friction? When the coefficient of friction is 1/4?

(6) If the force required to keep an ocean liner moving at the rate of 10 miles an hour is 1,000,000 lbs., how great, approximately, is the force required to keep the same vessel moving at the rate of 20 miles an hour? (See §112.)

CHAPTER XI.

MOTION, INERTIA, MASS.

114. Introductory.—It is well known that motion or change of motion is a very common result of the application of force to matter. We shall now begin a more careful study of the relation between matter, force, and motion than we have yet made. This study will require us to look sharply to the meaning of the words *motion* and *velocity*.

115. All Motion Relative.—It may seem at first sight that it would be a simple affair to define the real, or *absolute*, motion of a body; that is, to state just how it travels with reference to some fixed point. But there is no point which we know to be fixed. The earth and all the other members of the solar system, not excepting the sun itself, have very complicated motions of their own; and, since also the stars are probably drifting through space, it is quite impossible to determine the absolute motion of any object. The most that we can do is to describe its movement with respect to some chosen point which is, for the purpose we have in view, to be regarded as fixed. Thus we might describe the motion of a man along the deck of a ship, neglecting in our account the rolling of the ship from side to side, its pitching, and its general motion of translation along the water; neglecting, in fact, everything except the *difference* between the motion of the man and the motion of the point from which he started. This difference of motion is called *relative motion*. When nothing is said to the contrary, it will be understood that motions spoken of in this book are relative motions with respect to the earth regarded as fixed.

116. Composition of Motions.—As motions, like forces, have both magnitude and direction, they are, like forces, most conveniently described by means of lines, and accordingly they can be compounded or resolved as forces can be. Nearly all that was stated in Chapter VII, in regard to the parallelogram of forces, we might restate here, using the word *motion* instead of *force* and the words *rest* or *zero motion* instead of *equilibrium*.

An interesting and important class of recording instruments make use of this fact. Such are the self-recording barometer, the steam indicator, which traces the changes of steam pressure in the cylinder of an engine (Chapter XIX) during the stroke, and a number of instruments used in the study of sounding bodies. In all these pieces of apparatus a moving point is, sometimes by the aid of photography, made to record its motion upon a surface which is itself in motion. The manner in which the motion of the recording surface affects the trace left by the moving point is well shown by a simple phenomenon of frequent occurrence in every-day life. When rain falls on a comparatively windless day against the window of a railroad-car at rest, the drops trickle vertically down the surface of the pane. But when the car is in motion the line traversed becomes more and more oblique as the speed of the car increases. Fig. 100 contains a graphic record of the course of the drops for two different velocities of the car. The arrow shows the direction of the car's motion.

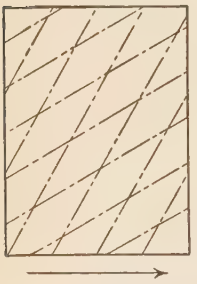


Fig. 100.—Resultant motion: course of rain drops.

117. Velocity.—*Velocity is rate of motion.*

When velocity is unchanging, it is measured by the number of units of space traversed in a unit of time. When

velocity is changing, it may be defined as equal, at any instant, to the distance the body would move in a unit of time if the rate of motion which it has at that instant were maintained unchanged. The choice of units depends somewhat upon the kind of motion and the purpose for which it is to be estimated or measured. The speed of railroad trains, of steamers, of pedestrians, and so on, is usually reckoned, in this country, in miles per hour, the speed of bullets or cannon-balls in feet per second, and the rate of transmission of electrical impulses along wires, or of light through air, in miles per second. For strictly scientific purposes, however, *centimeters per second* is the form of statement usually employed by physicists.

118. Change of Motion: Inertia; Newton's First Law of Motion.—We have now to study the behavior of objects which are neither at rest nor in uniform motion, but are changing from rest to motion, or from one state of motion

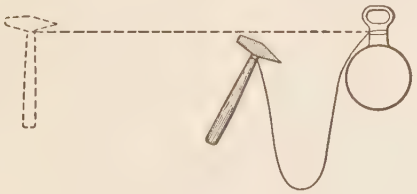


FIG. 101.—Illustration of inertia.

to a different state. In these cases we shall encounter a very important property of matter which we have almost totally disregarded thus far. Before we give a name to this property let us assure ourselves that it exists.

Suspend an iron ball, weighing not less than 10 lbs., from a firm support by a long, stout, cord, so that it may be free to swing at a slight push. Attach a thread to the ball, and with a gentle horizontal pull set it gradually in motion. Stop the ball, and again set it in motion by means of the thread, this time more suddenly.

Repeat, starting more and more suddenly each time, until the thread breaks.

Then take a string that will bear considerably more than the weight of the ball, and attach it to the latter. Tie the free end of the string securely to the handle of a good-sized hammer (see Fig. 101) held in the hand, and then, the string being long enough to give the arm free play, attempt to set the ball in horizontal motion with the greatest possible suddenness[1] by swinging the hammer. The string will probably be broken, while the ball will move but little.

The *hanging back* of the ball, which is so extremely obvious in these trials, cannot be accounted for by its weight; for weight, in its strict sense (§§9 and 10), is merely the earth's attraction for the ball, and this downward force cannot, save in some indirect way, oppose horizontal motion.[2] Nor is the behavior of the ball to be accounted for by the resistance of the air, nor by the action of any other opposing force applied to the ball by known outside agencies. We must conclude from the experiments described, and other similar ones in which the motion of bodies is arrested or changed in direction, that IT IS THE NATURE OF MATTER TO REQUIRE FORCE TO SET IT IN MOTION OR TO CHANGE ITS MOTION IN MAGNITUDE OR DIRECTION,[3] *little force if the change of motion is very gradual, greater and greater force as the suddenness of the change is increased.*

This is one of the most important facts that we know about matter, and it is convenient to have a single word which will call it to our minds. We therefore say that this behavior of matter manifests a certain property which we will call *inertia*. To say this, to give the name inertia

[1] Make sure that the handle of the hammer is strong and thd head firmly fastened to it.

[2] It is true that the ball begins to rise a little when it swings from its position of rest, but this rise is very slight at first, and that component of the earth's attraction which is parallel to the path of the ball is therefore very small.

[3] The words in small capitals are substantially equivalent to what is called Newton's *First Law of Motion.* See also § 55.

to a supposed property, does not *explain* the facts. It is merely a convenience, for it enables us to use the one chosen word, inertia, in place of phrases or sentences which would otherwise be required.

If we now suspend a much larger or much smaller iron weight than the one just used and repeat some of the experiments with it, we shall find that to start the larger weight with equal suddenness requires a greater force. This being the case, we naturally inquire whether we cannot use the property of inertia in comparing quantities of matter, in determining *how much* is the quantity of matter in one body compared with that of a second body, which process we call *measuring* the quantity of matter in the first body.

119. Quantity of Matter; Mass.—There are several characteristics belonging, so far as we know, to all kinds of matter,[1] which force themselves habitually upon our attention, and are of such a nature that we turn to them when we attempt to estimate the quantity of any aggregation of matter. These characteristics are:

matter occupies space;

matter attracts other matter;

matter requires the application of force to change its motion.

We shall call the three tests suggested by the properties just mentioned the *volume test*, the *weight test* and the *inertia test* respectively. Since they do not all agree, which one shall be selected and agreed upon as the best?

To a child the volume test is a natural one, perhaps the only one he can apply or imagine. As the individual grows older he observes that bodies may change in volume without addition or subtraction of substances and that in such cases the weight remains constant. He therefore comes to prefer the weight test to the volume test. Con-

[1] The luminiferous ether (Chapter XXVI) is a possible exception.

tinuing, he learns that a given body does not weigh the same, on a spring balance, at all parts of the earth's surface, and that in regions of space far from the earth, where nevertheless science has to deal with matter, the aspect of weight, if we regard it at all, is quite changed. He finds, however, that there is reason to believe that a given body, in whatever part of space it might be placed, would require the same force to give it the same velocity in the same time. He therefore in the end comes to regard the inertia test as more widely applicable than the weight test, and he now understands why physicists agree that two bodies which are equal in the inertia test shall be said to contain, or consist of, *equal quantities of matter.*

As an equivalent for the phrase *quantity of matter* the word *mass* is commonly used. *Equal masses,* then, are, *by definition,* quantities of matter which, *whatever their inequality in other respects,* are alike in this, that *they require equal forces to give them equal velocities in equal times.*

An example of the inertia test, or acceleration test, is given in Exercise 16; but, as experiment shows that bodies which are equal by the inertia test are equal by the weight test also, and as the latter is very much easier to apply, mass, though *defined* with reference to inertia, is practically measured by the weight test. There are many advantages in this practice, but there is one disadvantage. The disadvantage is that many people, some of whom write books and magazine articles, fail to distinguish between inertia and gravity, between mass, the quantity of matter in a body, and weight, the pull of the earth upon that body.

It is gravity that we have to do with when a bullet rests in the hand. It is inertia that is manifested when the bullet, fired from a pistol, forces its way through an obstruction without the help of gravity.

It is gravity that tends to draw the earth into the sun.

It is inertia that tends to keep the earth moving in a straight line past the sun. It is a balance between the two that keeps the earth moving around the sun, neither falling into it nor receding from it. (See § 129.)

CHAPTER XII.

UNITS FOR PHYSICAL MEASUREMENTS; MOMENTUM, ACCELERATION, ETC.

120. Fundamental Units.—We have in physics a great many kinds of quantities to be measured, distance, time, velocity, mass, force, work, energy, heat, electrical resistance, electromotive force, etc., and we need a well-defined unit for each.

Such quantities are not entirely independent of each other, but are so related that, if we choose units in which to measure any three of them, we can from these units *make up* units for all the others. It is common practice to start with the units of *length*, *mass*, and *time*, and call these the *fundamental*, or *primary*, units.

There is a wide range of choice as to the *size* of the fundamental units. We shall commonly take

for the unit of length the *centimeter*,
" " " " mass " *gram*,
" " " " time " *second*,

which gives us what is called the centimeter-gram-second, or C.G.S., system of units, the one most commonly used by physicists the world over. But evidently we might use a *foot-pound-second* or a *meter-kilogram-second*, or even a *foot-kilogram-hour*, system.

121. Derived, or Secondary, Units.—Units defined by use of the fundamental units are called *derived* units. For example, the unit of velocity is defined as a velocity of one *centimeter per second* or one *foot per second*, etc. Rate of delivery of water through pipes is defined as so many

cubic centimeters per second or so many *cubic feet per minute*, etc.

122. Units of Force.—All common units of force are derived units; but the most common ones, in every-day use, do not belong to any one of the three-unit systems mentioned or referred to in Article 120.

We have repeatedly measured forces in the Exercises of this book, using spring balances and reading in pounds or ounces, kilograms or grams. But these are precisely the names we use for our various units of *mass* (§ 119) [1] How does it happen that units of mass and units of force have the same names?

It is because the pound (force) is the pull of gravity upon a pound (mass), the gram (force) the pull of gravity upon a gram (mass), etc. These force units are *derived* units, of a kind. They are defined by reference to units of mass and to gravitation, and are therefore called *gravitation units* of force.

Such units are variable, for the pull of gravity upon a given mass is not equally great at all parts of the earth's surface. For this reason, and certain others, gravitation units of force are not satisfactory for all scientific purposes. Engineers, who have to give a great deal of attention to gravitation, find them convenient, using the larger units, pounds, kilograms, etc., and physicists are not likely to discard them altogether; in fact, nearly all instruments used in physical laboratories for the direct *measurement* of force are, in this country, marked off for gravitation units.

In the more refined and the more general uses of science, in discussions which are to hold good for regions where gravitation may be much weaker or much stronger than it

[1] Engineers in this country do not, as a rule, have the same names for units of force as for units of mass. For example, they call the pound a unit of weight but not a unit of mass. See § 123.

is upon the earth's surface, and particularly in cases of change of motion, it is convenient to use a force unit which is defined, without reference to gravitation, by means of the fundamental units of length, mass and time. Thus we have the

DYNE, *which is the force that, acting for* 1 *second upon mass of* 1 *gm., gives it a velocity*[1] *of* 1 *cm. per second.*

This is the C. G. S. (§ 120) unit of force, the favorite unit, for refined purposes, among physicists. So we have the

POUNDAL, *which is the force that, acting for one second upon a mass of* 1 *lb., gives it a velocity of* 1 *ft. per second.*

Such units as the dyne and the poundal, which are defined without reference to gravitation, are called *absolute* units of force. *Inertia* units, or *acceleration* units, would be a more expressive name for them.

The general definition of an absolute unit of force is this: *the unit force is that which, acting upon the unit mass for the unit time, gives to it the unit velocity.*

123. Momentum: Newton's Second Law of Motion; Acceleration.—Experiment shows that the following equation holds true:

$$f = \frac{m \times v}{t} \quad \ldots\ldots\ldots\ldots \quad (1),$$

where f is a force, expressed in dynes;

m " mass, " " grams;

t " time, " " seconds;

v " velocity, " " centimeters per second,

which *is given to* the mass m by the force f[2] in the time t.

[1] This does not mean that the gram will *move* 1 cm. during the 1 second for which the force is acting. It means that, if the gram is at rest when the force begins to act, it will gain such a velocity that, if the force ceases to act at the end of 1 second, the gram will move 1 cm. during the next second.

[2] The force f is the resultant of all the forces applied to the body, if there is more than one force applied.

The same equation would hold true with force expressed in poundals, distance in feet, mass in pounds, etc.

Definition.—The product $m \times v$ is called the *momentum* of the mass m with the velocity v.

Accordingly, equation (1) given above means that A FORCE IS MEASURED BY THE MOMENTUM IT CAN IMPART IN UNIT TIME. This is equivalent to Newton's *Second Law of Motion.*[1] (See § 118 for the First Law.)

If the force, f, were expressed in gravitation units, in grams or pounds, for example, the equation would not be so simple. It would then be

$$f \times g = \frac{m \times v}{t}, \text{ or } f = \frac{m \times v}{g \times t}, \quad \ldots\ldots \quad (2)$$

g being the number of dynes in the gram unit of force, about 980, or the number of poundals in the pound unit of force, about 32.2. See § 126.

Definition.—The ratio $v \div t$, the rate of change of velocity, is called the *acceleration.* Velocity being expressed in *centimeters per second* or in *feet per second,* acceleration is expressed in *centimeters per second per second* or in *feet per second per second.*

Evidently equation (1) is equivalent to

force (in absolute units) = mass × acceleration, . . . (1)′

if we call the number of grams or pounds, etc., in a body its mass, as most physicists, chemists, etc., do.

In order to write equation (2) in equally simple form, as

force (in gravitation units) = mass × acceleration, . . (2)′

we must agree to call $m \div g$ the mass of a body which contains, or consists of, m grains or pounds, etc. This is what many engineers[2] do. Such engineers, if they use English, take as their *unit of mass* the amount of matter in 32.2 lbs., instead of the amount in 1 lb.

[1] As stated and explained by Maxwell in his little book on *Matter and Motion.*

[2] They would probably use w for the number of grams or pounds, etc., in a body and m for $w \div g$.

We shall in this book follow the practice of the physicist rather than the practice of the engineer in this respect.

Problems.

(*In each of these problems the force mentioned is supposed to be the only one acting.*)

(1) The pull of gravity upon a certain stone, the mass of which is 1 gm., is 980 dynes. How fast will this stone be moving at the end of its first second of falling? at the end of its second second? its third? How great is the acceleration? (Gravitation pulls steadily upon a body with the same force whether the body be falling, rising, or at rest.)

(2) The pull of gravity upon a 5-gm. mass being 4900 dynes, what velocity will this mass acquire in the first second of its fall? in the first two seconds? the first three seconds? How great is the acceleration?

(3) The pull of gravity upon a mass of 1 lb. is about 32.2 poundals. If this be taken as the exact force, what velocity in feet per second will the body have at the end of the first second of its fall? at the end of the second second? the third second? the tenth? How great is the acceleration?

(4) What force in poundals would give to a 10-lb. mass in 5 seconds a final velocity of 20 ft. per second?

(5) The momentum of a certain mass, reckoned in the C.G.S. system, is 400. If this was given by a force of 80 dynes, how long a time was required?

(6) What velocity would a 1-gm. force give to a 1-gm. body in 1 second? in 2 seconds? in 5 seconds?

(7) What velocity would a 10-gm. force give to a 100-gm. body in 1 second? in 2 seconds? in 5 seconds?

(8) What velocity would a 1-lb. force give to a 1-lb. body in 1 second? in 2 seconds? in 5 seconds?

(9) What velocity would a 10-lb. force give to a 100-lb. body in 1 second? in 2 seconds? in 5 seconds?

124. Resistance: Loss of Momentum.—A resistance, friction for instance, is merely an opposing force; it tends to destroy motion. The equation

$$f = \frac{m \times v}{t}$$

holds as well for opposing forces as for accelerating forces, but, when the force is an opposing one, $m \times v$ stands for the momentum *lost* by the moving body in the time t.

Gravitation acts as an accelerating force upon a falling body and as an opposing, or retarding, force upon a rising body. A stone, if the resistance of the air may be neglected, gains as much momentum per second in falling as it loses per second in rising.

Problems.

(*In these and all similar problems in this book the resistance of the air is to be neglected unless expressly mentioned.*)

(1) A stone weighing 5 gm. is thrown upward with a velocity, at the start, of 2000 cm. per second.

(*a*) How much momentum will it lose in the first second of its rise?

(*b*) What will be its velocity upward at the end of this second?

(*c*) How many seconds will it continue to rise?

(2) If a 10-lb. body, starting with a velocity of 100 ft. per second, is constantly opposed by a force of 50 poundals, how many seconds will it continue in motion?

(3) If a 10-lb. body, starting with a velocity of 1000 ft. per second, is opposed by a constant force of 50 lbs., how many seconds will it continue in motion?

125. Relation Between Time, Distance, etc., in Cases of Uniform Acceleration.—We have already had a number of problems in which the acceleration (§ 123) is assumed to be uniform. This assumption is usually made in book problems on bodies falling or rising vertically, without help or hindrance except from gravitation and air resistance, though in fact air resistance does seriously interfere and prevent uniform acceleration in the case of all bodies moving very fast and in the case of light bodies moving slowly.

A number of important relations, important for one who

is to study accelerated or retarded motions, follow directly from the *definition* of uniform acceleration, which is

$$\alpha \text{ (\textit{the acceleration})} = v \div t = \textit{a constant quantity},$$

during the change of v and t, it being understood, as in §§ 123 and 124, that v is the velocity gained (or lost) in t seconds.

We will write this equation in the form

$$V = \alpha t. \quad \text{(A)}$$

The velocity being zero at the beginning of the t seconds and v at the end, and the rate of increase being uniform, the *average*, or *mean*, velocity during this time is evidently $v \div 2$, that is, $(\alpha t \div 2)$, and accordingly the distance, s, travelled by the body in the t seconds is

$$s = \tfrac{1}{2}\alpha t \times t = \tfrac{1}{2}\alpha t^2. \quad \text{(B)}$$

Eliminating t algebraically from this equation by means of equation A, we get

$$v^2 = 2\alpha s, \quad \text{(C)}$$

an equation which is frequently useful.

If the body, instead of having zero velocity at the beginning of the t seconds, has at that time a velocity v, in the direction of the acceleration, the first two equations will be as follows:

$$v = v_1 + \alpha t, \quad \text{(A')}$$

$$s = v_1 t + \tfrac{1}{2}\alpha t^2. \quad \text{(B')}$$

That is, the acceleration *adds* the same amount, αt, to the velocity, and the same amount, $\frac{1}{2}\alpha t^2$, to the distance traveled, in t seconds, whatever the initial velocity may be.

If the acceleration is in the opposite direction to v_1, as in the case of a body projected upward against the pull of gravity, we get

$$v = v_1 - \alpha t. \quad \text{(A'')}$$

and

$$s = v_1 t - \tfrac{1}{2}\alpha t^2. \quad \text{(B'')}$$

126. The Value of g, the Gravity Acceleration.—In gravitation cases of uniform acceleration, with unobstructed rise or fall in a vertical line, all the formulas of § 125 hold, but it is customary to use the letter g instead of α to indicate this acceleration.

We have now to ask how the numerical value of g has been found by experiment. The theory of this operation is very simple. We have merely to take one of the formulas (A), (B), or (C), each of which contains g (or α), find by experiment the numerical values of the other letters in it for any case of a free-falling body, put these values into the formula in question and so find the value of g. The best formula to take is (B), for each of the others contains v, and velocity, when it is rapidly changing, is a hard quantity to measure directly.

In using (B) we must measure both s and t in some actual case. There are various ways of doing this, some of which make use of electrical devices for noting short intervals of time. The apparatus of Exercise 17, which is non-electrical and comparatively simple, cannot be expected to give a very accurate result.

In the method actually used by those whose business it is to measure g with the greatest possible accuracy, no body is actually dropped, but the time of swing of a pendulum of known length is very accurately taken, and from the length and time of swing of the pendulum, which are related to the value of g by a simple formula (§ 129) the value of g is found.

This value, expressed in centimeters and seconds, is about 980, varying from about 978 at the earth's equator to about 983 at the poles. That is, a body falling freely in a vacuum, in medium latitudes and near the sea-level, will gain during each second of its fall a velocity of about 980 cm., about 32.2 ft., per second.

Problems.

(*In the following problems assume g to be* 980 *cm. per second per second or* 32 *ft. per second per second.*)

(1) A body is started downward with a velocity of 500 cm. per second and falls freely. How fast will it be moving at the end of 1 second? 2 seconds? 5 seconds?

(2) If the initial velocity were 2000 cm. per second upward, what would be the velocity and direction at the end of 1 second? 2 seconds? 5 seconds?

(3) If the initial velocity were 50 ft. per second downward, what would be the velocity, in feet per second, at the end of 1 second? 2 seconds? 5 seconds?

(4) If the initial velocity were 100 ft. per second upward, what would be the velocity and direction at the end of 1 second? 2 seconds? 5 seconds?

(5) A falling body has acquired a velocity of 3000 cm. per second.

(*a*) How long has it been falling?

(*b*) If it were started upward with the same velocity, how long would it continue to rise?

(6) How long must a body that starts downward with a velocity of 60 ft. per second fall in order to have a velocity of 100 ft. per second?

(7) A body starts upward with a velocity of 100 ft. per second.

(*a*) How long after this start will it have a velocity of 50 ft. per second upward?

(*b*) How long after the start will it have a velocity of 50 ft. per second downward?

(8) In Problem (1) what would be the mean velocity during the first second? during the first 2 seconds? during the first 5 seconds?

(9) In Problem (3) what would be the mean velocity during the first second? during the second second? during the fifth second?

(10) A body falls freely from rest. How many cm. does it fall during the first second? during the first 2 seconds? the first 3 seconds? the first 4 seconds?

(11) How many cm. does this body fall during the second second? the third second? the fourth second?

(12) How many feet does the body fall during the first second? the first 2 seconds? the first 3 seconds? the first 4 seconds?

(13) How many feet does the body fall during the second second? the third second? the fourth second?

(14) A bullet is shot upward with a velocity of 300 m. per second.

How far will it rise in the first second? in the first 2 seconds? How far will it rise in all? How many seconds after starting upward will it reach the starting-point again?

(15) A falling stone has acquired a velocity of 15 m. per second. From what height has it fallen?

(16) A boy throws a stone straight upward 100 ft. With what velocity does the stone leave his hand?

(17) A moving mass of 30 gm. is opposed by a force of 150 dynes. How great is the (negative) acceleration?

(18) If the initial velocity of this mass is 200 cm. per second,

(*a*) How long will it move against the given resistance before coming to rest?

(*b*) How far will it move before coming to rest?

127. Force Required to Change the Direction of Motion: "Centrifugal Force."—In many cases change of motion affects, not its magnitude, but its direction. Even this change requires the application of force. If a person revolves a ball, fastened to the end of a string, with uniform velocity in a horizontal circle about his head, he is conscious of exerting a continual pull in order to prevent the ball from getting away. The tendency of the ball to escape from its circular path, which tendency makes necessary the retaining pull, is usually called *centrifugal force.*

It must not be supposed that centrifugal force means a tendency to fly straight away from the centre. Experiment and observation will show that, if the weight is at any time released, it starts off in a straight line which is a "tangent" to the circular path at the point where the release occurs. It is this tendency of bodies in motion, to move on in a straight line, that prevents all the planets of our solar system from falling into the sun. The sun's attraction for them is like the pull between the hand and the revolving weight.[1]

[1] We say this now with great confidence, although two hundred and fifty years ago very different theories in regard to the motions of the planets were held. Sir Isaac Newton proved about 1680 that the earth's attraction, supposed to decrease in

It was no mere accident that made equilibrium between gravitation and the centrifugal tendency in our solar system. If, for instance, the sun's attraction for the earth were suddenly doubled, the earth would not in consequence be drawn into the sun. It would approach nearer to the sun, and in so doing would be subject to a stronger and stronger attraction, but on the other hand it would revolve faster and faster around the sun, and in a smaller circle, which would make the centrifugal tendency greater. The centrifugal tendency would increase more rapidly than the attraction, and at last would equal or even exceed it.

If, on the other hand, the sun's attraction became smaller than it now is, the opposite thing would occur. The earth would move somewhat farther away from the sun, losing velocity meanwhile, till the centrifugal force became as small as the attraction.

In fact, this process of self-adjustment is continually going on in the solar system. The earth, for example, is slightly nearer to the sun at one time in the year than at other times, the "orbit" in which it moves around the sun being not quite circular but slightly elliptical.

Certain comets, among them the one (Halley's) which was visible to us in the year 1909-10, travel in very long and comparatively narrow ellipses and are very much nearer the sun at certain times than at others. As they come near the sun, from a vast distance, they get so great a velocity that, not being aimed directly toward it, they

proportion as the square of the distance between its centre and that of the attracted body increases, is just sufficient to hold the moon in its orbit, if the moon acts like ordinary matter with which we are familiar. He showed, too, that the same natural supposition, a mutual attraction inversely proportional to the square of the distance between the attracting bodies, would account for the retention of the various planets in their orbits about the sun, provided these planets act like ordinary bodies upon our earth. We accept Newton's theories because of their simplicity—they do not call into action among the stars properties of matter unknown to us upon the earth—and because of their completeness—they account for the magnitude of the effects observed as well as for their general nature.

shoot past it, make a partial turn around it, and then shoot off again, to be out of our sight many years before their next return.

The fact that the polar diameter of the earth is somewhat shorter, about 13 miles, than the equatorial diameter is accounted for by the centrifugal action, which has stretched the globe somewhat away from the polar axis on which it turns.

(1) Half fill the small brass bucket (No. 11) with water and then, holding it by the wire handle, whirl it in a vertical circle at arm's length.

(2) Fill an ordinary set wash-basin, having a stopper at the bottom, with water, give this water a vigorous rotary motion and then pull out the stopper. Note the behavior of the water over the vent, and the number of seconds required to empty the basin.

Repeat the experiment, filling the basin as before, but allowing the water to come to rest, as shown by the stillness of particles floating upon it, before pulling out the stopper.

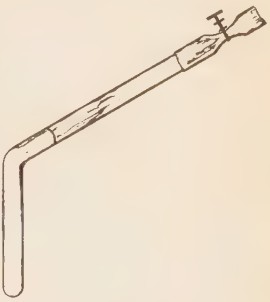


Fig. 102.—Illustration of centrifugal action.

(3) Fill the lower part of a glass tube (No. XLI, Fig. 102), with mercury to a point somewhat beyond the bend, and the upper part with water. Then whirl the tube rapidly around its vertical part and note the behavior of the two liquids.

When a liquid is to be separated from solid particles with which it is mixed, as molasses from the crystals in unrefined (brown) sugar, or oil from iron chips after certain machine-work processes, or even water from the fibres of coarse cloth, the operation can be hastened by putting the mixture into a rotating strainer, which throws out the liquid sidewise by centrifugal action, the solid being retained.

Experiment 3 above shows that it is possible by the use

of centrifugal action to separate two liquids of unequal density, provided they are merely mixed in an emulsion (§ 52) and not really dissolved in each other. In this way cream is removed from fresh milk in a few minutes, whereas the old-fashioned way, of waiting for the cream to rise to the top of the milk by mere difference of gravity, required many hours.

128. Combination of Horizontal and Vertical Motion.— The student may have some doubt as to the exact action of gravity upon a body having horizontal motion. It is probably a common belief that initial horizontal motion makes a body slower to fall than if it were dropped from

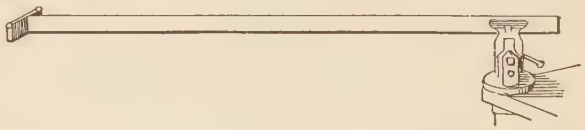


FIG. 103.—Apparatus for release of marbles.

rest. This is true, if the horizontal movement is great enough to bring into consideration the curvature of the earth. A very great horizontal velocity might carry a body completely around the earth without contact, as in fact the moon goes; but with ordinary velocities, up to some hundreds of feet per second, we may ignore the earth's curvature.

Fix apparatus No. XLII (Fig. 103) in a horizontal position several feet above a level floor. Put the two marbles in place in their notches, then pull the free end of the long bar quickly to one side. One of the marbles is shot out horizontally with considerable velocity. The other is merely deprived of its support and falls nearly straight down. Which strikes the floor first?

Problems.

If a rifle ball were fired at an inclination of 45°, with a velocity the vertical component of which is 640 ft. per second, and if the air resistance were zero, take $g=32$.

(*a*) How long would it be in reaching its greatest height?

(*b*) What would this height be?

(*c*) How far would it travel horizontally before striking the ground, assumed to be level and at the height of the rifle muzzle?

(In fact, the answers which are found for these questions when air resistance is ignored are very different from those which would be found by experiment in air. See § 112.

129. Relation of Pendulum Motion to g.—The motion of the pendulum is, of course, due primarily to the earth's attraction. If the earth's attraction for a given mass were greater than it is, the time of vibration of a given pendulum would be less than it now is. In fact, the time of vibration of a given pendulum is different at different parts of our earth, the earth's attraction for the matter of the pendulum being different in different regions, while the mass (§ 119) of the pendulum is everywhere the same.

We need to distinguish two classes, *simple pendulums* and *compound pendulums.*

A small, heavy, ball suspended by a long, light, thread is approximately a *simple* pendulum. The whole mass can be regarded as concentrated at the centre of the ball, and the *length* of the pendulum is taken as the distance from the point of support of the thread to the centre of the ball.

Pendulums, like those used in clocks, in which the weight of the suspending part cannot be neglected in comparison with the weight of the ball, or *bob,* are called *compound.*

In the case of the simple pendulum the relation between the time of a single swing, t, expressed in seconds, the length, l, and the acceleration of gravity, g, is

$$t = \pi\sqrt{\frac{l}{g}},$$

π being the familiar 3.14. . . .

The proof of this formula as a whole is too difficult to be attempted in this book, though in one particular, its assertion that *t is proportional to the square root of l*, it is easily tested by experiment. According to this statement, a pendulum 36 in. long should swing once while a pendulum 9 in. long swings twice, or while one 4 in. long swings three times. The trial is best made with two pendulums started at the same instant.

It is to be noted that the formula in question makes no mention of the length, the *amplitude*, as it is called, of the swing. In fact, the length of swing can be varied considerably without making any great difference in the time of swing, t, a property of the pendulum which is important in its use as a regulator of clocks, etc. (see § 130); but experiment with two equally long pendulums, one of which is swinging in a very short arc while the other swings in a much longer one, will speedily show that the longer swing takes a somewhat longer time. The formula is meant to hold for an exceedingly short arc.

The formula makes no mention of the mass of the pendulum, which seems to imply that a heavy pendulum, with a lead ball for example, would have the same time of vibration as one of the same length with a light ball, such as a clay marble. Experiment with two such pendulums swinging at the same time shows that this is nearly, if not quite, true.

The value of π being accurately known, it is evident that, if we can find t and l for a given simple pendulum, we can calculate g. Now both t and l are easy to measure, and therefore observations with the pendulum are the best means of finding g.

The measurement of g at many different parts of the earth's surface is of much interest, on account of the information which the variation of g from place to place gives

in regard to the shape and interior condition of the earth. Accordingly, various governments have employed skilled observers to make pendulum observations, and these observers have traveled far and wide over the earth in order that the same pendulum might be studied by the same person in widely different regions. The value of g at the poles is about one part in two hundred greater than its value at the equator, a fact which is accounted for partly by the shortness of the polar diameter of the earth as compared with the equatorial diameter, and partly by the slight tendency of bodies to fly off at a tangent, parting company with the rotating earth (see § 127), which tendency is greatest at the equator.

In such work as that just mentioned it is customary to use, not a simple pendulum, but a compound pendulum of peculiar shape. This is called a reversible pendulum, from the fact that it is provided with two cross-bars, or *axes of suspension*, one for supporting it with one end down, the other for supporting it with the other end down, the time of vibration being the same on each of the two axes. The time of vibration of such a reversible pendulum is the same as that of a simple pendulum having a length equal to the distance between the two axes of the reversible pendulum.

130. Use of Pendulum in Clocks.—In the works of an old-fashioned clock the motive power is a heavy weight working upon a kind of pulley by means of a long cord, but the device which governs the speed of the works and allows the motion to be neither too fast nor too slow is the pendulum. As a crowd of men at a turnstile, however they may try to force their way, can pass no faster than the swinging turnstile permits, so the clock-weight, which, if the control were removed, would run down at once with a furious buzzing of the wheels, is allowed by the pendulum

to descend only very slowly, a very little distance at every swing of the pendulum, and not at all when the pendulum does not move.

131. **Springs in Place of Pendulums.**—Many clocks and all watches are controlled by means of vibrating *springs;* for these, like pendulums, are very regular in their swings and so are good timekeepers. The controlling springs (see the "balance" of a watch, Fig. 123) must not be confused with the much larger *driving*-springs, or "*main*-springs," which are used in watches and in most clocks of the present day.

Problems.

(1) If one pendulum is 9 in. long and another is 64 in. long, what ratio will the time of vibration of the first bear to that of the second?

(2) If pendulum A, 39 in. long, vibrates once in a second and pendulum B vibrates once in 5 seconds, what is the length of B?

CHAPTER XIII.

ACTION AND REACTION.

132. All Force a Mutual Action.—We have discussed at some length the effect of a force applied to a body, in changing the direction or magnitude of the body's motion. But a force always requires the interaction of two bodies, and the two bodies are always affected in opposite directions by this interaction. Illustrations of such mutual action are common in the student's experience. Two sticks struck together will indent each other. A bat stops a ball and turns it back, but the ball stops, or at least checks, the bat. A weight pushes down upon a hand which pushes it up.

It is true that in many cases the effect is visible in one direction only. A pebble thrown against a massive stone wall may appear to make no impression upon it. But careful observations would probably detect in the wall a slight jar at the instant of collision.

The law, *action is equal to reaction,* is verbally so easy that many people have it on their tongues without being able to make any accurate use of it and without accepting its consequences. For example, imagine two men, A and B, pushing against each other, and so balanced that neither is gaining any advantage. It is easy to see that in this case A pushes against B just as hard as B pushes against A. But let A be gaining an advantage, so that B is pushed backward, as in Fig. 104. Is B now pushing against A as hard as A is pushing against B? Perhaps nine persons out of ten would answer no, although in so doing they would

virtually deny the truth of the law, reaction is equal to action.

The case stands thus: At the left-hand wall there is a force, F_{aw}, exerted by A against the wall and an equal force, F_{wa}, exerted by the wall against A. At the right-hand wall there is a force, F_{bw}, exerted by B against the wall and an equal force, F_{wb}, exerted by the wall against B. At the point of contact of A and B there are equal forces in opposite directions. We have, then, two forces, F_{wa} and F_{ba},

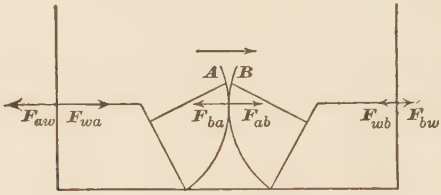


Fig. 104.—Illustrating action and reaction.

applied to A in opposite directions. If F_{wa} is greater than F_{ba} and F_{ab} greater than F_{wb}, as we suppose, A and B will move toward the right.

We do not say that A pushes no harder than B in this case, we say that A pushes no harder *against* B than B pushes *against* A. The push of A *against his wall* is greater than the push of B *against his wall*. The difference of the two forces F_{wa} and F_{wb}, applied to the two bodies as a whole from without, is used in giving momentum to one or both of the bodies, provided there is no friction or other opposing force.

133. Conservation of Momentum.—If we have two bodies, A and B, acting on each other, with no other forces coming into consideration than the two equal and opposite ones which the two bodies exert on each other, we must, according to §§ 123 and 124, have equal and opposite

changes of momentum occurring in the two bodies respectively. If both A and B are moving in the same direction, in the line of the forces acting between them, one will be gaining velocity and momentum while the other will be losing velocity and momentum. If both are, for the instant, at rest, one will be gaining velocity and momentum in one direction while the other is gaining velocity and momentum in the opposite direction.

We can shorten our statement of what happens, if we agree to call velocity and momentum in one direction positive while we call velocity and momentum in the other direction negative. We can then put our statement of what the law, *action equals reaction*, means, into the form, *the total change of momentum, that is, the algebraic sum of the positive and negative changes of momentum, in any interaction between two bodies, is zero.* An equivalent statement is, *the total momentum, the algebraic sum of the momenta, of any two bodies is unchanged by any interaction between these bodies.*

For example, if a mass of 20 gm., moving north at the rate of 50 cm. per second, which we will call a + velocity, should strike a mass of 15 gm. moving south at the rate of 40 cm. per second, which we will call a − velocity, and if the change of momentum suffered by each were 900, we should have

Momentum before			*Momentum after*
$(20\times50)-(15\times40)=[(20\times50)-900]+[-(15\times40)+900]$,			
or 1000 − 600	=	100	+ 300.

This must be true, if the law *action equals reaction* is true. We believe this law is true, partly because it seems *reasonable* to us, as soon as we understand what it means, and partly because no case is known in which accurate experiment or observation has shown it to fail, though there are

many cases in which inaccurate experiment or observation makes it appear to fail.

This is not an easy matter to test by experiment, for it is hard to find directly just how great the velocity of a body is at any given instant, or just how great its change of velocity is between two given instants. Accordingly, Exercise 18 is intended rather to help the student realize what the law in question means than to prove the law.

134. Newton's Three Laws of Motion.—The law discussed in § 133 is usually called Newton's *Third Law of Motion.* The other two have been previously given, the First in § 55, the Second in § 123. It is well to bring these all together as follows:

First Law.—*A body at rest continues at rest, and a body in motion continues in motion with uniform velocity in a straight line, unless it is acted upon by some external force.*

Second Law.—*The rate of change of momentum of a body is proportional to the magnitude of the net, or resultant, force applied to it.*

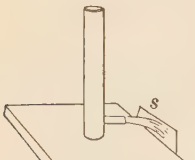


Fig. 105.—Action and reaction.

Third Law.—*Action is equal to reaction and opposite in direction;* that is, *the algebraic total momentum of any two bodies is not changed by any interaction that may take place between them.*

135. Illustrations of Action and Reaction.—Good illustrations of action and reaction are familiar, but are not always recognized as such.

Remove the plate *S* (Fig. 105) from No. XLV, close the tube at the bottom, fill the cylinder with water, float the whole in a tank of water, and then open the tube. The horizontal pressure within the water forces a horizontal stream from the tube, and the cylinder itself is driven in the opposite direction.

Repeat, with *S* in place.

This experiment is a type of many others that might be

shown, the soaring of a skyrocket, for example. The revolving lawn-sprinkler is also a case in point.

A bent tube through which water is flowing, as in Fig. 106, tends to straighten. The water entering at N, with a northerly velocity let us say, is, by the pressure P', exerted upon it by the tube at the angle, made to emerge with a westerly velocity; but at the same time the water reacts upon the tube with a force represented by P, tending to straighten it.

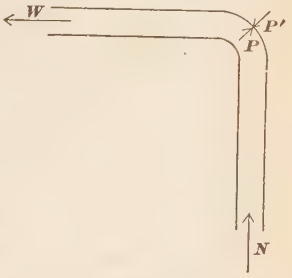


Fig. 106.—Action and reaction at bend of stream.

A swimmer pushes himself upward and forward by pushing the water downward and backward, and what water is to the swimmer air is to a flying bird. An air-tight box in which a bird is sustaining itself on its wings would press as heavily upon its support as if the bird were at rest upon the bottom; for a stream of air driven downward by the wings would press upon the floor.

An athlete can cover about a foot more in the "standing broad jump" by holding in each hand a stone of suitable size, which he flings swiftly behind him when in midair.

A child soon learns that he cannot move a sled, even upon the smoothest ice, by sitting upon the sled and tugging at his own rope. It is true that by standing upon the sled, drawing his body back upon one foot and projecting it forward upon the other foot, he can move the sled forward. The explanation of this latter case is, that there is some friction between the runners and the ice. This friction is enough to prevent the sled from moving backward while the body is slowly acquiring its forward

momentum but is not sufficient to resist the shock when the body is suddenly arrested by the forward foot.

When a charge of dynamite explodes in the open it leaves a great hole in the ground beneath the spot where it lay, and some observers seeing this permanent effect, after the tremendous temporary effect has vanished from the atmosphere, conclude that dynamite "spends its force downward." But dynamite has no such eccentric habit. It presses, like every other explosive, in all directions. The hole in the ground testifies merely to the suddenness and violence of the explosion, which are so great that the repelled air cannot give way swiftly enough to save the earth below from injury.

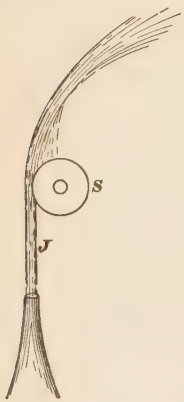


FIG. 107.—Action and reaction due to adhesion.

If a spool, *S* (Fig. 107), carried on a flexible horizontal support, is made to touch one side of a slender vertical jet of water, *J*, adhesion of the spool to the water deflects the stream, making it turn partly around the spool. The reaction for this action is a pulling of the spool toward and into the stream, so that it is presently hit on its under side by the rising water and is accordingly lifted. This phenomenon suggests an explanation of the fact that a small ball of cork or wood may be supported for a considerable time, perhaps many minutes, in such a jet of water as that just described, without falling out at the side.

Problems.

(1) A bullet weighing 10 gm. and moving 200 m. per second strikes horizontally into a suspended block of wood weighing 500 gm. and lodges there.

(*a*) How great is the momentum of the bullet before striking?

(*b*) How great is the momentum of bullet and block together just after the collision?

(*c*) How great is the velocity of the block just after the collision?

(The velocity of bullets and cannon-balls was formerly estimated by means of experiments and calculations like those here indicated.)

(2) If a bullet weighing 5 gm. is shot from a 4000-gm. rifle with a velocity of 300 m. per second, how great is the backward velocity acquired by the rifle itself during the discharge, if it hangs free?

(3) (*a*) If the discharge in the preceding problem lasted 0.005 second, how many dynes would be the average force exerted upon the bullet during that time?

(*b*) How many grams-weight would this force equal?

(4) A foot-ball crowd of 20,000 people, weighing 3,000,000 lbs., springs to its feet. If the average acceleration of the whole mass at the start is 5 ft. per second per second (§ 123), how great in lbs. is the equivalent extra load upon the structure which supports the crowd?

CHAPTER XIV.

WORK AND ENERGY.

136. Work: Units of Work.—The next physical quantity to be considered is *work*, which has been spoken of in §§ 71 and 88. The word *work* is used by the physicist to signify *the doing of something against opposition.* If a man pushes a saw through a board against friction and cohesion of the particles of wood, or if he raises a weight against the force of gravity, he does work. If the saw sticks, so that he pushes in vain, if after lifting the weight a certain distance he encounters some obstacle and ceases to raise it, he is no longer doing work, although he may be making more effort than before.

A statement like this frequently arouses a feeling akin to indignation in the mind of the student, who is apt to feel that scant justice is done to the supposed toiler. But the science of physics does not concern itself with emotions and purposes. Looking to the result accomplished, it says that a man who is merely sustaining a weight is doing no more for the weight than a post could do. He is not prevailing over the opposition of gravitation. He is not changing things. He is not doing work in the scientific sense of the word.

Measurement of work, then, always involves both force and distance. When the force is *parallel* to the motion maintained or assisted by it, the *work done by the agent exerting the force is reckoned by multiplying the number expressing the force by the number expressing the distance which the point acted on by the force moves during the action of the force.*

Certain units of work, the foot-pound and the gram-centimeter, for example, have been defined in § 71, and others will be defined in § 140.

137. Work Done by Forces Oblique to the Line of Mo-

tion.—When the force applied to any point is not parallel to the motion of that point, the calculation of the work done is not quite so simple.

For instance, suppose that a street-car moves 10 ft. along its track while a man pushes against the car, exactly at right angles with the track, with a force of 50 lbs. His effort does not help or hinder the motion of the car, save, possibly, by changing a little the friction against the sides of the rails. He does no *work* upon the car.

If the line of the force exerted by the man makes an acute angle with the direction of motion of the car, we

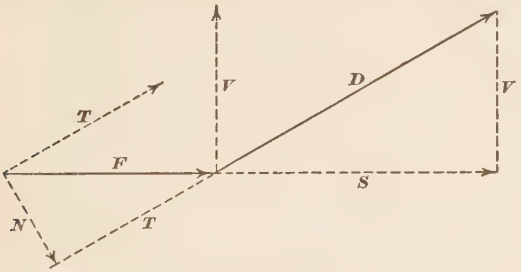


Fig. 108.—Illustrating the definition of work.

can resolve this force by means of the parallelogram of forces (Chapter VII.) into two components, a useful component parallel to the direction of motion of the car and a useless one at right angles with this direction.

Observe that, in such a case, *the product of the whole distance by that component of the force which is parallel to the motion* is equal to *the product of the whole force by that component of the distance which is parallel to the force. Either of these products shows the amount of work done.* Thus in Fig. 108, where the whole force is represented by F and the whole distance by D, we see from the similarity of the two triangles that $T:F::S:D$, whence

$T \times D = F \times S =$ *work done* by *the force F during the motion D.*

138. Rate of Work : Horse-Power, etc.—To express the *rate* at which work is done by a machine or an animal we have to tell the amount of work done in a certain time; for example, the number of foot-pounds done per second or per minute.

The term *one horse-power*, used to indicate a rate of work, is equivalent, in English-speaking countries, to 550 ft.-lbs. per second, or 33,000 ft.-lbs per minute. This rate of work is perhaps somewhat greater than that of an ordinary draught horse moving at his average pace and exerting his average pull. It is 76.04 kgm.-m. per second.

The following table, taken from one given by the Scotch engineering authority Rankine, compares the rate of work of a man and of a horse under various conditions. R stands for the resistance, expressed in pounds-force, overcome in each case, and V stands for the velocity of motion, in feet per second, against this resistance.

	R lbs.	V ft. per sec.	$R + V$ ft. lbs. per sec.
Man.			
1. Raising his own weight up stair or ladder.	143	0.5	71.5
2. Hauling up weight with rope.....	40	0.75	30
3. Lifting weights by hand.....	44	0.55	24.2
4. Shovelling up earth to a height of 5 ft. 3 in.	6	1.3	7.8
5. Pushing or pulling horizontally.....	26.5	2.0	53
6. Working a pump.....	13.2	2.5	33
Horse.			
7. Thoroughbred, cantering and trotting, drawing a light railway carriage.	30.5	$14\frac{2}{3}$	447.3
8. Draught horse, walking, drawing cart or boat.	120	3.6	432

The horse-power of an American railway locomotive engine may be 2000 or more. That of a great ocean steamship may be 30,000 or more.

Problems.

(1) How many ft. lbs. of work must be done in dragging a 100-lb. wt. a distance of 50 ft. along a horizontal surface upon which its coefficient of friction is 1/2?

(2) If a machinist exerts upon a file a force equivalent to 15 lbs. downward and 20 lbs. forward, how many ft.-lbs. of work does he do in 50 horizontal strokes of the file, each stroke being 8 in. long?

(3) A railroad train weighing 1,000,000 lbs. is drawn at a uniform speed along a level track, the coefficient of friction being 0.005. How many ft. lbs. of work are done in drawing it 1000 ft.?

(4) A mass of 30 lbs. is placed on an inclined plane which is 10 ft. long, the base being 8 ft. and the height 6 ft. Friction is to be called zero.—

(*a*) How large a force, parallel to the incline, is needed to keep the mass moving up the incline?

(*b*) How many ft. lbs. of work will be done by this force in drawing the mass the whole length, 10 ft.

(*c*) How large a force, parallel to the base, would be needed to keep the mass moving up the incline?

(*d*) How many ft. lbs. of work would be done by this force in drawing the mass the whole length of the incline?

(*e*) How many ft. lbs. of work would be done in lifting the mass straight up 6 ft., the height of the incline?

(5) The meter being, approximately, equivalent to 3.28 ft., and the kgm. to 2.2 lbs., how many kilogram-meters are equivalent to 100 ft. lbs.?

139. Energy: Kinetic and Potential.—Work and energy are so closely connected that they can hardly be discussed separately. In fact, by definition, *energy is the power of doing work.*

As work involves both force and motion, so energy involves not only the power of exerting force upon a body, but also the power of following up the body, for a greater or less distance, if it yields to the force. A compressed

spring, a charge of gunpowder, a flying cannon-ball, each of these is a body possessing energy. A weight raised above the surface of the earth is ordinarily said to possess energy, because it can do work during its descent, but strictly the energy in this case belongs not to the weight alone but to the system made up of the earth below and the weight above, which by their mutual attraction tend to approach each other and to overcome any resistance which opposes their approach.

Two classes of energy may be distinguished:—

A moving cannon-ball, quite apart from any attraction or repulsion, has energy by virtue of its inertia (§ 118) and its motion. This is called *kinetic energy*.

On the other hand, in the case of the raised weight described above, in a drawn bow, in an explosive mixture of hydrogen and oxygen, in a Leyden jar charged with electricity, etc., we have energy due to some attraction or repulsion between different parts of the same body or different bodies of a system. This is called *potential energy*.

Each of these kinds of energy is the power of doing work, and each is measured in the same units as work, that is, in foot-pounds, kilogram-meters, etc.

For example, let us find the number of foot-pounds of energy possessed by a body the mass of which is m lbs., and which is moving with a velocity of v ft. per second. Since experience shows that kinetic energy does not depend upon the direction of the motion, we shall for convenience suppose that the body is moving straight upward and that its kinetic energy is to be entirely expended in raising the body itself. If the body rises h ft. from its present position before stopping, its kinetic energy will have done $m \times h$ ft. lbs. of work. But how great is h? From § 125 we know that h (there called s) $= v^2 \div 2g$, g being equal to 32.2, nearly. Putting for h this value, we have as a general

expression, good for every case in which a body of m lbs. is moving with a velocity of v ft. per second,

$$kinetic\ energy = \frac{mv^2}{2g}\ ft.\ lbs.$$

Everywhere in the example just discussed we can, without impairing the reasoning in the least, write *kilograms* for pounds and *meters* for feet, g becoming in this case 9.80, nearly (see § 126). So we can write *grams* for pounds and *centimeters* for feet without altering the reasoning or the general formula obtained; but g in this case will have the value 980, nearly.

140. Statement of Work in "Absolute Units."—Sometimes work is expressed in foot-*poundals* instead of in foot-pounds. As the earth's attraction for a one-pound mass is g poundals (§ 123), it is evident that one foot-pound equals g foot-poundals. Hence, if we use this latter unit in expressing kinetic energy, we have, in the case of m lbs. moving with a velocity of v ft. per second,

$$kinetic\ energy = \left(\frac{mv^2}{2g} \times g\right) ft.\ pdls. = \frac{mv^2}{2}\ ft.\ pdls.$$

Physicists, as distinguished from engineers, are the world over in the habit of reckoning in grams and centimeters, rather than in pounds and feet or kilograms and meters. They generally take as their unit of force the *dyne* (see § 122), and as their unit of work the work done by one dyne acting a distance of one centimeter. This unit may be called a *dyne-centimeter*, but it is used so much that a special short name is given it. It is called the *erg*. If in the measurement of kinetic energy we use grams instead of pounds, centimeters instead of feet, and ergs instead of foot-poundals, we shall arrive at the general formula

$$kinetic\ energy = \frac{mv^2}{2}\ ergs.$$

In short, when we use a *gravitation* unit of work, that is, a unit of work based upon a gravitation unit of *force*, we must write

$$\textit{kinetic energy} = \frac{mv^2}{2g};$$

but when we use an *absolute* unit of work, that is, a unit of work based upon one of the so-called absolute units of force, we must write

$$\textit{kinetic energy} = \frac{mv^2}{2}.$$

141. Estimation of Potential Energy.—It would be difficult to give here any useful formula for the amount of potential energy contained in a body or system of bodies, in any case except that of bodies raised above the earth's surface. In such a case, if m is the mass of the body and if h is its height above the earth's surface, we see that the body in descending to the earth can be made to exert m *gravitation* units of force throughout a distance h. Hence its potential energy, reckoned in gravitation-units of work, is $m \times h$. In "absolute" units of work it is $m \times h \times g$.

Questions and Problems.

(1) How many ft. lbs. is the kinetic energy of a 500-lb. cannon-ball moving with a velocity of 2000 ft. per second? How many ft. pdls.?

(2) How many ft. lbs. is the kinetic energy of a ship weighing 10,000 tons and moving at the rate of 20 miles an hour?

(3) (*a*) How many ft. lbs. of potential energy are lost by a cu. ft. of water (see § 7) in falling 20 ft.?

(*b*) What is the horse power of a water fall in which 3000 cu. ft. per second drop 20 ft.?

(4) How many gm. cm. is the kinetic energy of a bullet weighing 10 gm. moving at the rate of 300 m. a second? How many ergs?

(5) A certain body has 800 ergs of kinetic energy.

(*a*) How *far* will this supply of energy carry the body against a constant resistance of 40 dynes?

(*b*) Can you tell from the information given how *long* the body would move against this resistance?

142. Conservation of Energy.—A body thrown upward spends its kinetic energy in doing work against gravitation, in pulling itself away from the earth, but the work thus done goes to store up potential energy. While the body falls this potential energy is turned back into kinetic energy. This is a familiar example of the change from one form of energy to the other which is continually going on about us, sometimes visibly, frequently unseen. Energy takes extremely subtle shapes—in heat, in light, in electric currents, in chemical conditions—but the two classes *kinetic* and *potential* include all forms of which we have knowledge. All such forms are so related that energy in any one of them may be turned into any or all of the others in succession, and back finally to its first shape.

In actually performing such a circle, or *cycle*, of transformations we observe an apparent loss. We bring back to the original form a smaller amount of energy than we started with. Thus, a ball thrown upward returns to the hand with diminished velocity. We must not, however, conclude from such trials that there is any real destruction of energy in the process of transformation.

If we were to take a quantity of water filling a certain vessel and pour it into a dozen vessels in succession until we reached the first vessel again, this vessel would not now be filled, but we should not infer from this any real destruction of the substance of the water. We should account for the apparent loss by the amount left clinging to the sides of the vessels, or absorbed by them, or spilled, or evaporated, or possibly changed by chemical action into some different form of matter; and we should have not the slightest doubt that, if all these small amounts could be collected, they would just fill the space now empty in the

vessel. This confidence is the result of long experience, which has taught us to believe that matter cannot be destroyed.

Equally long experience has been teaching the somewhat more subtle truth that energy cannot be destroyed; that its apparent annihilation in its transformation, or wasting away without transformation, is to be explained, like the disappearance of water in the process described above, merely as an escape in various ways from the immediate scope of our observation. But so numerous, and in many cases obscure, are these ways of escape, that their full extent and significance have not yet, it is likely, been perceived.

The belief that energy cannot be destroyed or diminished, and the converse belief that energy cannot be created or increased—which latter belief is practically rejected by those who attempt to "invent perpetual motion"—constitute the doctrine of the *Conservation of Energy*. All the familiar devices for the advantageous application of mechanical power, such as the lever, the inclined plane, and the hydraulic press, are simple examples of the truth of this doctrine, and their laws may be at once deduced from it.

It will now be seen that the law $P \times D_p = W \times D_w$, arrived at and used in §70, is merely a special example of the general law of Conservation of Energy, $P \times D_p$ being the work put into the machine by the power and $W \times D_w$ being the work done by the machine. Of course in this special case we assume that no increase or decrease of the energy of the machine itself takes place, the machine being merely a means of *transmitting* energy. If there is friction within the machine, a part of the energy stays in it as heat. (See §175).

143. Comparison of Momentum and Kinetic Energy.—Momentum and kinetic energy are often confounded or

mistaken for each other. The following comparison or contrast may therefore be of use:

Momentum is $m \times v$. There is no one-word name in common use for the unit of momentum.

Kinetic energy, in "absolute units," is $\frac{1}{2}mv^2$. It is reckoned in ft. pdls., ergs., etc., the same units in which work is measured.

Momentum is equal to the product of *force* and *time*. If we know the momentum only of a body, we can tell how long it would move against a given resistance, but not how far.

Kinetic energy is equal to the product of *force* and *distance*. If we know the kinetic energy only of a body, we can tell how far it would move against a given resistance, but not how long.

We have already seen (§133) that, by agreeing to call momentum in one direction + and momentum in the opposite direction − , we can frame the law, that *the (algebraic) total momentum of two bodies is unchanged by any interaction between them*. Thus, if two masses, each of m gm., one having velocity v cm. per second and the other being at rest, collide and then move on as one, the velocity, v', after collision will be one-half as great as v.

Since $(-v)^2$ and $(+v)^2$ have the same value, v^2, kinetic energy, which is proportional not to the velocity but to the square of the velocity, is always positive, even when the velocity, and so the momentum, of the body is negative.

The total kinetic energy of two colliding bodies is usually made less, and cannot be made greater,[1] by their collision. Thus, in the case just described, the kinetic energy before collision is $\frac{1}{2}mv^2$, and that after collision is $\frac{1}{2}(2m)v'^2 = \frac{1}{4}mv^2$.

[1] Unless their meeting releases some new store of energy by causing an explosion or some such action.

144. Internal Energy.—In the last paragraph preceding we considered merely the kinetic energy of the masses moving as wholes. We left out of account the energy, partly kinetic, of the vibratory or rotary motion of the particles within each mass. If this internal energy were considered, we should find the total energy to be unaffected by the collision, as the law of Conservation of Energy requires, but we should not necessarily find the total *kinetic* energy the same after collision as before.

One form of internal energy is *heat*, which will be considered in Chapter XV.

145. Sources of Mechanical Power.—Aside from the labor of men and animals, the great agents for doing mechanical work are wind-power, water-power, and heat-power. Electrical power, on any large scale, is usually produced from one of these three.

Wind Power.—To utilize wind power we have windmills and sailing vessels. Action of wind upon a sail, and the way of getting help from even an opposing breeze has been discussed in connection with Fig. 75 in § 89, and with very slight changes the whole of this discussion can be applied to the sails of a windmill. With the mill, however, the mere direction of the wind does not matter, as the wheel can be turned so as to face the wind from whatever quarter it comes.

Old fashioned windmills, of a century or more ago, though very picturesque features of the landscape, were clumsy and, for their size, inefficient. Modern windmills are not beautiful, but they are mechanically a great advance over their predecessors, and accordingly they are made and used in great numbers, though usually for small work, such as pumping water for domestic purposes.

The work done by sailing vessels, though all of the largest ships are now steam driven, is still very great, and it will

probably increase, for the price of coal is gradually rising, and the supply of it in the mines is not inexhaustible.

Water Power.—Old fashioned water wheels are of two classes, *undershot* wheels, the lower blades of which are struck and driven by the stream flowing beneath, and *overshot* wheels, the upper blades or buckets of which receive the impulse of the water.

Each of these forms is very wasteful of power, and, wherever economy of power is important, they have been replaced by some form of *turbine* wheel. The revolving lawn sprinkler, the operation of which was mentioned in § 135 as an example of action and reaction, illustrates also the principle of the turbine water-power wheel.

The water escapes in one direction, the wheel is driven in the opposite direction. If the wheel were prevented from revolving, the water would escape from it with a maximum of velocity and, except for the loss due to friction, would have as much kinetic energy as it would have acquired in falling freely from the level of the reservoir or mill pond to the level of its exit from the wheel. That is, if the height of fall is called h, the velocity in question would be $v = \sqrt{2gh}$. This formula for such a case was given by Torricelli (see § 26) and is called *Torricelli's law*. It is merely a special case under the general law of *conservation of energy*[1] (§ 142); for, if the water in falling a distance h under the pull of gravity does no work, it should acquire just enough kinetic energy to carry it back to the height h and this, we know from § 139, would require its velocity to be $\sqrt{2gh}$.

If, however, the wheel revolves, the water does work upon it in passing through it and will therefore escape from it with less velocity than the law of Torricelli gives.

[1] Torricelli did not know this general law, which is, indeed, merely an inference, a generalization, from many observed special cases of which his law is one.

In order to get from the water most of the work it is capable of doing in its descent, the wheel must revolve so rapidly that the escaping water has very little velocity, very little remaining kinetic energy.

Figure 109, giving one-half of a horizontal cross-section, illustrates the construction of a so-called *radial-flow* tur-

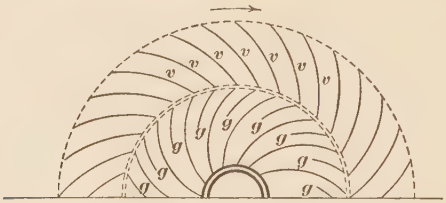


FIG. 109.—Radial-flow water-turbine.

bine, in which the course of the water is from within along the fixed guides, g,g, to the outer revolving part, the wheel, the *vanes*, or buckets, of which are v,v. The arrow shows the direction of revolution of the wheel.

Figure 110 illustrates an *axial-flow* turbine, the general course of the water being in the direction of the axis of the

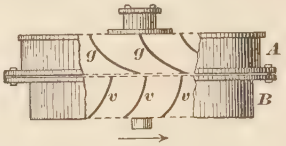


FIG. 110.—Axial-flow water-turbine.

wheel. The water passes downward through the casing A along fixed curved guides, g, g, which give it a nearly horizontal direction, and then strikes the vanes, v, v, of the wheel, which revolves on a vertical axis within the casing B.

Questions and Problems.

(*Take g as* 980 *or as* 32, *according to the system of units used.*)

(1) The momentum of a certain body is 600. It encounters a constant resistance of 50 absolute units.

(*a*) How long will it continue to move?

(*b*) Can you tell how far it will move?

(2) A bullet weighing 10 gms. is shot with a velocity of 300 m. a second from a rifle weighing 6 kgm. and hanging free. How great a backward velocity is given to the rifle by the discharge?

(3) A horse-car is moving northeast along a track. The team is pulling exactly north with a force of 800 lbs., and continues to do so while the car moves far enough along the track to go 12 ft. north. How much work does the team do during this movement? (State the unit in which the work is reckoned.)

(4) A block weighing 10 lbs. rests upon an incline such that the block must move 5 ft. in order to rise 3 ft. The pressure of the block against the incline is in this case 8 lbs. Let the coefficient of friction be 1/4. How much work must be done against gravity by a force parallel to the incline in drawing the block 5 ft.? How much work against friction?

(5) A body weighing 100 lbs. rests upon an incline. How much work must be done, by a force parallel to the incline, in drawing the body 10 ft. up this incline, the vertical ascent being 6 ft. and the coefficient of friction being 1/5?

(6) A hammer hangs vertically, head downward. The centre of gravity of the head, which weighs 16 lbs., is 3 ft. below the point of support. The centre of gravity of the handle, which weighs 1 lb., is 1 ft. below the point of support. How much work is necessary to lift the hammer into a horizontal position at the level of the point of support?

(7) The kinetic energy of a certain body is 600 ft. lbs. How far would this energy carry it against a constant resistance equal to the weight of 50 lbs.? Can you tell how long it would continue to move?

(8) The mass of a steamer is 5000 tons. It is drifted against a wharf by the tide with a velocity of 2 ft. per minute. What is its kinetic energy?

(9) A mass of 10 lbs. is thrown upward with a velocity of 64 ft. per second. (*a*) How many foot-pounds of kinetic energy has it? (*b*) How many foot-poundals? (*c*) How much work was done in giving this velocity?

(10) (*a*) How long will the mass mentioned in (9) rise? (*b*) How far will it rise? (*c*) What can you say of its energy when the mass reaches the greatest height?

(11) A 16-lb. hammer is 20 ft. above the earth's surface and is moving with a velocity of 50 ft. per second. (*a*) How many foot-

pounds is the potential energy? (*b*) How many foot-pounds is the kinetic energy?

(12) A bullet which travels at the rate of 1200 ft. per second is found to penetrate a wooden target to the depth of 6 in. What velocity would be required to enable a bullet of the same size and shape to penetrate 8 in. into the same target, if the resistance encountered is the same at all depths?

(13) An engine pulls with a force of 4000 lbs. upon a train of cars weighing 400,000 lbs. upon a horizontal track. If the friction of the train were zero, how great a velocity would it acquire in one second, starting from rest? How far would the train move in this second? How much work (name the unit) would the engine do in this time?

(14) What would the answers be in (13) if the coefficient of friction were 0.002? How much of the work done in this case would be spent in overcoming friction? How much in giving kinetic energy to the train?

(If in either of the following problems the application of the law of conservation of *momentum* (§ 133) leads to the result that the total kinetic energy is greater after collision than before collision (see § 143, last paragraph), we must conclude not that our general laws are false or in conflict with each other, but merely that the data suggested for this particular problem are in some way wrong).

(15) If a 40-gm. ball having a velocity of 200 cm. per second should strike centrally an equal ball at rest, and rebound with a velocity of 50 cm. per second,

(*a*) What would be the velocity of the second ball after the collision?

(*b*) What would be the total kinetic energy of the two balls before collision?

(*c*) What would be the total kinetic energy of the two balls after collision?

(16) If a 40-gm. ball moving north with a velocity of 200 cm. per second should strike centrally an equal ball moving south with the same velocity, and after the collision should rebound southward with a velocity of 100 cm. per second,

(*a*) What would be the velocity of the second ball after collision?

(*b*) What would be the total kinetic energy of the two balls before collision?

(*c*) What would be the total kinetic energy of the two balls after collision?

CHAPTER XV.

HEAT: TEMPERATURE, CONVEYANCE OF HEAT.

146. Sensation of Warmth.—The bodies which we touch and handle in every-day life might be roughly classified as warm, cold, or in a neutral condition giving us neither the sensation of warmth nor that of cold. This classification is an exceedingly variable one at best, depending largely upon the condition of the observer at the time. To the hands of any one coming indoors on a cold winter's day, a basin of water at the temperature of an ordinary living-room feels warm, while the same water at the same time would feel cool to any one just risen from a warm bed.

But all of us agree in a general way as to the meaning of the words hot and cold.

147. Historical.—We have now to consider the nature of the agent, heat, which is added to bodies in order that their temperature may rise, and is taken away from them, in some measure, in order that their temperature may fall.

During the first half of the nineteenth century heat was generally supposed to be a fluid, invisible and without weight, capable of working its way into bodies somewhat as water enters a sponge, thereby causing expansion, and capable, too, of being forced out of bodies by friction or blows. Experiments which should have been fatal to this belief were made about the year 1798 by Count Rumford, who in boring cannon for the Bavarian government showed that there appeared to be no limit to the amount of heat which continued grinding would bring out of a body, and by Sir Humphry Davy, who showed that two

pieces of ice could be melted by rubbing them together.[1] The theory survived these attacks for a long time, but at length, between 1840 and 1850, the careful experiments of Joule, who measured the amount of heat developed in many different processes, gave it the death-blow, and it steadily, though still gradually, lost its hold upon the minds of scientific men.

148. The Present Theory.—In order to understand the present theory, which is called the *kinetic*, or *motion*, theory, we must think of the very little particles of which all bodies, whether solids, liquids, or gases, are made up. These particles, or molecules, which are too small to be seen one by one, even with the most powerful microscope, are believed to be all the time in a state of more or less rapid motion. In solids, this motion may be compared to the movement of people seated close together in a large company, each individual having a certain freedom and a certain activity of his own, although confined to a particular seat and kept in close contact with his neighbors. In liquids, we may think of the particles as moving about more or less from one place to another, the crowd in which they are packed being less rigid, though perhaps not less dense, than that of the previous case. In gases, we consider the particles as moving about with very great freedom, like skaters on a wide field of ice, occasionally bumping against each other, or against the boundary of the space in which they move, but most of the time free from such disturbances.

We do not mean that a gas particle goes any long time without hitting something. We mean that the time between two hits is usually long compared with the time a hit lasts. In fact, the particles are believed to move

[1] See Tyndall's *Heat as a Mode of Motion*. Rumford (Benjamin Thompson) was an American by birth.

Benjamin Thompson, Count Rumford

so fast that any one of them is likely to hit thousands and even millions of times a second, and, as each cubic centimeter of an ordinary gas contains millions upon millions of such particles, the continual pelting of this multitude against the walls of the containing vessel makes what seems to be a perfectly steady pressure, tending to burst the vessel or make it larger.

The following experiment gives evidence of the incessant activity of gas particles, the phenomena observed being due to the fact that the particles of the illuminating-gas move, under like conditions of pressure and temperature, with a greater average velocity than do those of the heavier air.

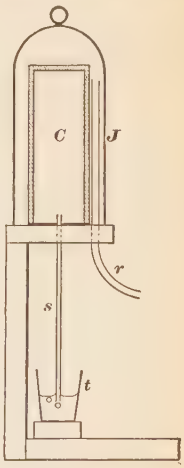


FIG. 111.—Activity of gas particles.

In Fig. 111 (App. No. XLVIII) the porous cup *C* and the glass jar *J* at first contain air at the ordinary pressure. When gas is allowed to flow through the rubber tube *r*, it soon drives most of the air out of the space between the cup and the wall of the jar. Then bubbles of air come out from the stem *s* and rise through the water in the tumbler. This is because the particles of gas are working their way into the cup through its porous wall faster than the air particles can work their way out by the same passages.

When the bubbling has continued for a while, if the flow of gas through the tube *r* is stopped and the jar is lifted, exposing the porous cup to the outside air, the gas which has entered the cup will begin to work its way out faster than air can work its way in, and the water will begin to rise in the stem.

By pushing the jar, which should be held mouth downward all the time, down over the cup again, the water can be made to descend promptly in the stem.

149. Velocity of Gas Particles.—The aggregate weight of all the particles in a given volume of gas being known,

it is possible to make an estimate of the mean, or average, velocity with which they must be moving in order to account for the expansive pressure exerted by the gas. In hydrogen at 0° C. the average velocity of the particles is believed to be about 1800 m., more than a mile per second.

Moreover, all gases are supposed to have the same average kinetic energy per molecule at any given temperature. That is, if m_a is the weight of a molecule of a gas A, and v_a the velocity of this molecule, while m_b and v_b are the corresponding quantities for a gas B, then, taking average values, we have

$$\tfrac{1}{2} m_a v_a^2 = \tfrac{1}{2} m_b v_b^2, \text{ or } v_a : v_b : \sqrt{m_b} : \sqrt{m_a}.$$

Oxygen being about 16 times as heavy, molecule for molecule, as hydrogen, the average velocity of oxygen molecules is about one-quarter of the average velocity of hydrogen molecules.

150. Relation of Heat and Temperature.—Two bodies are said to have the same *temperature* if there is no flow of heat from one to the other when they are brought into contact with each other, but two bodies in this condition would not necessarily contain the same amount of *heat*.

The case is somewhat like that of two communicating vessels containing water. If the water surface is at the same height in both vessels, there will be no flow from one to the other, though the first may contain a thousand times as much water as the second.

We shall in the next chapter begin to study the effects of change of temperature and shall undertake to measure, after a fashion, differences of temperature, but the *measurement* of heat will be taken up later, in Chapter XVII.

CONVEYANCE OF HEAT.

151. Three Modes.—Before attempting any strict measurement of either heat or temperature, it will be well to

consider the modes by which heat is conveyed from one place or body to another. These are three—*conduction*, *convection*, and *radiation*.

152. Conduction.—The following experiment will give some rude idea of the relative freedom with which heat travels through the four solids examined.

Heat with a Bunsen burner a central metal block (see *M* in. Fig. 112, which gives a view, from above, of the apparatus, No. XLIX) from which project rods of equal length and size but of different materials, copper, iron, lead, and glass, for example, until small balls fastened to these rods by means of soft wax are loosened and fall. Notice the order in which the balls fall.

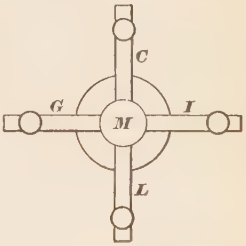


Fig. 112.—Conduction of heat.

Transmission of heat in this way is called *conduction*. It is a flow of heat *as such;* that is, without change into any other form of energy, and without the help of any movement except of course molecular heat-movement, in the substance through which the heat is passing.

Most ordinary substances have been examined in regard to their power of conducting heat, and it is found that solids, and particularly metals, are the best conductors, while liquids, except mercury, and gases are very poor conductors.

Push a number of lumps of ice to the bottom of a test-tube nearly filled with ice-cold water, and fasten them down by means of a loose wad of wire. Heat the water at the top of the tube, by holding the tube inclined in the flame of a Bunsen burner, and notice whether the ice melts rapidly as the upper portion of the water is made to boil. See Fig. 113.

Good and bad conductors of heat can frequently be recognized as such by merely touching them. If a number

of bodies all of the same temperature are touched in succession, the good conductors will feel warmer than the bad conductors, if all are warmer than the hand, and will feel cooler than the bad conductors, if all are, as is generally the case, cooler than the hand. For instance, the coated and uncoated sides of a photographic dry-plate can often be distinguished in the dark by the different degrees of rapidity with which they cool the palm of the hand or the tip of a finger.

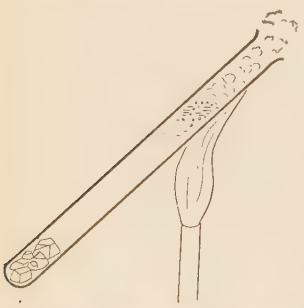


Fig. 113.—Water a poor conductor of heat.

The explanation is that, when one touches a bad conductor, the spot in contact with the hand, not being freely assisted by the other parts of the conductor, soon rises or falls to a temperature approaching that of the hand at the point of contact. In a good conductor the part touched is not so readily brought to the temperature of the hand. In this test, however, as in the experiment of Fig. 112, the effect depends upon the specific heat (§ 177) and density of each substance as well as upon its conducting power.

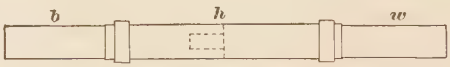


Fig. 114.—Metal a better conductor than wood.

The following familiar experiment gives striking proof of the great effectiveness of brass, as compared with wood, in carrying off heat from a body, paper, in contact with the two substances.

The junction of the compound rod (No. LI), wrapped with paper secured by rubber bands (Fig. 114), is held over a small flame and

moved back and forth at such a height that the paper begins to char. It will presently be noticed that the charred area has a pretty definite boundary on one side, and a knife-blade cutting through the paper at this line will enter the niche between the brass and the wood.

The density and conducting power of gases being small, it is possible to hold one's hand for some little time without injury in gases much hotter than the hottest solid or liquid that one would care to handle. It is a well known fact that dry steam coming in contact with the skin does less harm than steam mixed with water, although the dry steam may be the hotter.

Very bad conductors, like cork, felt, and non-metallic powders, are used to impede the flow of heat into or out of bodies; that is, to keep hot bodies hot and cold bodies cold. The worst heat conductor, or the best *non*-conductor, is a vacuum, and a highly rarefied air space between two solid walls, usually of glass, is now commonly used in making vessels intended for holding liquid air.

To lessen the transfer of heat by radiation (see § 154) the glass walls enclosing the vacuum are covered with a bright coating of silver.

Coefficients of Conductivity.—The amount of heat that will pass by conduction in one second from one face to the opposite face of a unit cube of any substance, when there is one degree difference of temperature and a uniform temperature-gradient between the two faces, is called the *thermal conductivity*, or *coefficient of thermal conductivity*, of the substance.

Taking the centimeter as the unit of length and using the heat unit which is defined in § 176, the *calorie*, we have the following approximately correct table of thermal conductivities, θ, of a few important substances at ordinary room-temperature:

Substance	θ	Substance	θ
Aluminium	0.48	Platinum	0.17
Antimony	0.04	Silver	1.00
Bismuth	0.02	Steel (1% carbon)	0.11
Brass	0.25	Tin	0.15
Constantan	0.54	Zinc	0.27
Copper	0.89	Brick	0.0015
Gold	0.70	Glass	0.0025
Iron	0.15	Rock	0.003 to 0.008
Lead	0.08	Rubber (hard)	0.0005(?)
Nickel	0.14	Water	0.0015

The value of θ for still *air* at 0° C. is about 0.00005; and for certain fibrous[1] materials, wool, cotton, silk, hair, feathers, etc., loosely packed, it is perhaps even less.

Evidently n cubes of any substance placed side by side would, other things being equal, transmit n times as much heat as one cube. That is, the amount of heat carried by conduction is proportional, other things being equal, to the area of cross-section of the transmitting body. It is approximately proportional also, and for ordinary purposes may be taken as strictly proportional, to the steepness of the gradient of temperature in the direction of the flow. Conductivity varies somewhat with the mean temperature of the body, being for metals somewhat less at high temperatures than at low temperatures.

[1] Even fibres of metal, loosely wadded masses of fine curled wire, are used as a packing or covering to prevent flow of heat. The fibres proper occupy but little of the whole space and so carry but little heat, while they hold the air within their meshes nearly still and so lessen *convection*. (See § 153.)

Problem.

How much heat, Q, would pass in 1 min. through an iron plate 2 cm. thick and 20 cm. square, the difference in temperature between the two faces being 10° C. and the gradient uniform, if the thermal conductivity of the metal is 0.15?

Solution: $Q = 0.15 \times 20^2 \times (10 \div 2) \times 60 = 18000$ calories.

The difference in temperature between the two faces of a metal plate through which heat is flowing is often much less than one at first would suppose it to be. For example, if flame is playing on the bottom of a kettle containing boiling water, the difference of temperature of the two faces of the iron may be only a few degrees, even if the iron is clean and bright. The accumulation of soot or ashes on metal plates intended to transmit heat greatly obstructs the flow, as these substances are very bad conductors. Kitchen ranges and the heating-tubes of boilers (see § 198) should accordingly be frequently cleaned. To leave them foul is like leaving the chimney of a lamp black with smoke.

Davy's Safety-Lamp.—This is an instrument which makes use of the fact that the flame of a burning gas will, under certain conditions, not pass through a sheet of wire gauze, the wire carrying off the heat so rapidly that the gas within the meshes will not burn.

Light a Bunsen burner and press down upon the flame a sheet of fine wire-gauze. Observe that the *flame* does not, until the wire becomes red-hot, pass through the gauze. To show that the gas passes through unburned, hold a lighted match above the gauze.

The safety-lamp is used by miners in regions where they are likely to meet with inflammable gases, which an ordinary lamp or candle would ignite. It has (see Fig. 115) a cylinder and cover of wire gauze over the flame. The inflammable "fire-damp" of the mines works its way into

the lamp through the meshes of the gauze and causes the flame to grow larger and change color, and the miner, seeing this, leaves the dangerous region before the flame can set fire to the gas outside the gauze. This lamp was the invention of Sir Humphry Davy (§ 147), a famous English chemist who lived from 1778 to 1829.

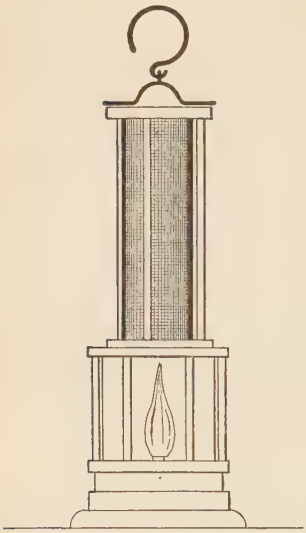


Fig. 115.—Miner's safety-lamp.

153. Convection.—This is the conveyance of heat by movement of the body containing the heat. Such a movement occurs naturally in a liquid or a gas which is unsymmetrically heated; for, as we shall see later, the hotter parts are, in general, less dense than the colder parts, and so the former tend to rise while the latter tend to fall. The movements thus produced are called convection-currents. It is evident that they should be especially strong when heat is applied at the bottom. It would be a difficult matter, as the experiment of Fig. 113 has shown, to boil water at the bottom of a vessel by applying heat at the top.

(1) Light a small piece of touch-paper (No. LII) and place it under a bell-jar held mouth downward. Currents, made evident by the smoke, will be seen moving up and down within the jar.

This experiment illustrates not only the circulation of air in domestic heating and ventilation, up the chimneys, down outside, in at doors and windows, but also the large-scale movements of the atmosphere, upward in the hot equatorial regions, toward the poles at great elevations, downward at certain latitudes, etc.

(2) To show convection currents in water, apparatus LIII, shown

in Fig. 116, may be used. Fill the apparatus completely with water, hold it as shown in the figure, and drop in through the tube *a* a small amount of some colored liquid. Then gradually heat one angle, *b*, of the apparatus by cautiously applying the flame of the Bunsen burner, and note the evidence of currents in the tube.

This illustrates the method of circulation in the warming of buildings by means of hot water or in the hot-water system of an ordinary kitchen cooking-range.

Even when currents are set up in a fluid that is being warmed, they do not work independently of conduction, but assist it rather. This they do by constantly bringing colder particles into contact with warmer ones. But the actual transference of heat from one portion of the fluid to adjacent portions is effected mainly by conduction.

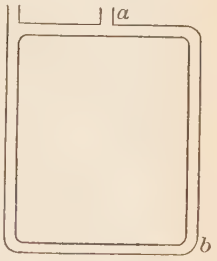


FIG. 116.—Convection current of water.

154. Radiation.—One body may be heated at the expense of another without either conduction or convection between them. The earth is warmed by the sun, but if we were to start from the earth's surface toward the sun with a thermometer, we should very soon find that we were not following an ascending scale of temperature in the medium between us and the sun, as we should be if heat came to us through this medium by conduction. On the contrary, we should, as a rule, get into colder and colder regions above the general level of the earth. If, on the other hand, we look for streams of matter coming from the sun to the earth, and bringing us heat by convection, we do not find them.

Physicists believe, on the evidence of experiments not to be described here, that the heat we get from the sun is transmitted to us by means of a wave motion of the intervening medium, whatever that medium may be, that fills the space between the heavenly bodies. As waves in deep

water travel straight forward, although the single water particles go but short distances and then return, so waves travel swiftly from the sun to the earth, while the particles of the substance that transmits them merely vibrate, it is supposed, to and fro.

Now wave motions, in which every particle repeats the same movements again and again, not jarring against its neighbors irregularly but moving in system and harmony with them all, are not what physicists call heat vibrations. We shall therefore speak of the medium between us and the sun as containing, not "radiant heat," as books formerly said, but *radiant energy*, which, coming from the heat of the sun, may be turned back into heat when it strikes the earth.

It is not from the sun alone that radiant energy is transmitted. On a small scale the same kind of action is noticeable at moderate distances on the earth's surface, from fires, masses of heated metal, and so on; it is the principal means of heating in rooms warmed by open fires.

Radiant energy may traverse a medium without appreciably warming that medium. Its velocity between us and the sun is about 30,000,000,000 *cm., about* 186,000 *miles, per second.*

This is the velocity of light. In fact, light is radiant energy, though not all radiant energy is light.

Hold a convex lens 6 or 8 cm. in diameter in the sunlight above a sheet of paper, at such a distance as to throw upon the paper as small and bright a spot of light as possible. The radiations passing through the lens set the paper on fire. Is the lens itself much heated?

The discussion of radiation is continued in Chapter XXVI.

CHAPTER XVI.

EXPANSION WITH RISE OF TEMPERATURE: THERMOMETRY.

155. Expansion Caused by Rise of Temperature.—The hotter a body is the more active its particles are. The hotter a gas, for example, becomes, the more it tends to expand; for, if it is not allowed to expand, its particles will hit harder and oftener against the walls of the containing vessel than they did before the heating.

The expansion of gases is so great, even with moderate changes of temperature, that it would be a familiar fact to every observing person, if gases were not, as a rule, invisible. The following simple experiment makes this expansion evident to the eye.

(1) Take an "air-thermometer bulb" (No. LIV) 4 or 5 cm. in diameter. Invert this bulb, and immerse the open end of its tube in a glass of water colored red. Heat the bulb by applying the hand, or cautiously by means of a Bunsen flame, and note the effect. Then allow the bulb to cool and observe what occurs, the end of the tube remaining all the time in the water.

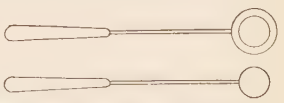


FIG. 117.—Expansion of a metal.

So, too, in solids and liquids a rise of temperature generally, though not always, makes the substance tend to expand.

(2) Take apparatus No. LV, which consists of a metal sphere and a metal ring just large enough to encircle it easily at ordinary temperatures (Fig. 117). Heat the sphere in the Bunsen flame and try to drop it through the ring.

While the sphere is still hot, heat the ring and see whether the sphere will now drop through it.

(3) One flask of apparatus No. LVI, shown in Fig. 118, is filled with colored water to a point which is marked by a rubber ring on the tube, all large bubbles of air being excluded. When the flask thus filled is plunged into warm water, the first visible effect is a slight fall of the water surface; but presently the liquid becomes warmed and expands enough to raise the surface much higher than it was at first.

What is the cause of the fall at first?

Fig. 118.—Expansion of liquid and vessel.

This last experiment illustrates the action of a mercury thermometer. What we observe in the thermometer is not the total expansion of the mercury. It is the excess of the expansion of the mercury over the expansion of the space which it fills within the glass.

156. Different Rates of Expansion.—It is easily seen that air expands more under such conditions as those of the preceding experiments than water or solids. But different liquids and different solids may have different rates of expansion.

(1) Take the two flasks of No. LVI and fill each to a height of 1 or 2 cm. of its stem, one with water and the other with kerosene or with alcohol. Then, after marking the height of the liquid surface in each stem, immerse both bulbs in a vessel nearly full of warm water and note the height of each liquid surface in its tube after the bulbs have been immersed some minutes.

Fig. 119.—Expansion of different metals.

(2) Move the compound metal bar (No. LVII, Fig. 119) back and forth in the flame of a Bunsen lamp until it becomes hot throughout, taking care not to heat it to redness at any point. Observe the bending caused by inequality of expansion, and note which of the metals expands the more.

157. Thermometry.—Further experiments like those preceding would show that, with few exceptions, solids, liquids, and gases expand with rise of temperature, but that each solid or liquid substance has a rate of expansion peculiar to itself. Moreover, these rates of expansion do not bear a constant ratio to each other; that is, if liquid A expands just twice as much as liquid B between certain limits of temperature, it probably does not expand just twice as much as B between other limits of temperature.

Accordingly, although the expansion of almost any substance can be used to indicate a rise of temperature, it is by no means a matter of indifference what substance is selected and used commonly for this purpose. The general experience of mankind has fixed upon mercury, contained in a glass bulb and graduated stem, as the best *thermometer*, or gauge of temperature, for common use.

In adopting the mercury thermometer as a standard instrument we practically say this: *Every increase of temperature which produces a certain apparent*[1] *expansion of the mercury, shall be called one degree, and all such degrees shall be considered equal, though they might not produce equal expansions of any other substance.*

Evidently this is a very arbitrary standard, and scientific men do not consider it a final standard for their work. See § 172.

158. The Mercury Thermometer.—One method of making this instrument will be here described.

First, a piece of thick-walled, small-bore, tubing, known as thermometer tubing, is tested for uniformity of the bore by means of a short column, or "thread," of mercury, which is placed in various positions in the bore and measured as to length in each position. A bulb, R (Fig. 120),

[1] That is, the real expansion of the mercury minus the expansion of the glass envelope. See Exp. 3 under § 155.

is then blown upon one end of the tube, and at the other end a reservoir for mercury, B, with a slender prolongation and a minute opening. B is heated until the air within it is somewhat rarefied and then plunged into mercury. The cooling effect produced by the latter causes the inclosed air to contract, and considerable mercury enters the reservoir B. The end B is now raised, R and the stem are somewhat heated to drive out some of the contained air and then cooled to allow the mercury to sink into R. The mercury in R is then boiled so that its vapor shall drive out the air in the stem and in B. This and similar operations are continued until nothing is left in the bulb and tube but mercury and mercury vapor. Then, finally, the tube is strongly heated just below B, drawn out to a point, and sealed.

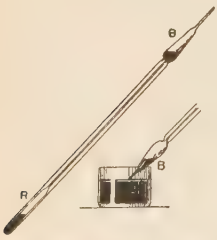


FIG. 120.—Filling a thermometer bulb.

Inexperienced students should not try to boil mercury, as its vapor is poisonous.

159. Graduation of a Mercury Thermometer.—It has been ascertained by an immense number of observations that the temperature of melting ice, and also that of boiling water, is substantially invariable, provided certain conditions are maintained.[1] These conditions are chemical and mechanical purity and a standard, unvarying, pressure. Slight chemical or mechanical impurity affects these temperatures but little. Ordinary changes of atmospheric pressure affect the melting temperature extremely little, but they affect the boiling temperature of

[1] A like statement can be made concerning other solids and liquids, but ice and water are the most convenient for use in graduating thermometers. Some solids gradually soften before melting, so that they have no definite melting temperature.

water very perceptibly. These changes are easily observed by means of the barometer, and allowance for their effect is, as we shall see, easily made.

Advantage is taken of the peculiarly constant behavior of water to establish "fixed points" on the thermometer-tube. The instrument is first placed upright, with the bulb and the stem buried in a mixture of ice and water to such a depth that the top only of the mercury column is visible. The position of the top of the mercury column, when it has ceased to descend, is marked by a transverse scratch on the tube. The thermometer is then removed from the ice and placed upright in a vessel in which the bulb and the greater part of the stem can be perfectly surrounded by an abundant supply of freely escaping steam. After the mercury has ceased to rise in the stem, a fine scratch across the latter is made at the top of the mercury column, if the barometer at the time reads 76 cm.; otherwise, the scratch is made where the top of the column would be if the barometer read 76 cm.

If the thermometer is to be graduated by the Centigrade system (§160), the melting-point mark is called the 0° mark, and the boiling-point mark is called the 100° mark. The space between these two marks is divided into equal[1] parts, usually one hundred parts, each called a degree. Two other systems of graduation are described in §160.

All thermometers, however carefully made, are likely to be more or less in error; for the glass changes somewhat after manufacture, thus altering the capacity of the

[1] If the thermometer is intended for any accurate work, the tube before or after filling with mercury should be "calibrated," a short column of mercury being forced through it a few millimeters at a time, and the length of this column measured in successive portions of the tube. By this means, even if the tube is not of uniform diameter throughout, it may be divided into successive portions, all of equal *capacity*, and the graduation afterwards made in accordance with these; or, if the scale is made in parts of equal *length*, the calibration helps toward the making of necessary corrections in the ordinary use of the thermometer.

bulb and stem. Cheap thermometers—and inexperienced workers should use no others—frequently have errors of half a degree or more at various points. The easiest points to test are the zero and the boiling points, and these are the only ones that will be attempted in Exercise 19.

A number of interesting effects will be noticed incidentally in connection with the tests. Thus, if the thermometer has been kept at the ordinary temperature of the room for some weeks preceding this Exercise, it is quite likely that the zero point found at the beginning of the Exercise will be notably different from that found at the end, the glass not recovering at once, upon cooling from the boiling temperature, the same volume that it had before heating.

160. Comparison of Thermometer Scales. — Unfortunately, there are three thermometer scales in somewhat general use—the Centigrade, already described, the Réaumur, and the Fahrenheit. The Réaumur scale differs from the Centigrade only in the fact that the space between the two standard points, of freezing and boiling water, is divided into 80 instead of 100 equal parts, so that 1 Réaumur degree equals 1.25 Centigrade degrees. The Fahrenheit scale, which is the one commonly used in this country except in laboratories, bears a less simple relation to the Centigrade. Fahrenheit, its inventor, divided the space between the temperature found in melting ice and that found in boiling water into 180 equal parts, and placed the zero[1] of his scale at a point 32 like divisions below that of the melting-point of ice.

The annexed diagram (Fig. 121) shows the relation between the three scales more simply than any verbal statement. In order to find the equivalent of a Centigrade

[1] The lowest temperature he reached with the freezing-mixture which he used.

degree in Fahrenheit degrees, we need only remember that 100 Centigrade degrees = 180 Fahrenheit degrees. Therefore 1 Centigrade degree is $\frac{180}{100}$ (= $\frac{9}{5}$) of a Fahrenheit degree, or 1 Fahrenheit degree = $\frac{5}{9}$ of a Centigrade degree. In order to reduce *temperatures* from one scale to another we must take account not only of the different values of the degrees of the two scales, but also of their different starting-points.

Suppose that it is required to reduce a temperature of 15° C. to the corresponding temperature on the Fahrenheit scale: 15 Cent. degrees = $15 \times \frac{9}{5}$ (= 27) Fahr. degrees, but the Cent. temperature was measured from a point coinciding with 32° Fahr.; so the required temperature is 27° + 32° (= 59°) Fahr.

Suppose that it is required to reduce the temperature 68° Fahr. to the corresponding Cent. temperature: 68 Fahr. is 68° − 32° (= 36°) Fahr. degrees above the Cent. zero, so that the required temperature is $36 \times \frac{5}{9}$ (= 20°) C.

Stated as formulæ, the rules for reduction of these two scales of temperature are as follows:

$$F.^\circ = (C.^\circ \times \tfrac{9}{5}) + 32^\circ;$$
$$C.^\circ = (F.^\circ - 32^\circ) \times \tfrac{5}{9}.$$

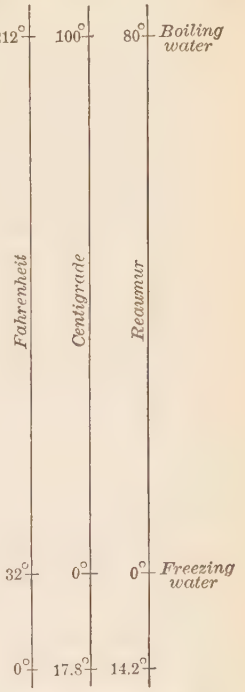


FIG. 121.—Comparison of thermometeric scales.

Problems.

(1) If a change of 2.7 cm. in the barometer makes a difference of 1 Cent. degree in the boiling point of water near 100° C., at what temperature will water boil when the barometer reads 78 cm.?

(2) If the barometer column falls 1 cm. for a rise of 100 m. from the sea-level, at what height will water boil at a temperature of 99° C.?

(3) What temperature on the Cent. scale corresponds to 20° Fahr.? to −20° Fahr.?

(4) What temperature on the Fahr. scale corresponds to 40° C.? to −40° C.?

161. Linear Expansion of Solids.—Nearly all of the solid substances which have been tested are found to expand in all directions as their temperature rises. If the increase of size is observed or estimated in one direction only, as in the lengthening of a bar of metal, the increase is called the *linear expansion*. In all ordinary cases this lengthening is so slight as to be nearly imperceptible, although in many situations it must be taken into account. For example, spaces are left between the ends of the rails in laying a railroad track, to allow of the expansion which may be produced by hot weather. In the spans of long iron bridges the motion of the ends under the influence of changes of temperature may become considerable, as in the case of the iron and steel railroad bridge across the Forth in Scotland. The total length of the bridge, including the approach viaducts, is 8098 ft., and the allowance for expansion of ironwork over the whole length is 6 ft.

The amount of expansion of most metals and of various other solids per degree Centigrade has been ascertained by many accurate experiments, and it is found that the expansibility varies greatly with the substance, no two ordinary metals, for example, expanding the same amount for the same increase of temperature.

162. Coefficient of Linear Expansion.—Experience shows that, if we represent the length of an ordinary solid at $t°$ by l and the length at $t'°$ by l', the relation between these quantities can be shown with sufficient accuracy for ordinary purposes by the equation

$$l' = l + l(t' - t) \times k = l(1 + k(t' - t)),$$

where k is a quantity that does not vary with the temperature. This quantity is called the *coefficient of linear expansion*. It has different values for different substances.

From the equation just given we may get

$$k = (l' - l) \div l(t' - t),$$

which shows that we can find k, for any particular substance, by finding its length, l, at any convenient temperature $t°$, and its increase of length $(l' - l)$, in rising from $t°$ to another known temperature $t'°$. The most convenient temperatures for our use are the ordinary temperature of the room and the temperature of steam. These temperatures are used in Exercise 20.

If in the formula just given we take $l = 1$ and $(t' - t) = 1$, we get $k = (l' - l)$. This shows that *the coefficient of linear expansion is equal to the increase of length which a piece of the material 1 cm. long undergoes with a rise of one degree Centigrade.*

163. Coefficient at Different Temperatures.—For metals between 0° and 100°, the coefficient of expansion is usually nearly uniform, but through wider ranges of temperature this uniformity does not hold good in the case of metals or of other solids. It is found that, as the temperature grows higher, expansion usually proceeds at a more rapid rate. In the case of a specimen of glass whose mean coefficient per degree was ascertained, first from 0° to 100° and then from 0° to 300°, the mean coefficient k was found in the second case to be about 20 per cent. larger than in the first.

164. Applications.—The expansion of most substances with rise of temperature and the inequality of expansion of different substances are facts which must be taken account of in many kinds of work.

The warmth communicated from the hand to a small

steel cylinder or to the gauge which measures it may cause a misfit in the works of a watch.

The need of allowing room for expansion of steel car-rails has already been mentioned. Iron frame-works within brick or stone walls, sometimes in the construction of "fire proof" buildings, have proved dangerous, because in case of fire the expansion of the iron ruins the walls. In steam boilers having steel shells, or bodies, and brass internal tubes, allowance must be made for the greater expansion of the brass. Steam pipes placed nearly horizontal along the walls of a room are not rigidly fastened to the walls at both ends, but are supported in some way that allows them to expand and contract. The expansion is easily seen in long pipes when the steam is turned on, and part of the noise then heard is due to the slipping of the pipes upon their supports.

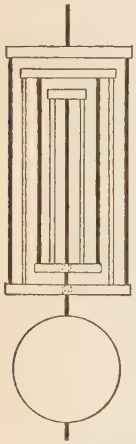


Fig. 122. "Compensation pendulum."

The steel tires of carriage wheels and the steel jackets which strengthen barrels of large guns, are put into place while hot, and their contraction in cooling gives the necessary snugness of fit.

The unequal expansion of two metals, already illustrated in § 156, is made useful in a number of ways.

Thermometers are made (No. LVIII) in which an index is caused to move by the bending or unbending of a compound bar consisting of two strips of unequally expansive materials, placed side by side and fastened close together.

The "gridiron-pendulum" (Fig. 122) is so constructed that the expansion of the brass bars (shown light) offsets or "compensates" the expansion of the steel bars (shown

dark), so that the effective length of the pendulum is nearly unchanged by change of temperature.

In the "balance-wheel" of a watch or clock (No. LIX, Fig. 123) the adjustment for changes of temperature is made by setting the change of curvature of the rim, *RR*, which is open at two places, *C* and *C*, to counterbalance the change of length of the diameter, *DD*. The outer arc of *RR* is of brass, the inner arc of steel, so that heating turns the free ends inward.

If a body to which heat is rapidly applied is thicker in some parts than in others, or if the heat is unsymmetrically

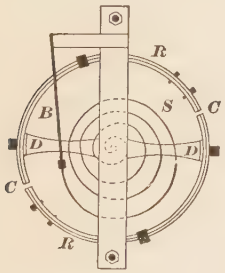


Fig. 123.—"Balance-wheel" of a watch.

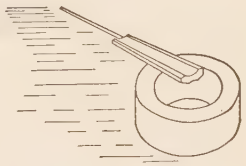


Fig. 124.—"Singing" rocker.

applied to it, some parts grow hot faster than others, and the result is an unsymmetrical expansion, which may break the body. Glass vessels are often broken in this way by pouring hot water into them. Like results may come from unsymmetrical cooling of hot bodies.

165. Trevelyan's Rocker.—One of the most interesting effects of heat expansion is shown in the following experiment, in which the slight impulses given, first to one edge and then to the other of the "rocker" (No. LX, Fig. 124), by the lead expanding at the points of contact, keeps the rocker in rapid oscillatory motion for a considerable time.

The result is a more or less musical note, subject to eccentric changes of pitch when the rocker is disturbed by a touch.

166. Coefficient of Cubical Expansion.—Since each edge of a cube whose length at 0° is 1 becomes $1+k$ (§ 162) at 1°, the volume of the cube, which at 0° was 1, will at 1° become $(1+k)^3$, or $1+3k+3k^2+k^3$.

Now the value of k is always a very small fraction, and consequently its square or its cube is so extremely small a quantity that it may for common purposes be altogether neglected, so that the value of $(1+k)^3$ is practically equal to $1+3k$. Suppose that for some particular substance $k=0.0001$: then, if the object experimented upon is a cube 1 cm. on an edge, it will, after being heated 1°, have a volume of $(1.0001)^3=1.000300030001$; so that, reckoned to seven places of decimals, the cubical expansion, expressed as a fraction of the original volume, is just three times the linear expansion expressed as a fraction of the original length.

The *coefficient of cubical expansion* is defined as the ratio which the increase of volume for 1° rise of temperature bears to the original volume.[1] In the case supposed it is 0.0003, which is three times as large as the coefficient of linear expansion, 0.0001. If we had not 1 cu. cm. only, but a body of V cu. cm., having k as its coefficient of *linear* expansion, the volume after a rise of temperature from $t°$ to $t'°$ would be $V'=V+3k(t'-t)V$, the increase being $3k(t'-t)\ V$. We shall represent the quantity $3k$, the coefficient of *cubical* expansion, by K, and so write

$$V'=V+K(t'-t)V.$$

167. Expansion of Liquids.—Liquids, like solids, vary greatly among themselves in the amount of expansion

[1] In the case of gases, which are very expansible, it is customary to divide the increase per degree by the volume at 0° C., even when the actual starting point of the expansion is some other temperature.

which they undergo for equal increments of temperature. In liquids we generally have to do with cubical expansion, and we may, as appears from §155, in measuring this take account either of apparent or of real expansion.

The real expansion of a liquid can be determined by using the device of balancing columns described in Exercise 6. This method gives the ratio between the density of the liquid cold in one branch of the tube and the density of the same liquid hot in the other branch, and from this ratio the rate of expansion is easily found. Mercury is the liquid that has been most carefully tested in this way.

The real expansion of other liquids can be determined by a like test, but the more common method is to measure first their apparent expansion in glass bulbs with slender stems attached, and then add to the apparent expansion the expansion of the bulb. The expansion of the bulb is measured by means of the already determined real expansion of mercury. That is, a known weight of mercury is heated through a given temperature interval in a bulb having a narrow stem attached, and the apparent expansion of the mercury in this glass envelope is thus found. But the real expansion of the mercury with the same rise of temperature can be calculated, and the difference between this real expansion and the apparent expansion is a measure of the expansion of the glass vessel. The hollow space of a bulb expands just as much as a solid piece, of the same material as the wall, would expand.

For equal increments of temperature liquids in general expand much more than solids.

168. Irregular Expansion of Water and other Liquids.— If a thermometer bulb and tube of suitable proportions were filled with boiling-hot water and then allowed to cool slowly to the temperature of melting ice, the surface of water in the tube would fall slowly until a certain point

was reached, after which it would rise until the water became of the same temperature as the melting ice. The turning point, at which contraction ceases and expansion begins, when the water reaches its temperature of maximum density, is very near 4° C. Water taken at this temperature, then, becomes less dense upon being either cooled or heated. Its density at 0° C. is substantially the same as at 8° C. If the volume of a given portion of water is 1.00000 at 0° C., it would be about 0.99987 at 4° C., 1.0118 at 50° C., and 1.0430 at 100° C.

On the approach of freezing weather the surface water of a lake is cooled to the temperature of greatest density sooner than the lower layers, and accordingly the surface water begins to sink, its place being taken by the warmer water from below, which in turn is cooled to the sinking point. In this way the whole depth of the lake is finally affected, if the cold continues long. Further cooling at the surface will then cause an expansion, and the top layer will remain at the top and freeze there, while the water at the bottom may still be at or near 4° C. Large bodies of fresh water in which there is little disturbance by deep currents or the presence of springs—deep lakes like Tahoe in California, for example—may show a bottom temperature of about 4° C. at all seasons.

Salt water continues to contract below 4° C. and, if it is salt enough, down to and below its ordinary freezing-point, which is itself below 0° C.

Most liquids expand more rapidly at high temperatures than at low temperatures, according to the mercury thermometer. Accordingly, if a thermometer were made after the fashion of a mercury thermometer, but with a different liquid—glycerine, for instance—and if the expansion of this liquid between the freezing and boiling temperatures of water were divided into 100 equal parts, each called 1°,

this thermometer would generally not agree with a mercury thermometer, if the two were equally warm. According to the mercury thermometer, glycerine expands irregularly. According to the glycerine thermometer, mercury expands irregularly. We take as a standard for ordinary purposes that thermometer which we find most convenient, the mercury thermometer; but for more refined purposes we find a better standard in some form of gas thermometer. See § 172.

Problems.

(1) What is the excess of length at 30° C. of a steel meter-rod which is correct at 16° C., K, the coefficent of *cubical* expansion, being, we will suppose, 0.000036?

(2) An iron ring, of 10 cm. internal diameter at 20° C., is to be slipped around a cylinder which is 10.001 cm. in diameter at 20° C. If k for the ring is 0.000012, to what temperature must it be heated in order to be placed around the cylinder?

(3) If the cylinder in the preceding problem is of brass, with $k = 0.000018$, at what temperature would the cylinder slip into the ring, if both were cooled together and equally from 20° C.?

(4) If K is 0.000025 for glass, what is the capacity at 100° C. of a flask that contains 800 cu. cm. at 0° C.? (See § 167.)

(5) If this flask held 10,848 gm. of mercury at 0° C., how many grams would it hold if both glass and mercury were at 100° C., the mean value of K for mercury between these two temperatures being 0.00018?

169. Expansion of Gases.—Accurate measurement of the expansion of gases is complicated by the fact that the volume of a gas is greatly dependent upon the pressure it bears. In fact, the pressure upon a gas which is being heated may be so manipulated as to make the rate of expansion anything we please, large or small.

It is customary, however, in studying the effect of rise of temperature in gases to follow, as closely as may be, one of two courses: 1st. The volume may be kept unchanged

during the heating, in which case we find the *increase of pressure with volume constant;* 2d. The pressure may be kept unchanged during the heating, in which case we find the *increase of volume with pressure constant.* Both methods are used in Exercise 21.

From the data obtained in this Exercise two coefficients of expansion are to be calculated. One is the coefficient of expansion of *pressure,* with volume constant; the other is the coefficient of expansion of *volume,* with pressure constant. The first, α_1, is obtained by dividing the increase of *pressure* per degree rise of temperature *by the original pressure, which held when the gas was at the same volume at* 0° *C.* The second, α_2, is obtained by dividing the increase of *volume* per degree rise of temperature *by the original volume, which held when the gas was under the same pressure at* 0° *C.*

Reference to 0° C. was not made in defining the coefficient of expansion of a solid (§ 162). It made practically no difference there whether the lower temperature was 0° or the temperature of the room. But in the case of gases this does make a good deal of difference. So, in accordance with the definitions just given, Exercise 21 begins by cooling the gas to 0° C.

170. Discussion of Results.—From the equations in Exercise 21 we get, putting v for l,

$$p_t = p_o(1+\alpha_1 t), \text{ when } v \text{ is constant,}$$
$$\text{and} \qquad v_t = v_o(1+\alpha_2 t), \quad \text{“} \quad p \text{ “} \quad \text{“}$$

Careful experiments show that each of the coefficients, α_1 and α_2, is about 0.003665,[1] or nearly $\frac{1}{273}$, for air and for various other gases; and henceforth we shall write each of them as α, with the value $\frac{1}{273}$, or 0.00367.

[1] For proof that α_1 and α_2 must be equal if Boyle's law is strictly true, see Appendix V.

This means that a gas at constant volume, in being heated from 0° C. to 100° C., suffers an increase of pressure equal to $\frac{100}{273}$ of its pressure at 0° C.; and that a gas at constant pressure, in being heated from 0° C. to 100° C., suffers an increase of volume equal to $\frac{100}{273}$ of its volume at 0° C.

But would it be true to say that a gas at constant volume will expand in pressure $\frac{1}{273}$ part of its 0° C. pressure for *each degree* C. rise of temperature, as noted on the *mercury thermometer;* or could we make the similar statement concerning a gas at constant pressure? No: neither statement would be quite true. If we take the mercury thermometer as our standard, the expansion of gases is not quite regular.

Sometimes, however, the mercury thermometer is discarded as a standard, and the "air thermometer" is taken in its stead. When this is done, we take as the *definition* of one degree centigrade on the air thermometer such a change of temperature as will change the pressure of the air, at constant volume, $\frac{1}{273}$ part of its 0° C. value, or will change the volume of the air, at constant pressure, $\frac{1}{273}$ part of its 0° C. value.

For ordinary purposes it is unnecessary to distinguish between the degrees of the air-thermometer scale and those of the mercury-thermometer scale.

171. Behavior of Different Gases: Law of Charles.—It is found that all gases, at temperatures far from their points of condensation into liquids, expand nearly alike; so that, if the student had used coal gas, carbonic-acid gas, or hydrogen, in Exercise 21, he would have obtained nearly the same results as with air. The equality of gases in regard to expansion is set forth in a law known from its discoverer as the *law of Charles.* It is variously worded by different authors. Clerk-Maxwell in his *Theory of Heat* states it as follows:

The volume of a gas under constant pressure expands

when raised from the freezing to the boiling temperature [of water] by the same fraction of itself, whatever be the nature of the gas.

172. Relative Merits of Gases and Mercury as Thermometric Substances; the Gas Thermometer.—The uniformity of the behavior of all gases within a wide range of temperature is in striking contrast to the behavior of liquids and solids, no two of which, so far as we know, agree exactly in their rates of expansion, or even have rates that maintain a constant ratio at different temperatures.

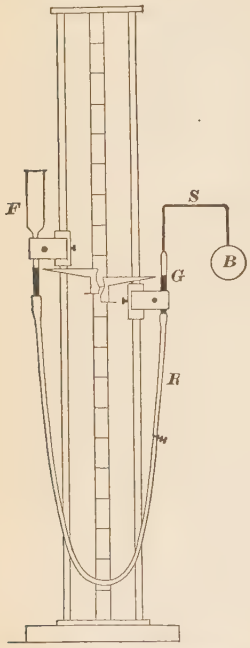


FIG. 125.—Air thermometers.

In assuming that the "apparent expansion" (§ 157) of mercury in glass is regular, as we do in making and using mercury thermometers, we have to assume that all other substances expand irregularly. Yet mercury as a thermometric substance has much in its favor. A very low temperature is required to freeze it, and a very high temperature to boil it. Its excellent conductivity for heat enables it to become heated or cooled quickly. It does not wet the tube which incloses it and leave an adhering film, when the column descends, as most liquids would. If a liquid is to be used, mercury is the best one for general purposes.

But as all the permanent gases agree very closely in their rates of expansion, it seems better to take a gas as the standard thermometric substance for the most refined

purposes, and to test the mercury thermometer by comparing it with a gas thermometer.

As sometimes constructed, the gas thermometer (No. LXI) consists of a bulb, B (Fig. 125), filled with carefully dried air, connected by means of a slender stem, S, and a short glass tube, G, with one end of a rubber tube, R, containing mercury, this tube connecting with a glass funnel, F, at its other end. The mercury column confines the dry air and, the outer end being raised or lowered at will, serves to exert upon it the varying pressures required to keep the volume of the air unchanged when its temperature is raised or lowered. The temperature of the air at any time is calculated from its pressure.

The bulb may have a capacity of one or two hundred cubic centimeters, and the whole instrument is too cumbrous for common purposes. It can be used, however, for testing and correcting mercury thermometers intended for accurate work. Far below the temperature at which mercury freezes and far above the temperature at which it boils, the gas thermometer can be used; and, when we read of $-150°$ C. or $+1000°$ C., we may be sure that such temperatures, if determined accurately, have been determined by means of a gas thermometer or some instrument, usually some electrical device, tested and approved by it.

173. The "Absolute" Thermometer-Scale.—If we have two temperatures, $t_1°$ C. and $t_2°$ C., in mind, we may, in accordance with § 170, write, for a gas at constant volume,

$$p_1 = p_0\left(1 + \frac{t_1}{273}\right) \quad \text{and} \quad p_2 = p_0\left(1 + \frac{t_2}{273}\right),$$

whence

$$\frac{p_1}{p_2} = \frac{273 + t_1}{273 + t_2}. \quad \ldots\ldots\ldots (1)$$

Similarly we get, for a gas at constant pressure,

$$\frac{v_1}{v_2} = \frac{273 + t_1}{273 + t_2}. \quad \ldots \ldots \ldots \quad (2)$$

Here t_1 and t_2 are reckoned from the freezing-point of water. But, in dealing with gas problems, there is a certain convenience in reckoning temperatures from a point 273° below the freezing point of water. If we do this, and use capital letters for this new scale, we get

$$T_1 = 273 + t_1 \text{ and } T_2 = 273 + t_2,$$

which puts equations (1) and (2) into the form

$$\frac{p_1}{p_2} = \frac{T_1}{T_2} \quad \ldots \ldots \quad (3) \text{ and } \frac{v_1}{v_2} = \frac{T_1}{T_2}. \quad \ldots \ldots \quad (4)$$

The scale of temperature to which T_1 and T_2 belong is called the *absolute* scale of the gas thermometer, and the zero of this scale is called the *absolute zero* of the gas thermometer.

Hydrogen boiling at atmospheric pressure has a temperature about 20° above this zero. For the measurement of temperatures so low as this thermometers containing a rarefied gas are used.

It is proved in Appendix V, that, if p_1, v_1, T_1 are the pressure, volume, and temperature (absolute) of any body of gas in any state 1, and p_2, v_2, T_2 the pressure, volume, and temperature (absolute) of the same body of gas in any state 2, the following relation holds:

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}. \quad \ldots \ldots \ldots \quad (5)$$

If we are told any five of the six quantities in this equation, we can find the sixth. Accordingly, this equation is very useful in dealing with cases, such as are given in some of the following problems, in which p, v, and T all change together.

The same general formula, (5), holds when the Fahrenheit absolute scale is used, each T being multiplied by $\frac{9}{5}$ in going from the centigrade absolute to the Fahrenheit absolute scale.

174. Some Effects of Cubical Expansion.—The draft of a stove or an ordinary furnace, the unforced circulation of hot air through a building, and of water through the pipes of a heating system, all are due to the unequal density of air or water at different temperatures. Winds are due, directly or indirectly, to unequal heating in different portions of the atmosphere, and ocean currents in part to the expansion of water heated by a tropical sun. The importance of this joint circulation of air and water in equalizing the temperature of the earth's surface is incalculable.

In all of these cases there is movement of the fluid, gas or liquid, in a more or less definite circuit, heat being added at one or more places, usually at or near what we may call one of the lower *corners* of the circuit (see *b* in Fig. 116), from which place the ascent proceeds, while cooling occurs elsewhere, mainly in the upper horizontal or descending vertical parts.

Draft of Chimneys.—A chimney "smokes," fails to "draw," till the column of air within it gets warm, and a handful of paper, burning over a slowly kindling mass of wood or coal, may greatly improve the draft at first.

The greater steadiness and straightness of a lamp flame when protected by a chimney, is due to the fact that the chimney confines the heated air in a vertical column and compels the fresh air to approach vertically from below. The objectionable tall blades of flame, which reach up over parts of a lamp wick that are left a little higher than other parts, are due to the greater draft at these parts, a small original superiority of flame and excess of draft growing into something of surprising magnitude.

The draft of a chimney being due to the difference between the weight of the hot air-column within it and the weight of a column of like size and shape out of doors, it is evident that, other things being equal, this draft will be the stronger the taller the chimney. Making the flue of a chimney smooth and straight increases the flow of air.

The draft of a chimney may be checked in either of two ways: 1st. By putting some obstruction in the course of the air, as by shutting a "damper," or allowing ashes to accumulate in the fire space; 2d. By allowing a stream of cold air to enter above or beyond the fire space, thus reducing the temperature of the air column within the chimney.

Various devices, depending usually on the expansion and contraction of bodies with changes of temperature, are used for the "automatic" control of the draft in furnaces. For example, a bar of metal in a room which is to be kept at 70° F., may be so arranged that at this temperature it will be just long enough to close an electric circuit or work a valve and thus put into operation some power which will close or partly close a damper.

Questions and Problems on Chapter XVI.

(*See Appendix II for Coefficients of Expansion.*)

(1) How could a mercurial Centigrade thermometer be constructed so as to make a degree very long—for instance, a centimeter or more?

(2) What is the temperature of boiling water when the barometer reads 77 cm.?

(3) What is the error of the boiling point in a thermometer that reads in free steam, when the barometer stands at 77.8 cm., 100.1°?

(4) The standard platinum meter-rod of France is correct at 0° C. What is its length at 20° C.?

(5) Indicate the calculations by means of which you found the coefficient of expansion of brass from your experimental data in Exercise 20.

(6) Tell what measurements in Exercise 20 should be made with the greatest care, and give reasons for your statement.

(7) What force would be required to prevent contraction of a rod of brass of 1 sq. cm. in area of cross-section, cooling from 30° to 0°? (It would be the same as the force required to stretch the rod the amount that it would naturally contract in cooling 30°. Take 10^9 as Young's modulus (§ 98), reckoned in grams and centimeters, and take 0.000019 as the value of k.

(8) The barometer out of doors at 0° stands at 75 cm. What will be the reading after it has been brought into a room in which it attains a temperature of 18° C., the atmospheric pressure remaining unchanged? (Neglect the expansion of the scale.)

(9) What temperature of the ordinary Centigrade scale corresponds to 400° of the absolute scale? to 250°?

(10) A certain quantity of air has a volume of 273 cu. cm. at 0° C. under a pressure of 546 cm. of mercury. It is heated under the same pressure till its volume is 293 cu. cm. What is its final temperature?

(11) The same quantity of gas is cooled under the same pressure till its volume is 253 cu. cm. What is the final temperature?

(12) The gas is heated until its pressure is equal to 819 cm. of mercury, its volume being 273 cu. cm. What is its final temperature?

(13) It is cooled with the same volume, 273 cu. cm., till its pressure is equal to only 500 cm. of mercury. What is its final temperature?

(14) What is the volume at 400° absolute of a quantity of gas which occupied 1000 cu. cm. at 300° absolute, the pressure being unchanged?

(15) What is the pressure at 500° absolute of a quantity of gas which bore a pressure of 76 cm. of mercury at 400° absolute, the volume being unchanged?

(16) What is the volume, when $t=200°$ C., of a quantity of gas that occupied 500 cu. cm. when t was 50° C., the pressure being unchanged?

(17) If with a certain quantity of gas $p=100$ and $v=500$, when $T=300$, what must T' be in order that p' may be 150 and v' 400?

(18) If a quantity of gas under a pressure of 76 cm. at 500° absolute has a volume of 800 cu. cm., what pressure will make its volume 400 cu. cm. when its temperature is 300° absolute?

(19) If a room 20 m. long, 10 m. wide, and 5 m. high contains 1290 kgm. of air at 0° C. when the barometer reads 76 cm., how much air will it contain when the temperature is 30° C. and the barometer reads 74 cm.?

(20) A room measures $3\times3\times3$ m. How many cubic centimeters

of air, supposed unexpanded, will escape from it when the remaining air is warmed from 10° C. to 20° C.? From 10° F. to 70° F.?

(21) A chimney 20 m. tall and 50 cm. square inside is filled with air at a temperature of 300° C. The outside air is at 0° C. and the barometer reads 76 cm. The top of the chimney is covered by a board. How much does the total upward pressure upon this board exceed the downward pressure? (The case is like that of holding a cork under water.)

(22) If a quantity of air occupying 10 cu. ft. at 20° C. under atmospheric pressure has in the first operation of a liquid-air machine been cooled to − 20° C. and raised in pressure to 300 atmospheres, what is its present volume?

(23) Why, in the hot-water method of heating buildings (see § 174), is it customary to have an open tank connected with the water pipes, and where should this tank be placed?

(24) Describe, with sketches, the system of heating (not steam-heating, which will be considered later, § 192) and ventilation in some building well known to you from personal observation.

CHAPTER XVII.

CALORIMETRY; TRANSFORMATION OF HEAT; MECHANICAL EQUIVALENT OF HEAT.

175. Heat as Energy.—As we by doing work can set bodies in motion, thus endowing them with *energy* by means of which they in turn can do work, so we can by friction or blows, or other purely mechanical means, set into more violent motion among themselves the invisibly small particles of which bodies are made up, thus adding to their energy, their power of doing work. We now believe, in short, that *heat is energy, the energy of individual molecules*, as distinguished from the energy of visible bodies. It is like the energy of a mob, each individual of which may be in motion, though the crowd as a whole does not move, while the energy of visible motion is like that of an army moving as a unit.

Heat energy, like any other energy, can be measured in foot pounds. The experiments of Joule, which have already been alluded to and of which more will be said farther on, showed very exactly the number of foot pounds of work which must be done in order to heat a pound of water one degree by stirring, and we have similar information concerning the heating of many other substances.

Lay one end of a rod of lead or solder upon some firm support, pound it briskly with a hammer, and note by touch the rise of temperature produced.

176. Measure of Heat.—Heat is not commonly measured in foot pounds or other similar units, the *unit of heat*, or *thermal unit*, adopted for convenience being *the amount of*

heat required to raise the temperature of unit mass of water one degree.

The thermal unit used in this book is the amount of heat required to raise the temperature of 1 gm. of water 1° C. It is called the *calorie*. This is the unit most used by physicists. It does not have exactly the same magnitude for all degrees, but the variations are small, at ordinary temperatures, and in this book will be disregarded.

The most common unit of heat among English-speaking engineers is the amount required to raise one *pound* of water one degree *Fahrenheit*. It is called the *British thermal unit*, or *B. T. U.*

Let us now inquire by means of an experiment whether the amount of heat required to raise the temperature of a certain number of grams of another substance is equal to that required to raise equally the temperature of the same number of grams of water.

Make ready 100 gm. of water 10° C. colder than the air of the room, and 100 gm. of mercury 10° C. warmer than the air of the room. Pour both, the water first, into a thin glass beaker which has the same temperature as the air, stir the two liquids thoroughly with a thermometer for one-half minute, then note the temperature of the mixture. Which liquid appears to have had the greater influence in producing the final temperature?

Make a similar experiment with water warmer and mercury colder than the air. Which liquid has the greater influence in this case?

177. Specific Heat; Thermal Capacity. — The *specific heat* of any substance is equal to the number of thermal units required to heat unit mass of the substance one degree.

It follows from this definition and the definition of the *thermal unit* (§ 176) that the specific heat of water is taken as 1. The specific heat of most substances is less than 1, but that of hydrogen is greater than 1.

The amount of heat required to raise a given body, large

or small, one degree is called the *thermal capacity*, or "water-equivalent," of that body; that is, the number of grams of water which would absorb as much heat is rising one degree as the body in question. If the mass is m grams, and if the specific heat of the material of which the body consists is called h, the thermal capacity of the body is mh.

178. Calorimetry: Measurement of Specific Heats.—The measurement of heat, as distinguished from the measurement of temperature, is known as *calorimetry*.[1]

One of the objects of calorimetry is the determination of specific heats. For this determination several methods have been employed. One of the earliest of these was to place a portion of the heated substance in a hole scooped in a cake of ice and cover it with a slab of ice. The amount melted from the ice in this way, the quantity of heat required to melt a given quantity of ice being known, served to determine the specific heat required. This general method has been brought to very great perfection by means of an ingenious piece of apparatus known as Bunsen's ice calorimeter.

The method most commonly used for the determination of specific heats is called the *method of mixtures*. In this method a known mass of the substance to be tested is plunged at a known temperature into a known mass of some liquid, usually water, at a different known temperature, and the resulting temperature, called the temperature of the mixture, is noted. In the use of this method there are various opportunities for error, even if the balances and thermometers are correct and are correctly read. Some of these will now be considered:

1st. Any substance when its temperature is taken may not have the same temperature throughout. If the sub-

[1] When heat was regarded as a weightless substance, this supposed substance was frequently called *caloric*.

stance is a liquid or a finely divided solid, it should be thoroughly stirred before its temperature is taken.

2d. A substance may gain or lose considerable heat while it is being poured from one vessel to another. The pouring should be prompt and quick, and through the shortest practicable air space.

3d. The substances in the mixture may not reach the same temperature before the "temperature of the mixture" is noted. They should be stirred well together, and the thermometer should read the same whether its bulb be near the bottom or near the top of the mixture, before the final temperature is taken.

4th. The vessel in which the mixing takes place will probably be heated or cooled by the substance or substances put into it. Allowance must be made for this in the calculations, and in order that this allowance may be small and readily made the vessel should be thin-walled, not unnecessarily large, and, in general, made of metal.

5th. Heat may be lost or gained by the mixture, to or from the surrounding air and other bodies, before its temperature is taken. An attempt is usually made to keep this loss or gain small by having the liquid into which the heated substance is to be plunged about as much below the temperature of surrounding objects before the mixing as it will be above that temperature at the end of the mixing. As a further precaution, the temperature of the mixture should be taken as soon as it can, by stirring, be made the same throughout.

Exercise 22 serves as an example of calorimetric work. Other calorimetric experiments will be found in the next chapter in connection with a study of changes of physical state.

Problems.

(See Appendix II for specific heat of iron, glass, etc.)

(1) Into 110 gm. of water at 15° C., contained in a vessel the thermal capacity of which is equal to that of 10 gm. of water, are put 200 gm. of a certain solid at 100° C., and the resulting temperature of the whole is 25° C. Calculate the specific heat of the solid.

(2) From the following data find the temperature after mixing:

Weight of water used	100 gm.
" " mercury "	1000 gm.
Original temperature of water	10° C.
" " " mercury	100° C.
Specific heat of mercury	0.0333
Number of heat units absorbed by the calorimeter.	80

(3) What is the water equivalent of a thermometer which is made of 20 gm. of glass and contains 10 gm. of mercury?

(4) If 1 gm. of good coal yields 7000 calories in burning under a boiler and if 60 per cent. of this heat is transmitted to the water in the boiler, how much coal, approximately, is required to heat 1000 kgm. of water from 20° C. to the boiling temperature?

(5) If 1000 kgm. of water must be heated at the rate of 2 degrees C. per minute, the heat coming through an iron plate 0.2 cm. thick and 1 m. square, of thermal conductivity 0.15 (see § 152), how great must be the difference of temperature between the two faces of the plate?

(6) If the specific heat of copper is 0.093 when the Centigrade scale is used, what would it be if the Fahrenheit scale were used? (Consider carefully the definitions of thermal unit and specific heat before answering.)

(7) If the specific heat of iron is 0.11 when the gram is the unit of mass, what would it be if the pound were taken as the unit of mass?

(8) How many calories are equivalent to 1 *B. T. U.* (See §176).

179. Mechanical Equivalent of Heat.—The first precise measurements of the amount of work necessary to produce a given amount of heat were made by Joule, an English physicist, about 1845. His experiments were varied in several ways, but one method which he used was to cause a known weight *W* (Fig. 126), falling through a known distance,

to turn paddles inside a vessel, V, containing a weighed quantity of water at a known temperature. The friction of the paddles raised the temperature of the water, and from the data obtained in this experiment the value of a thermal unit in foot pounds could be calculated. According to Joule's observations 772 ft. lbs. would raise a pound of water 1° Fahr.; that is to say, a pound of water falling

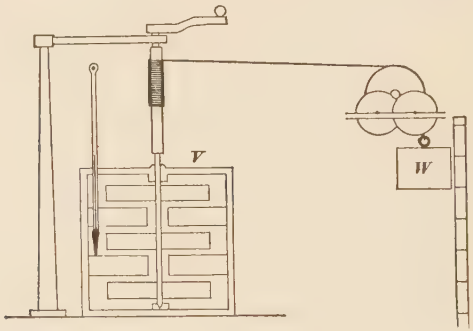


Fig. 126.—Joule's heat-work apparatus.

772 ft. would be raised 1° Fahr., if all the energy of its fall could be turned into heat and all the heat could be kept in the water. To raise a pound of water 1° C., about 1390 ft. lbs., according to Joule's experiments, were required.

Later experiments by Professor Rowland of Baltimore showed that the quantities 778 and 1400 are more nearly correct than 772 and 1390. Using the C. G. S. system (§ 120), we find, according to Rowland's experiments, that the mechanical equivalent of 1 gram-degree C., one calorie, is about 42,700 gm. cm.

A striking example of the transformation of mechanical energy into heat is found in the following experiment, which

requires a good piece of apparatus, good tinder, and considerable physical vigor:

Ignite tinder by heat generated by sudden compression of air in a "fire-syringe" (No. LXIII).

As mechanical work can produce heat, so heat can, on the other hand, be used up in doing mechanical work. For instance, an expanding gas pushing something forward, and so doing work as it expands, becomes cooled by this act.

It is easier to turn other forms of energy into heat than to make the opposite transformation (see § 183), but experiment shows that one thermal unit wholly turned into work will yield just so many foot pounds as in the reverse process would give one unit of heat. This is an illustration of the conservation of energy (§ 142).

180. How a Gas is Cooled in Expansion and is Heated in Compression.—The behavior of gases is best explained (§ 148) by supposing them to consist of very small elastic particles of matter which are flying about in very rapid motion in all directions, frequently bumping against each other and against the walls of the vessel that contains them, but rebounding in a very active manner. The tendency of a gas to expand, shown in the pressure which it exerts upon the walls of the containing vessel, is due to this activity of its particles. The energy of these motions is the heat energy of the gas. To increase the velocity of the particles as a whole is to raise the temperature of the gas, and *vice versa*.

When the gas is expanding, some part of the containing wall must be moving outward while the air particles strike it. When it is being compressed, some part of the wall must be moving inward while particles strike it. Why does the gas become cooled in one case and heated in the other case? The following experiment may throw some light on the matter:

Suspend an ivory ball by means of a string several feet long. Hold in the hands a flat weight of two or three pounds. Give the ball a slight push, and on its return through the lowest point of its path let it strike the weight held at rest. Swing the ball again, and on its return let it at the same place strike the *receding* weight. Repeat, letting the ball strike the *approaching* weight. In each case note the effect of the collision upon the velocity of the ball, as indicated by the distance it swings after the collision.

When, *during the collision*, the weight recedes before the ball, the latter does *work* upon it and loses energy to it. When, during the collision, the weight keeps moving forward, pushing the ball back, the weight does work upon the ball, and the latter gains energy.

181. Heat Engines.—In all heat engines, whether they are ordinary steam engines, hot-air engines, gas engines, or other forms, we have mechanical work done at the expense of heat, the working substance, whatever it may be, using up heat as it expands, because it is doing work. The steam engine is dealt with at some length, and certain other engines more briefly, in Chapter XIX.

182. The Sun Our Main Source of Energy.—Nearly all our available sources of energy on the earth's surface are due to the energy of the sun. Wood is produced by the growth of trees, which growth is maintained by solar light and heat. Coal and petroleum are stored up results of the solar activity of past geological ages. All animals depend for their food either directly or indirectly upon vegetable substances, and therefore upon solar energy. Finally, since natural streams depend upon the evaporation of water and its subsequent fall in the shape of rain or snow, and since winds are due mainly to the unequal heating of different portions of the atmosphere, water power and the motive power of windmills must be referred to the sun as their source.

The amount of heat which the earth receives from the

sun in a day is enormous, and when we reflect that the earth, viewed from the sun, would look no larger than the planets appear to us, it is plain that we receive only a very minute part of the sun's radiation. How can the sun continue to give out this great flow of energy year after year and century after century without cooling?

Much ingenuity has been spent in attempting to answer this question, and the following theories, with perhaps many others, have been proposed:

The Combustion Theory.—This assumes that the surface of the sun is a great furnace in which coal, hydrogen, etc., are burning, as these substances burn upon the earth,[1] it being known (see § 305) that the materials of the sun's surface are the same as those found on the earth. This theory must be rejected because, if it were true, the sun's diameter would be diminishing rapidly, and no diminution is observed.

The Meteoric Theory.—In the space between the planets there is considerable solid material in pieces too small to be seen at any great distance. Such pieces, called meteorites, frequently plunge into our atmosphere, and, becoming heated by friction with the air, glow and burn as the familiar objects which we call "shooting stars."

Even if there were no friction, these bodies, falling from enormous heights under the influence of gravity and striking the earth's surface with very great velocity, would be made very hot by the shock. The sun's mass being vastly greater than that of the earth, meteorites approach its surface under a much greater attraction and strike it with a correspondingly greater generation of heat. In this way some have imagined the heat of the sun to be maintained.

It is now seen that a shower of meteorites sufficiently dense to produce this effect would, if not counteracted by

[1] One gram of pure carbon burning with oxygen yields about 8000 calories; and one gram of hydrogen, burning in the same way, about 34,500 calories.

some other agency, increase the diameter of the sun at a rate so rapid that the change would soon be visible. Therefore a meteoric bombardment, although it no doubt is maintained and may be of considerable importance, is not the only, and is probably not the main, source of the sun's heat.

The Contraction Theory.—If the fall of meteorites under the sun's attraction may do much, the falling in of the sun upon itself under the mutual attraction of its parts may do still more. It is estimated that a rate of contraction sufficient to maintain the present outflow of energy from the sun would diminish its size so gradually that centuries would elapse before the change could be made out by astronomers.

Accordingly the contraction theory is now accepted as accounting for a considerable part of the sun's radiation. This theory came from Helmholtz, a great German physicist of the nineteenth century.

The Radium Theory.—Near the end of the last century two Parisian chemists, M. and Mme. Curie, discovered a new substance, to which, because of its marvellous radiations, the name *radium* was given. It was soon found that a small quantity of radium is capable of keeping itself sensibly warmer than its surroundings, and therefore giving off heat to them continually, for a very long time, probably for hundreds of years, drawing upon some internal store of energy vastly greater than science had previously discovered in so small a compass of weight and volume. It is now known that this energy is of a chemical nature and that a chemical change in the radium is constantly, though very slowly, occurring. It is known, too, that radium is very widely distributed in nature, being a constituent of many materials in the body of our earth; to such an extent, indeed, that the rise of temperature which is observed as

deep borings are followed from the earth's surface down are now supposed to be possibly due, in the main, to the heat given out by this substance.

If the sun also contains, as we have good reason to suppose it does, a vast amount of radium, vast in the aggregate though a very small part of the total mass, we can account for the present solar radiation, and the very slow rate of change of the sun's condition, by taking thought of this store of chemical energy.

183. Tendency of Other Forms of Energy to Become Heat.—Whenever the transformations of any given portion of energy are traced as far as possible, it will be found that they tend to take the form of heat. The flight of a cannon ball, the music of an orchestra, the white light of an electric arc-lamp, the muscular energy of a race horse, all end in slightly warming the air and surrounding objects. It may be said that heat is the lowest form of energy. Other forms turn into it as naturally as all material structures turn into dust.

On the other hand, the transformation of heat into work is attended with considerable difficulties. While savages know how to produce heat and even fire by the expenditure of work, only civilized peoples know how to use heat in engines for the performance of work. The best steam engines yet invented can turn into work far less than one-half of the heat yielded by the coal they burn. The remainder is, of course, not destroyed. It remains as heat, but in such a condition that it cannot be utilized.

184. "Availability" of Heat Energy; "Efficiency" of Heat Engines.—Heat alone is not enough for the working of a heat engine. Difference of temperature between bodies is necessary.

For example, if we have air confined in a cylinder fitted with a piston, heat duly applied to the cylinder may cause

the air to expand, thus pushing forward the piston and doing work; but presently the piston must come back and start anew, if this engine is to continue its work, and during this return it must push against the air which a moment before was pushing it forward. If we can take away heat from the cylinder, by bringing it into contact with some cold body, we can reduce the "back pressure" of the air, so that the work required to compress it will be less than the work done by its expansion, and therefore at the end of the back stroke we shall have got a certain advantage from the use of the engine. But if there is no way of cooling the air, if all bodies about it are at the same temperature, all the work gained from the forward stroke is lost in the back stroke, and the engine will be of no use.

The theory of the steam engine, and of most other forms of apparatus which do work at the expense of heat, is less simple than that of the air engine here imagined; but the same general restriction holds for all of them. If T_1 is the "absolute" temperature (§ 173) of the hottest part of the apparatus and T_2 the absolute temperature of the coldest part, no engine can turn into work a greater fraction than $\frac{T_1 - T_2}{T_1}$ of the heat which it takes in. No proof of this law is here attempted.

The ratio,

$$\frac{\textit{work done by an engine}}{\textit{mechanical equivalent of the heat taken in by the engine}},$$

is called the *efficiency* of the engine. The law stated in the preceding paragraph declares that no heat engine, no contrivance depending on heat for doing work, can have an efficiency greater than $\frac{T_1 - T_2}{T_1}$

Efficiency can evidently be made large by making T_1 large or by making T_2 small. Practically, for ordinary purposes, T_2 cannot well be less than the temperature of the earth, air, or water, about us. In the steam engine T_1 is sometimes made higher than 400° F., about 200° C., though it is usually much lower. Certain other forms of heat engine, which burn gas or oil inside the cylinder, have a much higher T_1 and, accordingly, a higher efficiency than any steam engine.

The heat which is supplied, from the boiler or other *source*, to an engine and is not *used up* there in doing work is, in the regular operation of the engine, transferred to some colder body, the condenser or other *sink* of heat. That is, the normal working of an engine, in which work is produced at the expense of heat, is necessarily accompanied by the passage of heat from some body to some colder body.

CHAPTER XVIII.

CHANGES OF PHYSICAL STATE; HEATING AND VENTILATION.

185. Change of Properties in Solids by Addition or Subtraction of Heat.—Besides the expansion discussed in Chapter XVI. a number of other changes are usually produced in solids by the addition of heat. Rigidity and tenacity are lessened by heating. Iron pillars and beams which are amply sufficient to support the floors of a building become so much weakened upon being heated, if the building becomes thoroughly on fire, that they sometimes yield and fall sooner than fireproofed wooden ones, that is, wooden ones coated with plaster or tiles, would have done under the same circumstances. Zinc, which is not very malleable at ordinary temperatures, may be easily rolled into thin sheets between heated rollers at a temperature of 100° to 150° C., while at a temperature of 200° C. it is so brittle as to be readily powdered in an iron mortar. The power of metals to conduct electricity undergoes such diminution with rise of temperature that this diminution is used as a means of estimating very high temperatures. Sir William Thomson (Lord Kelvin) said: "Every known property of a piece of matter, except its gravity and inertia, varies with variation of temperature."

FUSION AND SOLIDIFICATION.

186. Fusion: Melting-Points.—Most of the solid *elements* —that is, substances which consist of only one kind of matter—pass at a more of less definite temperature from

the solid to the liquid state. So do many chemical compounds and mixtures of compounds, such as common salt, paraffin, beeswax. This change of state is called *fusion*, or *melting*.

Melting Points.—The temperature at which a substance fuses, or melts, is called the *melting point*. Appendix III. shows that the melting points of many substances have been pretty definitely determined.

There is a noticeable difference in the abruptness of the transition from solid to liquid in the case of different substances. Ordinary glass becomes plastic at temperatures below redness, while it melts only at a temperature producing an orange or straw-yellow color; wrought iron and mild steel act in the same way, while cast iron, antimony, and many other substances, notably water, pass abruptly from the solid to the liquid condition.

Variations of the Melting Point.—The melting point of a substance is affected by the presence in it of impurities and by change of pressure. Increased pressure lowers the melting point of a substance which contracts upon melting, and raises that of one which expands upon melting (§ 187). The change from this cause is, however, usually very slight. For instance, one atmosphere of additional pressure lowers the melting point of ice only about 0.0075 of a degree Centigrade.

Slight as it is, the change of melting point caused by change of pressure may produce very curious results, for a change to the extent of 0.001° or less will determine, under certain conditions, whether freezing or melting shall take place.

Freeze two lumps of wet ice together by pressing one hard against the other between the hands.

In this experiment the pressure is borne by a few points of contact, and is therefore rather intense at these points.

Melting occurs, the points are flattened down, making somewhat larger surfaces of contact; and the water in the narrow chinks near these surfaces (see Fig. 127), being shielded somewhat from the atmospheric pressure by its own concave surface (§ 44), is in a condition to freeze, even if its temperature were a trifle above 0° C. Accordingly, freezing at once occurs.

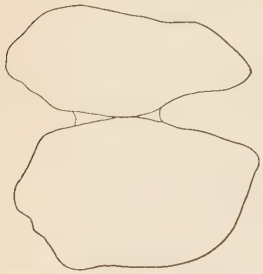


Fig. 127.—*Welding* of ice.

The apparently *flowing* movement of glaciers, the great masses of ice in mountain ravines, is explained by melting under great pressure at the points of resistance and freezing again at places of less pressure beyond the obstacles.

Take a block of ice about 15 cm. long and 4 cm. square, support it in a horizontal position at the ends, and suspend from it, by means of a fine wire passing over its middle, a weight of a kilogram or more. Observe the rapidity with which the wire passes through the ice and the condition of the ice after it is cut through.

187. Change of Volume during Fusion or Solidification.—Most solids change their volume during the melting or the solidifying process.

Most metals and alloys contract in solidifying, but a few, as cast iron and type metal, expand, and these alone can be readily and successfully cast when it is necessary to obtain a sharp, clear, impression of the mould in which the cast is made. In the casting of steel cannon the melted metal is submitted to the action of a powerful hydraulic press which forces the steel into every portion of the mould and at the same time greatly diminishes the size of any contained air bubbles.

The fact that ice has a lower specific gravity than water and will therefore float, coöperates with the fact that the

maximum density of water is reached somewhat above 0° C (§ 168) to prevent large bodies of water in cold climates from freezing solid. One cu. cm. of ice weighs about 0.92 gm.

If water continued to contract all the way down to 0° C, the continual sinking of the surface layer in very cold weather would in time bring the whole body of water nearly or quite to 0° C. But when the upper layer becomes lighter, as well as colder, than the water below, further cooling of the under layers proceeds only by the slow operation (see Fig. 113) of conduction of heat upward through the water. In troubled streams, where the surface water is carried to the bottom in certain places, this bottom may become so cold that ice will form upon or adhere to it. Such a formation is called *ground ice* or *anchor ice.*

188. Latent Heat of Fusion or Melting; Freezing-Mixtures.—The student may have noticed that a melting body absorbs heat, even when it shows no rise of temperature. A kilogram of crushed ice and a kilogram of water at 0° put into similar vessels, and exposed to such equal sources of heat as would be furnished by adjacent lids of an ordinary hot cooking-stove, would be found, at the end of the few minutes necessary to melt the ice, to be many degrees apart in temperature, the water in the vessel which contained ice being little above 0°, while that in the other vessel would be hot. The heat which disappears in the melting is said to become *latent*, that is, *hidden.* The phenomenon is not confined to water, but occurs whenever a solid is liquefied by true melting.

When a solid is liquefied by *dissolving* in another substance, the action is more complicated, and frequently produces a rise of temperature. Acids in which metals are being dissolved may rise from the temperature of the room to a temperature of 100° C. or over, as one can see by putting strips of zinc into strong hydrochloric acid.

On the other hand, many cases are known in which two solids or a solid and a liquid have sufficient attraction for each other to form a liquid mixture when brought together, but which do not in uniting furnish sufficient heat to provide for the work of liquefaction without fall of temperature. The result is a greater or less cooling, not merely of the mixture itself, but of the surrounding objects as well. Such combinations of substances are therefore known as *freezing-mixtures*.

The commonest and most convenient freezing-mixture is that of ice and common salt (about two parts by weight of the former to one of the latter), so generally employed in ice-cream freezing. This mixture can easily be made to produce a temperature of $-20°$ C. Fahrenheit took for the zero of his thermometer scale the lowest temperature which he obtained by means of it.

With a mixture of properly prepared calcium chloride and snow a temperature of $-48°$ C. can be reached, and mercury rapidly solidified.

To illustrate the demand for heat which the process of liquefaction involves, the following experiments can be tried, in one of which alcohol is mixed with *liquid* water while in the other it is mixed with *solid* water:

(1) With 50 cu. cm. of water in a thin beaker-glass mix about 10 cu. cm. of alcohol, taking both at the temperature of the room, and note the resulting change of temperature.

(2) Pour into a mass of snow or finely broken ice enough alcohol to moisten it, and note the resulting change of temperature.

In numerical terms, the *latent heat of melting* of a solid is the *number of heat units required to melt unit mass of the substance without change of temperature*. This quantity can most readily be determined by an application of the "method of mixtures" (§ 178). It is illustrated in the case of ice in Exercise 23.

Questions and Problems.

(1) If the melting point of lead is 330° C., its specific heat 0.031, and its latent heat of melting 5.6, how many units of heat are required to raise 500 gm. of lead from 300° C. to the melting point and then melt it?

(2) (*a*) If the latent heat of melting of ice is 80, how many degrees C. will 10 gm. of water be cooled by melting in it 1 gm. of ice, the first temperature of the water being 30° C.?

(*b*) What would the answer be, if pounds were substituted for grams in (*a*)?

(3) (*a*) If the latent heat of melting of ice is 80, when the Centigrade scale is used, what is it when the Fahrenheit scale is used? (Be sure of the definition of *latent heat of melting* and *unit of heat* before answering.)

(*b*) Would the numerical value of the latent heat of melting of any substance be changed by changing from the gram to the pound as the unit of mass?

(*c*) How many *B. T. U.* (§176) are required to melt 1 lb. of ice?

(4) How many degrees F. would 10 lbs. of water, taken at 80 F., be cooled by melting in it 1 lb. of ice.

VAPORIZATION AND CONDENSATION.

189. Vaporization: Ordinary Evaporation.—Some solids, such as sugar and glue, and some liquids, such as most oils, cannot be vaporized, that is, converted into vapor, without suffering chemical change, by which they are split up into new substances. But a great many substances can, like water, exist in all three of the physical states—the solid, the liquid, and the gaseous.

Vaporization, or evaporation, is not for any particular substance confined to a particular temperature. Water vaporizes at all ordinary temperatures, even below its freezing point. It is a fact well known to housewives that wet clothes hung out in very cold weather will "freeze dry," that is, dry without thawing; and it may be observed that icicles and patches of snow and ice waste away even in severe cold weather, when no thawing can occur.

Certain other solids, iodine and camphor, for example, may pass directly and freely, when heated, from the solid into the gaseous condition, without melting. This special process of evaporation is called *sublimation.*

Ordinary quiet evaporation occurs only at the free surface of liquids or solids, and it can be greatly hastened by increasing this surface. In a process of salt-making formerly a good deal used, the brine was concentrated by allowing it to trickle over piles of brushwood, on the surface of which it spread out and rapidly dried away. Increase of temperature, renewing the air in contact with the evaporating liquid, and rarefying the air, all aid evaporation.

190. "Non-Saturated" and "Saturated" Vapors.—A century or two ago evaporation was explained as the absorption of liquid particles by the air, and the presence of air was supposed necessary for the process.[1] After a time it was

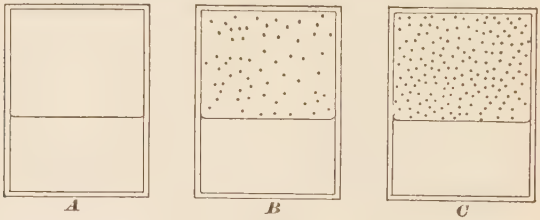


FIG. 128.—States of vapor.

found that evaporation takes place with especial rapidity in a vacuum, as a later experiment will show, but the terms "saturated" and "nonsaturated," as applied to vapors, continue to be used.

The present theory of saturated and nonsaturated vapors may be illustrated as follows: Let *A*, in Fig. 128, represent a closed vessel containing a liquid with a vacuum above it.

[1] See Whewell's *History of the Inductives Science*, vol. II.

The particles of the liquid are in a state of invisible motion; some of them are naturally more agitated than others. The more lively particles at the surface break loose and fly off into the vacuum above, which therefore ceases to be a vacuum (see *B*). These particles are now vapor. They fly about, bumping against each other and against the wall of the vessel, exerting pressure.

Some of the flying particles plunge back into the liquid, but at first others, more numerous, escape at the same time from the liquid. So the pressure in the space above the liquid increases. We have still a *nonsaturated* vapor there.

After a time, if the temperature remains unchanged, the swarm of particles above the liquid becomes so dense (see *C*) that just as many go back into the liquid per second as escape from it; *Condensation* balances *evaporation*. Now the vapor no longer increases in density and pressure. It is a *saturated* vapor. The maximum pressure which the vapor of any given liquid can exert, that is, the pressure of its saturated vapor, increases with rise of temperature.

191. Boiling.—The student is probably well aware that a liquid disappears more rapidly when boiling than when not boiling. He has seen, too, in some of the laboratory work, that boiling, in the case of water at least, takes place at a definite temperature, which is, however, somewhat affected by the pressure to which the liquid is subjected.

Boiling is the formation of bubbles of vapor of the boiling substance (not of air) within the body of the liquid, and *it cannot occur unless the expansive pressure of the saturated vapor is great enough to withstand the pressure tending to collapse these bubbles and condense their vapor.*

The reason why a liquid, after it begins to boil, no longer rises in temperature, if the pressure remains constant, is that the rapid formation of vapor absorbs, renders *latent*, the heat which the flame continues to supply. See § 197.

192. Boiling Points of Solutions.—When a substance which does not itself give off vapor appreciably, sugar, salt, or sulphuric acid, for example, is dissolved in water, the maximum vapor-pressure of the solution is less at any given temperature than that of pure water, and so the boiling temperature under any given pressure is higher than that of pure water.

When liquid which is more volatile than water, alcohol, for example, is dissolved in water, the maximum vapor-pressure from the solution may be greater at any given temperature than that of pure water, and so the boiling temperature under any given pressure may be lower than that of pure water. In some cases, however, the opposite effect is seen. Thus a small addition of pure hydrochloric acid, which is a gas or vapor at ordinary temperatures under atmospheric pressure, produces with water a mixture which has a lower vapor pressure and a higher boiling point than pure water has.

193. Increase of Volume during Evaporation.—One gram of water would make about 1660 cu. cm. of steam, at the standard barometric pressure, at a temperature of 100°. When water is boiled at a temperature higher than 100° C. the saturated steam generated from it has a greater density, as well as a greater pressure, than saturated steam at 100° C.

Many liquids increase in volume less than water upon evaporating, so that their vapors are heavier than that of water. For example, at a pressure of 1 atmosphere, the density of saturated alcohol-vapor is about 2.5 times, while that of saturated ether-vapor is about 5 times, as great as that of saturated water-vapor.

194. Mixtures of Gases and Vapors.—When vapors and gases which have no especial attraction for each other are mixed, the pressure of the mixture is the sum of the pres-

sures of its components. For example, if a cubic foot of oxygen at a certain pressure, a cubic foot of nitrogen at a certain pressure, and a cubic foot of aqueous vapor at a certain pressure, the temperature being the same for all, are crowded together into one cubic foot without change of temperature, the resulting pressure will be equal to the original pressure of the oxygen *plus* that of the nitrogen *plus* that of the vapor. Each of the three constituents is to be regarded as exerting still the same pressure that it exerted before the mixing. This is called *Dalton's law*, from its discoverer. It is not *strictly* correct.

195. Atmospheric Vapor: Dew Point; Relative Humidity. —Our atmosphere is a mixture of gases and vapors, each bearing its part of the total pressure. When the air contains aqueous vapor in a nearly saturated condition it is commonly called moist air, although, strictly, aqueous *vapor* is not called *moisture*. The proportion of such vapor in the atmosphere near us can be found by exposing a measured portion of air to the action of a weighed amount of some substance, calcium chloride for example, which will absorb the water vapor from the air, and then calculating the amount of this vapor from the increase in weight of the absorbing substance. A more rapid process consists in the use of some sort of *hygrometer*,[1] one of the best known of which is the *dew-point* hygrometer (see Exercise 24). *Dew-point* is the name given to the temperature at which the water vapor in the air begins, when cooled, to turn into liquid.

This temperature is higher the greater the amount of vapor in the air, and it differs greatly from time to time. If the room in which Exercise 24 is performed is not very large, it will be interesting to compare the dew point found after several boilers have been in operation for a considerable time with that found before boiling begins.

[1] Greek *ὑγρός*, moist.

Observations of the dew point are continually made by those whose business it is to foretell the weather.

In numerical terms, the *relative humidity* of the atmosphere at any time is the actual amount of aqueous *vapor* (not *liquid*) in it divided by the maximum amount there could be in it at the given temperature. This is commonly called in Weather Reports simply *the humidity*.

Relative humidity is found from the dew point and the temperature of the air by means of carefully prepared tables or diagrams based on long observation of the relation between these quantities.

The so-called *wet- and dry-bulb hygrometer* is much used. Its most important parts are two thermometers, one of which is exposed bare and dry to the air, while the bulb of the other is surrounded by a wick wet with water. If the air is not saturated with water vapor, evaporation from the wick cools the bulb within (see § 197). From the reading of the two thermometers the dew point and relative humidity of the air can be found. It is important to keep the air moving in the vicinity of the wet bulb.

196. Distillation.—A most important practical application of the diversity of substances in regard to their boiling points is found in the operation of distilling. In many cases a volatile liquid may be separated from less volatile solids or liquids with which it is mixed or combined by heating the mixture to boiling and cooling the escaping vapor until it condenses, as in Fig. 129, where C is a jacket tube, through which cold water flows, surrounding the tube by which the vapor to be condensed comes from the heated flask.

This is essentially the way in which distilled water is obtained for laboratory or other purposes, the impurities of ordinary water being left behind in the boiler.

If a liquid is found to change, as crude petroleum does, its boiling point upon long heating, it is highly probable

that it consists of a mixture of substances of various boiling points. These substances can be separated and collected by slowly distilling the mixture, with a thermometer in the path of the escaping vapor, and changing the receiving vessel every time a rise in the thermometer is noted.

Sometimes two or more vapors distil over together, but in proportions different from those in the original mixture, and many successive distillations may be needed to make the separation practically complete.

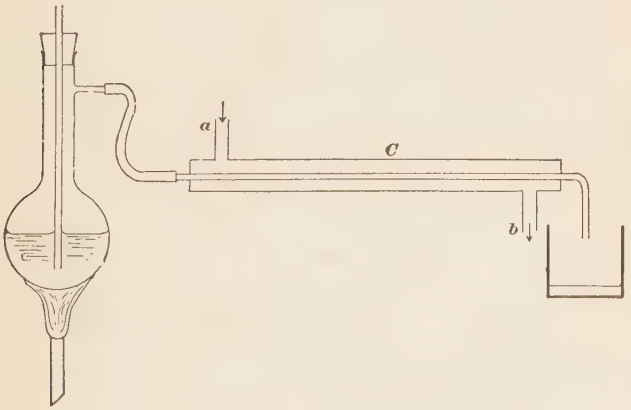


FIG. 129.—Distillation.

197. Latent Heat of Vaporization; Cooling by Evaporation.—The absorption of heat by a boiling liquid has already been alluded to in explanation of the fact that liquids cease to rise in temperature when they begin to boil. A like absorption of heat occurs in the process of quiet surface-evaporation, and, if heat is not continually supplied in sufficient quantity to an evaporating liquid, the liquid itself and any object in contact with it is cooled.

The fact probably is that the more active particles break

loose first from the liquid, leaving behind them a lower average of molecular activity, a lower temperature.

Many important and curious effects are produced by this method of cooling.

Every one has noticed how much more the hands feel the out of door cold, especially on a windy day, when they are wet.[1] The sensation of cold is still stronger when they are wet with alcohol, which evaporates more readily than water. In the heat of summer the evaporation of perspiration from the skin tends to keep the body cool. When the air contains a great deal of water vapor, when it is, in common speech, moist, this evaporation from the skin takes place but slowly, and we miss the cooling effect. Weather which is both hot and moist is therefore peculiarly oppressive.

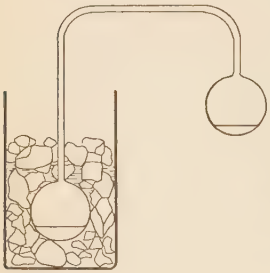


Fig. 130.—Freezing by evaporation.

It has already been stated that evaporation takes place with especial rapidity in a vacuum. Advantage is taken of this fact in the following experiment. In the glass bulbs and the connecting tube (see Fig. 130) there is no air, but water or water vapor. When one of the bulbs is cooled by the freezing-mixture, much of the vapor within it is condensed, and thus gives room for more, which is rapidly formed at the surface of the water in the other bulb. The apparatus (No. LXVII) is called a *cryophorus:*[2]

Get all of the water into the upper bulb of the cryophorus, wrap this bulb with cotton wool, place the other bulb in a good freezing-mixture, then leave the whole apparatus to itself for several minutes.

[1] Sailors find the direction of a light wind by holding up a wet finger and noting upon which side the cool sensation is strongest.

[2] Greek *carrier of cold.*

When the covering is removed from the upper bulb the water within it will probably be found frozen at the surface.

The absorption of heat by evaporation of water at the earth's surface or in clouds, and the giving up of heat in the reverse process of gathering dew or forming mists, cannot but have important atmospheric effects, which it is the duty of science to trace out and make useful in weather predictions and possibly in other ways.

In numerical terms, the *latent heat of vaporization* of a liquid is *the number of heat units* (§ 176) *required to evaporate unit mass of the liquid without change of temperature.* This quantity is different at different temperatures.

Exercise 25 undertakes to find the amount of heat required to evaporate 1 gm. of water at 100° C., or as near that temperature as the atmospheric pressure prevailing at the time may permit the boiling point to be. The actual process employed, however, in this Exercise is to *condense* the vapor at this temperature and find the amount of heat *given out* in this operation, which is the reverse of evaporation by boiling.

Carbonic-acid gas, which for convenient use in "charging" "soda-water" fountains is forced into steel cylinders under high pressure, can be liquefied by the application of a somewhat higher pressure. When this pressure is removed evaporation is very rapid, and a temperature as low as −78° C. may be thus attained. Liquid air (§ 207), which also is now a not uncommon article of commerce, may give a temperature about −185° C.

198. Steam-Heating of Buildings.—The great latent heat of steam, shown in Exercise 25, makes this fluid an effective agent for the heating of buildings. A comparatively small flow, reckoned by weight, through the radiating pipes is sufficient, for every pound of steam that condenses in these pipes yields a very large amount of heat. The main

features of a simple steam-heating system are shown in Figs. 131 and 132.

Coal is burned in the fire-box *G* (see Fig. 131). The heated air and the hot gases given out by the combustion pass from front to back under the body of the boiler and from back to front through tubes, between and above which lies the water, the upper surface of which is at *W*. *D* is a damper to control the draft.

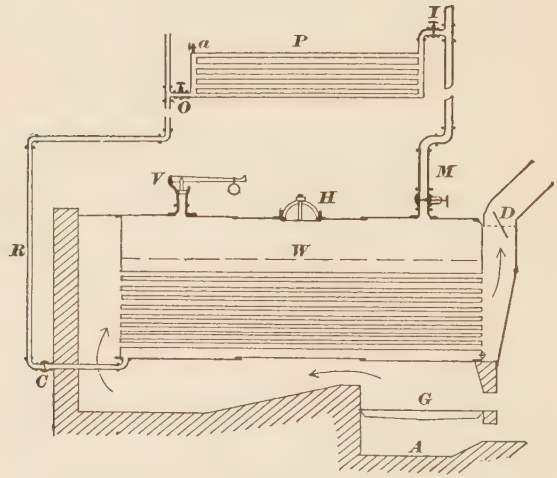


FIG. 131.—Steam-heating system with flue-tube boiler.

The steam passing out through the main pipe, or *main*, *M*, goes to the radiating tubes *P*, where it is gradually condensed by loss of heat to the air. The water thus formed in the tubes drains into the return pipe *R*, where it accumulates until its weight, assisted by the steam pressure upon its upper surface, forces open the "check valve" *C*, leading back to the boiler.

If the inlet cock *I* is open and the outlet cock *O* is shut,

the tubes P gradually fill up with water, which presently cools, and the apparatus becomes useless for the time. If the cock O is open while I is shut, the water from the return pipe R may back up into the radiating tubes.

The accumulation of cold water in these tubes is troublesome. When both cocks are opened and the steam begins to flow, after such an accumulation, it has to drive this water out, and in this operation many sudden condensations occur, like those observed in Exercise 25, but far louder and more violent.

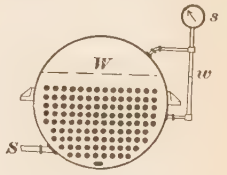


Fig. 132.—Boiler; end view, front.
S = inlet-pipe for cold water;
w = water-gauge;
s = steam-pressure gauge.

Both cocks should be shut, therefore, when the tubes are not in use. Most of the steam then left in them is soon condensed, leaving a partial vacuum, into which air is pretty sure to find its way after a time. When steam is to be readmitted this air must be driven out. Accordingly, the vent a and the cock I are opened, O remaining closed, until steam begins to flow from a. Then a is closed and O is opened.

Figs. 131 and 132 illustrate what is called a *fire-tube* or *flue-tube* boiler. In so-called *water-tube* boilers, which are much used, the flames or hot gases pass across or between tubes containing water to be heated. Fig. 133 illustrates a common form, the Babcock and Wilcox, of the water-tube type of boiler. The flames and hot gases follow the course of the curved arrows and pass out by way of the damper, D, to the chimney. SS is a steam drum, a long cylinder open at certain places for admission of steam from the vapor space above the water level W. From SS the steam passes out to be used. The object of the steam drum is to lessen the amount of liquid water. in the form of small drops, carried out by the steam.

The system of steam heating illustrated in Fig. 131 furnishes no fresh air to the rooms which it warms. Often, however, the steam instead of entering the rooms serves merely to heat a current of fresh air which, drawn into the basement of the building from out of doors, passes at once through a sheet-iron box, surrounding pipes or hollow castings through which steam is flowing. This is called the *indirect* method of heating by steam.

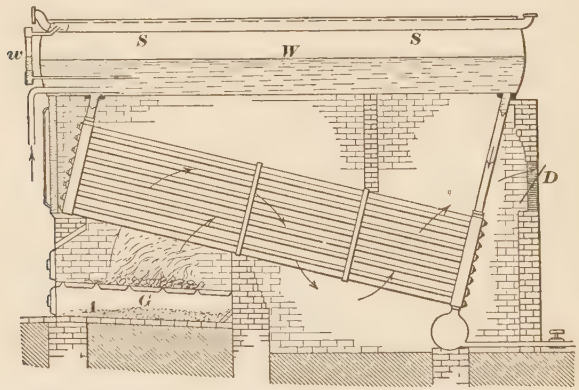


Fig. 133.—Dabcock and Wilcox water-tube boiler.

199. Ventilation of Buildings.—The proper ventilation of a room requires something more than the mere supply of fresh air. Care must be taken to have this air get to the places where it is needed, the nostrils and mouths of the occupants, and, moreover, it should in cold weather be heated somewhat above the desired temperature of the room to which it is admitted. If the inlet is in or near the floor and the outlet in or near the ceiling, the stream of warm fresh air may rise at once above the heads of the occupants of the room and go off, doing little or no good. A device sometimes used with good results is to admit the

fresh air at the top of the room and allow it to escape, after slowly falling, through many small openings in the floor; but this plan, if the fresh air is heated, requires it to be *forced* through the room by some form of pump or fan, as it would not otherwise move downward.

The amount of fresh air required for each person per minute depends, then, somewhat on the system of ventilation which is used; 30 cu. ft. per minute is an estimate given on good authority, but with a thoroughly good arrangement of inlets and outlets a smaller supply is doubtless sufficient; for no human being under ordinary circumstances takes into his lungs half as much as 30 cu. ft. of air per minute; it is well, however, to be on the safe side, and the safe side is the liberal side in the matter of air supply.

The size of the inlet, if the air coming through it is to strike directly upon the occupants of the room, should be such that the velocity of the stream need not be more than 4 or 5 ft. per second.

If the air in a room is very still, breathing and perspiration may make it bad close by the occupants, while it is good elsewhere. And so the mere gentle stirring of the air by fans or otherwise may serve, to a considerable degree, the purpose of ventilation.

Questions and Problems.

(Take 540 calories as the latent heat of evaporation of 1 gm. of water at 100° C.)

(1) Let an open glass vessel contain cold water. Let a flame be applied to the bottom of the vessel until the water boils. Describe and explain the phenomena seen in the water during the heating process and the boiling.

(2) Describe, with a diagram, the construction and action of the device used in the experiments on boiling to prevent the cold water in the outer vessel from rising into the boiler after heating of the latter has ceased. (This refers to experiments described in the Teachers' Edition, which are supposed to have been shown.)

(3) If 20 lbs. of steam at 100° C. flow into 500 lbs. of water taken at 20° C., contained in a vessel of copper, sp. ht. 0.093, weighing 100 lbs., what is the resulting temperature?

(4) If the specific heat of air is 0.237, and the latent heat of evaporation of water at 20° C. is 585, how many kgm. of air would be heated 10 Centigrade degrees by the condensation of 1 kgm. of water vapor at that temperature?

(5) (*a*) If the latent heat of evaporation of a substance is E units when the Centigrade scale is used, how many units is it when the Fahrenheit scale is used?

(*b*) Would the numerical value of the latent heat of evaporation of any substance be changed by changing from the gram to the pound as the unit of mass?

(6) (*a*) How many B. T. U. (§176), approximately, are required to raise 1 lb. of water from the freezing temperature to the boiling temperature, under ordinary atmospheric pressure.

(*b*) How many to evaporate it after the boiling temperature is reached?

(7) (*a*) The density of air being 0.08 lb. per cu. ft. at 32° F. under atmospheric pressure, what is the weight in lbs. of 10,000 cu. ft. of air at 82° F.?

(*b*) The specific heat of air under atmospheric pressure being 0.237, how many lbs. of steam, in condensing at atmospheric pressure and then cooling as water to 132° F., would be required to heat this amount of air from 32° F. to 82° F. on its way from out of doors to a room which it is to heat and ventilate?

(8) Name all the familiar instances that occur to you of the conversion of mechanical energy into heat, omitting examples given in the preceding pages.

(9) Assuming the average height of Niagara Falls to be 50 m., how much warmer should we find the water at the foot of the falls than at the top? (Assume that all the heat generated by the collision of the falling water with other water and with the river bed below is imparted to the falling water.)

(10) From what height, approximately, above the earth's surface would a block of ice have to fall in order that the heat generated by the fall should be sufficient to melt it?

(11) From what height would a body of water have to fall in order that the heat generated in the fall should be sufficient to heat the water from 0° C. to 100° C. and then evaporate it?

(12) A piece of iron weighing 15 kgm. is cut in two by a saw.

The force required to push the saw through the iron is 8 kgm., the length of the stroke is 12 cm., and the number of cuts required to finish the work is 500. If all the heat generated remains in the iron, what will be its temperature at the close of the operation, if the temperature at first was 10° C.?

(13) A loaded sled on ice at 0° C. is drawn by a pull of 25 lbs. How much ice would be melted by the friction in drawing the sled 1 mile?

(14) An iron cannon-ball is shot against an iron target with an amount of energy sufficient to have carried it vertically upward 10 miles. If the cannon ball weighs 50 lbs. and the target 5 tons, how much rise of temperature would be noted in the target if it received all the heat produced by the blow? (Take 0.11 as the sp. ht. of iron.)

(15) (*a*) The heat of combustion of 1 lb. of good hard coal being 12,000 B. T. U. (§ 176) nearly, how high could a ship weighing 10,000 tons be lifted if all the energy of combustion of 10 lbs. of such coal were used up in the work?

(*b*) How high could the same ship be lifted by the same amount of coal used in a perfect heat-engine having a boiler temperature of 800° F. absolute and a condenser temperature of 600° F. absolute?

CHAPTER XIX.

HEAT ENGINES; REFRIGERATORS; LIQUEFACTION OF GASES.

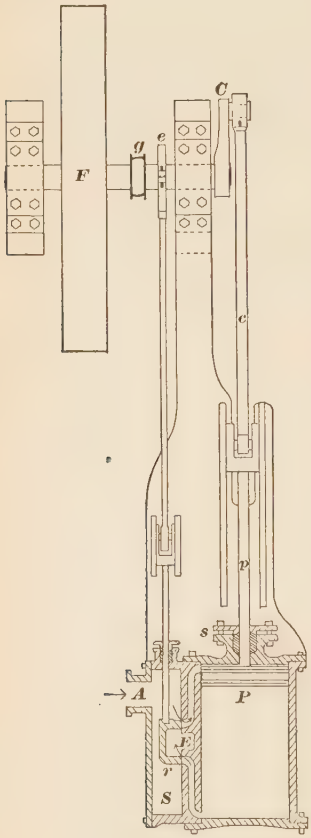


FIG. 134.—Steam engine, simple form.

200. Description.—Fig. 134 shows many of the principal parts of a simple steam engine. The steam from the boiler passes through *A* into the *steam-chest, S.* The *slide-valve, r,* is moving backward (toward the bottom of the page), and the steam is just beginning to pass by it from the steam chest through the narrow channel called a *port* into the *cylinder,* where it will push the *piston, P,* backward (downward in the figure) to the other end of the cylinder. From that part of the cylinder that is behind (below) *P,* steam, which has already done its work in pushing *P* forward to its present position, is beginning to escape under the slide valve into the *exhaust-pipe* at *E.*

As the piston *P* moves back and forth, it carries the *piston-rod, p,* which is jointed to the *connecting-rod, c,* and thus

makes the *crank*, *C*, revolve. With this crank the *main shaft* revolves, carrying the eccentric, *e*, and the pulley, *g*, from which a belt, not here shown, runs to the governor (Fig. 136). Upon the main shaft is fixed also the *fly-wheel*, *F*, which steadies the motion of the other parts and often serves, too, as a driving pulley, communicating power by means of a belt from the engine to the machinery driven by it.

A different view of the piston rod, connecting rod, crank, and fly wheel, is given in Fig. 135.

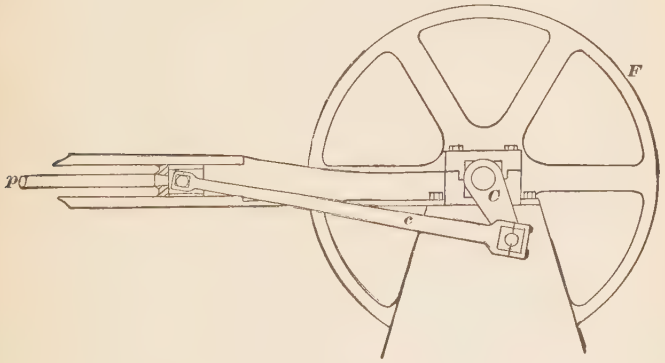


Fig. 135.—Connecting rod, c; crank, C; fly-wheel, F.

The *governor* of an engine is a device for controlling the admission of steam to the cylinder in accordance with the magnitude of the load, so that the speed may be nearly uniform, although the load may vary. It is usually some revolving contrivance, varying in speed with the main shaft of the engine, which spreads out, under centrifugal action, with increasing speed, and in spreading reduces the size of the passage by which steam is admitted to the valve chest, or controls the valves in such a way that "cut off" occurs earlier in the stroke. See Fig. 136, where *p* is a pulley made

to revolve by a belt, not shown, leading from a pulley, like g in Fig. 134, on the main shaft. The revolution of p causes the revolution of the balls b and b, and the outward motion of these balls is accompanied by a downward motion of the rod r, which leads to a valve controlling the inlet of steam to the steam chest.

201. Action and Economy of the Steam; Types of Steam Engine.—The pressure of the steam escaping from the cylinder, resisting the piston that is driving it out (see Fig. 134), is called *back pressure.* It is evidently a resistance to the action of the engine, and the smaller it can be made the better the action is.

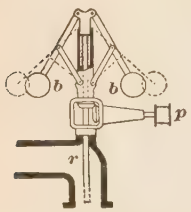


FIG. 136.—Fly-ball governor.

If the escape is into the open air, the back pressure must, on the whole, be greater than the atmospheric pressure. By connecting the exhaust pipe with a chamber, called a *condenser* (not shown here, but see B in Fig. 142), from which the air is excluded and which is kept cool by means of running water, the steam which has passed through the pipe is rapidly condensed. This action maintains in the condenser a more or less perfect vacuum, into which the cylinder discharges with great freedom, reducing the back pressure far below the pressure of the atmosphere.

If steam were freely admitted to either end of the cylinder during the whole of the stroke which begins at that end, the termination of each stroke would find the cylinder filled with steam under a pressure nearly as great as that of the boiler, and this steam, passing off into the condenser, would be wasted.

High pressure steam should not be thrown away. It should be made to expand and give up a considerable part of its heat energy in doing useful work before it is released.

Accordingly, the valves of most engines are so contrived that the admission of steam to the cylinder is stopped, or *cut off*, before the end, and usually before the middle, of the stroke, after which the steam already admitted "works expansively," falling in pressure and temperature till the stroke is ended. By this device more work is accomplished by a given weight of steam used.

It is evident that the driving power upon the piston of an engine using steam expansively is irregular, varying during each stroke. The fly wheel is intended to prevent the irregularity of movement which this might cause.

The to and fro motion of the piston in an engine cylinder is called *reciprocating* motion, and engines having such an operation are called *reciprocating* engines, to distinguish them from the so-called *turbine* engines, in which the motive parts continually revolve. See § 204.

Some engines take steam from the boiler at a pressure of 250 lbs. or more per square inch, and before sending it to the condenser allow it to expand in work till its pressure is only 3 or 4 lbs. per square inch. This expansion takes place not all in one cylinder, but in two or more, the steam entering one after another. Engines having such an arrangement are called *compound engines.*

202. **Locomotive "Link."**—There are many devices for varying the time at which cut off (§ 201) shall occur in the admission of steam to the cylinder. Some of these make use of two or more valves having rather complicated relations to each other. The slide valve of the ordinary locomotive is very simple, not very different from that shown in Fig. 134, but an ingenious device called a "link motion," indicated by Fig. 137, gives the engineer the power of controlling at will the distance the slide valve shall move at each stroke, and thus admitting steam for a greater or less part of the stroke, according to need.

The rod V, in Fig. 137, which works the slide valve (not shown), is so controlled that it can move only back and forth in the direction of its own length. A stud at the end of V connects with the slot in the *link*, LL'. The point L is connected by means of the rod r with the *eccentric* E, the point L' by the rod r' with the eccentric E'. These eccentrics are disks fixed upon the main shaft, but not centred with it. The centre of each eccentric therefore revolves in a small circle around the axis of the shaft. Each eccentric turns in a collar which connects it with the

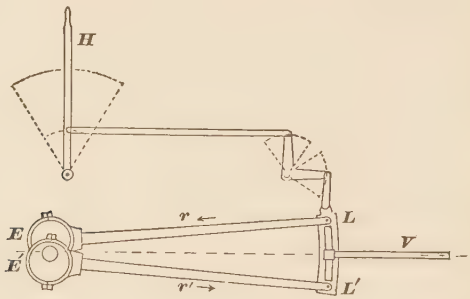


FIG. 137.—Locomotive "link" and reversing device.

rod r or r', and these two rods therefore move back and forth a short distance at every revolution of the shaft.

If, as we will suppose, the shaft is revolving counter-clockwise, the rod r and the point L are moving toward the left, while the rod r' and the point L' are moving toward the right. When the stud at the end of V is half-way between L and L', as in Fig. 137, the rod V and the valve at the other end of it have no motion. No steam is admitted to the cylinder. If, however, the handle H were pulled far to the left, L' would rise close to the end of V, and this rod and the valve would share the horizontal motion of the rod r' and the eccentric E'. If H were pushed far to the right,

L would approach the end of V, and the eccentric E would control the motion of the valve.

The link motion is, then, not merely a device for admitting steam during a longer or shorter part of the stroke. It is also a device for admitting steam to either end of the cylinder at will during almost any instant of the stroke. It therefore enables the engineer to reverse his engine, sending it forward or backward, as he pleases.

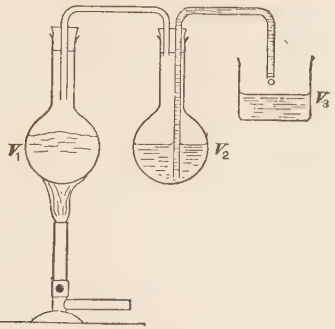


Fig. 138.—Work done by direct pressure of steam on water.

203. History of the Steam Engine.—Certain devices for doing work on a small scale by means of steam were described by Hero of Alexandria, who lived somewhat before or a little after the birth of Christ. One of these, the *principle* of which is shown in Fig. 138, was used by priests to work an apparent miracle. The two vessels, one containing water, the other wine, were concealed from the populace. A fire, represented by the lamp, was built upon the altar of some god. Steam, generated in the unseen boiler, pressed upon the surface of the wine in the second vessel and forced it through a tube contained in the hand of the god.

In seventeen centuries little progress was made. In 1698

Captain Thomas Savery obtained a patent for a mechanism to do work "by the impellent force of fire," which, though on a larger scale, was very much like the engine, if we may call it such, of the Egyptian priests. Savery's engine (Fig. 139) was used for pumping water by direct pressure of the steam upon its surface, but his device for getting the water into the vessel where steam pressure could act upon it showed an advance over the practice of the priests. He connected the vessel, P_1 or P_2, by means of a tube with a body of water at a lower level, filled the vessel with steam from a boiler A, closed it, and condensed the steam within it by pouring cold water upon the outside. Into the vacuum thus formed water from the tank was forced by the atmospheric pressure through the valve R_3 or R_4, which could open upward only. Then steam from the boiler was again admitted and the water was driven up through S.

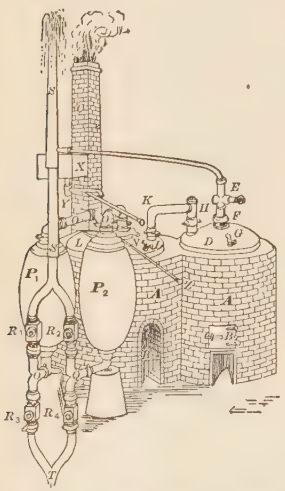


FIG. 139.—Early form of steam pump, without piston.[1]

Before many years the vessels which Savery had filled with steam and water alternately were replaced by a steam *cylinder* with a solid piston, by means of which a pump-piston was made to work in another cylinder, which received and discharged the water. Steam which had done its work under the first piston was still condensed by pouring cold water upon the outside of the cylinder. But one day, by accident it is said, some water got into the cylinder,

[1] From Bourne's *Steam-engine.*

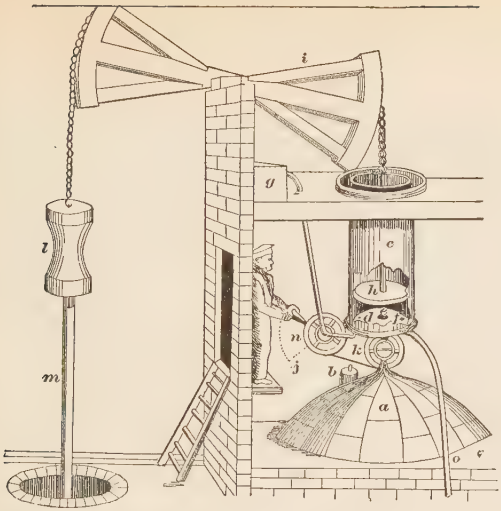


FIG. 140.—Early form: cylinders and piston, admission valve worked by hand.[1]

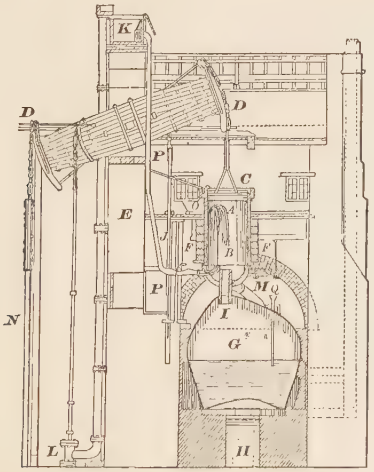


FIG. 141.—State of steam engine as Watt found it.[2]

[1] From Galloway's *Steam-engine*. [2] From Bourne.

and the engine was found to work much better than before. Presently it became the regular practice to inject water into the cylinder, as in Fig. 140, where a boy is employed in operating the water and steam cocks.

In 1775, when steam engines were nearly in the stage of development represented by Fig. 141, which is a picture of Smeaton's engine, James Watt, instrument maker to the University of Glasgow, was employed to repair one. He

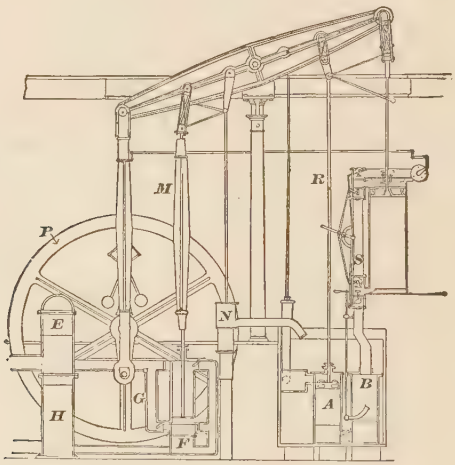
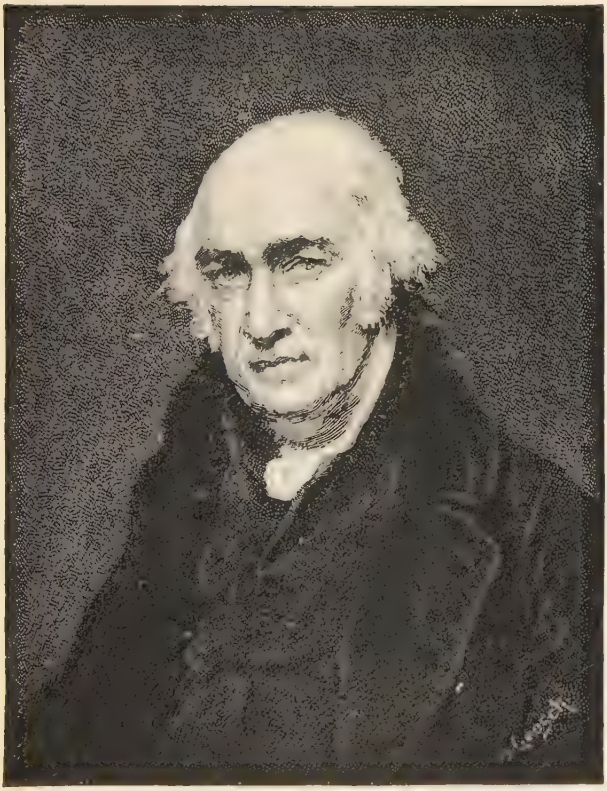


FIG. 142.—State to which Watt brought the steam engine.[1]

saw the great possibilities of steam power and devoted the remainder of his life to the construction and improvement of the steam engine. The enormous advances which he made are to be seen by comparing his own perfected engine (about 1800), shown in Fig. 142, with the rude contrivance (Fig. 141) which preceded it.

Watt found the steam engine an instrument of local and

[1] From Bourne.



James Watt

special use, pumping water from mines. He made it one of the great factors of civilized life.

Fig. 142 shows or indicates the following very important features, all due to Watt:

(1) *Crank connection,* for driving rotary machinery.

(2) *Governor,* see § 200.

(3) *Double action,* steam being forced, by a contrivance obscurely indicated at *S*, to act on the two faces of the piston alternately.

(4) *Expansive working* (§ 201).

(5) *Condenser, B,* distinct from the cylinder.

204. Turbine Engines.—Hero of Alexandria, who described a steam engine which is mentioned in §203, described also another, a revolving affair, the principle of which is essentially the same as that of a revolving lawn-sprinkler. It is sometimes represented as a spherical boiler, mounted so as to revolve about a vertical axis, and furnished along its equatorial belt with a number of outlet tubes from which the steam escapes horizontally at right angles with the radii of the sphere. Seen from above, the apparatus probably looked something like Fig. 143, in which the small arrows indicate the direction of outflow of the steam and the long arrow shows the direction of revolution of the boiler. This engine, which usually bears Hero's name, was regarded only as an interesting toy until near the end of the last century, when De Laval, a Swedish engineer, invented and put to industrial use, as the motor of centrifugal milk and cream separators, a steam turbine similar in principle to this ancient form. Such an engine is called a *reaction* turbine. The force of the reaction cannot be very great unless the relative velocity with which the steam and

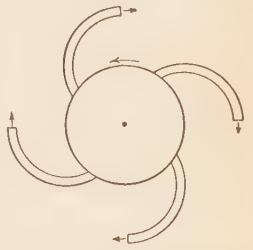


Fig. 143.—Illustrating the reaction-jet engine described by Hero.

the revolving wheel part company is very great. But the steam, if it left the engine with much velocity, would carry away much kinetic energy (§ 139), and so would waste power; accordingly, economical working requires the wheel of the reaction turbine to revolve very rapidly[1] indeed, so rapidly that it may fly apart by mere centrifugal action. Purely-reaction steam turbines are, therefore, not used on a large scale.

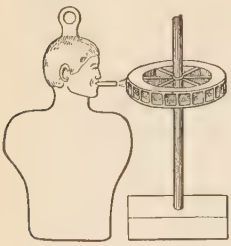


Fig. 144. — Illustrating Branca's impulse-jet engine.

Another type of engine, called the impulse turbine, was foreshadowed in the machine invented about the year 1629 by Branca, an Italian. Fig. 144 shows the principle of his apparatus, a stream of steam issuing with great velocity from a tube connected with the boiler and striking the vanes or blades, of the wheel. This engine, in its original form, did not prove to be important; but De Laval, who invented a flaring tube or nozzle (see Fig. 145) for imparting very great velocity[2] to the jet of steam issuing from the boiler, and curved the blades of the wheel, made the impulse turbine a commercial success. Here also, a very high

Fig. 145.—Illustrating the De Laval impulse-jet turbine.

[1] In one "purely-reaction steam turbine" the English engineer Parsons used about 80 revolutions per second.

[2] The maximum velocity with which a gas or any fluid closely resembling a gas can be driven through a *cylindrical* orifice is approximately equal to the velocity of sound in the fluid. With a flaring orifice of the right shape the velocity of the expanded gas at the outlet may be considerably greater than the velocity of sound in such a gas.

speed of rotation is required, and the pure impulse type is not used in the largest turbines.

A combination of impulse and reaction, with several stages of expansion from the high pressure to the low pressure part of the apparatus, corresponding in a way to the successive expansions in reciprocating compound engines (§ 201), is now much used. The number of revolutions per second in such turbine engines, though large compared with the number of strokes per second in reciprocating engines, need not, for economical working, be dangerously great.

One of the great merits of turbines, as compared with the old types of steam engines, is their relatively very small size for a given horse power. On the other hand, they have the disadvantage that they cannot readily be reversed and so made, for example, to drive a ship backward.

205. Gas Engines.—This name does not stand for steam engines using gas instead of coal for fuel, but for engines in which some gas is used, instead of any readily condensable vapor, as the working substance in the cylinder. This gas may be air, heated at one part of the stroke by flame and cooled at another part by water, from without, in what is commonly called a *hot-air* engine, or it may be some combustible gas or vapor, which is actually taken into the cylinder, together with the air needed for its combustion, during one part of the stroke, or cycle, is burned there during another part of the stroke, and is let out or driven out during another part. Engines of this latter type are said to have *internal combustion.*

In stationary gas-engines, which are sometimes used for industrial purposes, the fuel may be ordinary illuminating gas or a special kind of permanent gas, not vapor, but in the most familiar types of small engines for locomotive purposes, such as are used in automobiles and small "power-

boats," the fuel is commonly the vapor of gasoline or alcohol.

On account of the highly combustible, almost explosive, character of such fuel, the reservoir in which it is carried should be as far as is practicable from the engine, to which it is conveyed in liquid form along a narrow pipe. Before entering the engine cylinder it is vaporized by and mixed with a stream of air in the so-called *carbureter* (see Fig. 146).

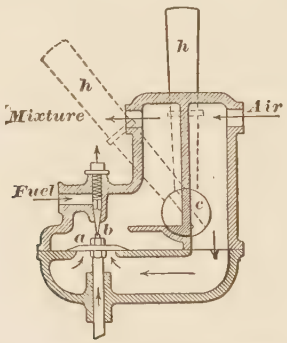


FIG. 146.—One form of "carbureter" for an internal-combustion engine. The machinery lifts the valve *a* at intervals, allowing air to pass; *a* lifts the needle-valve *b*, allowing fuel, gasoline or alcohol, to enter. The air and fuel mix and pass on to the cylinder. By a movement of the handle, *h*, air can be admitted at C, thus changing the composition of the gaseous mixture.

This mixture is in many engines *drawn* into the cylinder by a forward motion of the piston, compressed during a backward motion, exploded by an electric spark during the next forward stroke and ejected, burnt and useless but still very hot, in the next backward stroke. The two forward and two backward strokes of the piston, which must occur for each explosion, make a *four-part-cycle*, often rather confusingly called a *four-cycle action*.

Since the motive power of any internal-combustion engine is generated within the engine itself, and only while

the engine is running, and since it fails unless an igniting spark[1] is applied to the gas mixture at the right instant in each cycle, the art of starting such an engine and keeping it going is much more troublesome than the management of an ordinary steam engine. The more powerful gasoline engines, such as are used in automobiles, are sometimes started by means of compressed air, provided by the previous action of the engine in normal operation or by the muscular labor of the chauffer in "cranking" the machine. Smaller engines of the same general type may be started without the compressed air device by giving the fly wheel one or two revolutions by hand, thus setting the carbureter at work, by drawing a stream of air through it, and filling the cylinder with the proper mixture to be ignited by the electric spark. When ignition occurs the rapidity of revolution increases suddenly, and for safety it is necessary to have some device for releasing the hand automatically from the wheel as soon as the explosions begin. Some engines, when stopped, retain an explosive charge for hours and can be started by merely applying the spark.

It is evident that an actual flame, acting on the bare metal of the piston face and the cylinder wall, is likely to make it dangerously hot, unless some measures are taken to prevent such a result. Accordingly, it is customary to make the wall of the cylinder hollow and to force water through the jacket thus formed. With a stationary engine, or in a motor boat, water can be used freely for this purpose as the supply is practically unlimited; but an automobile cannot take a very large amount at one time, or get more while in motion, and so must economize in the use of water. Therefore a device has been found for cooling the jacket

[1] During the first few revolutions of the engine the spark is usually supplied from a small "battery" of "dry cells" or storage cells; but after speed is established a small dynamo, a "magneto," driven by the engine itself, furnishes the electric power for the ignition.

water rapidly while the car is in motion. The trunk-shaped affair carried at the front of an automobile contains an ingenious arrangement of pipes, through which the jacket water circulates in one part of its course. These pipes present a large surface to the air, which rushes between them at great speed when the car is in rapid motion and carries away heat so fast that the water in the pipes, though it remains warm enough to give off vapor slowly through a small opening at the top of the radiator, does not waste away very fast.

The very high value of the combustion temperature in gas engines, which is the T_1 of the efficiency formula (§ 184), makes the efficiency of such engines greater than that of most steam engines, and this merit, combined with their very high speed—that is, great number of revolutions per minute, gives them a large horse power for their size and weight. This has made them especially useful for automobiles, small power-boats, and flying machines, and the consequent multiplication of such vehicles has, on the other hand, stimulated inventors to improve the engines as fast as possible, so that the two decades following 1890 saw a great development of their powers and their uses.

206. Reversed Heat-Engines: Heat-Pumps; Refrigerating Machines.—Certain machines, which we may call *reversed heat-engines*, produce heat at the expense of work, and their operation is often accompanied by a passage of heat from some body to some warmer body. They may take heat from some cold body, add to it the heat equivalent of their own work and deliver the whole to some warmer body. The working substance in such *heat pumps* may be air, which absorbs heat from the cold body in expanding and gives out heat to the warmer body during or after compression, or it may be some volatile fluid, ammonia for example, which absorbs heat from the cold body during evaporation and

gives out heat to the warmer body during condensation. In either case the heat-pumping part of the apparatus is usually driven by a regular heat engine working in the usual way, the whole making a rather complicated "plant," in one part of which heat goes down hill in temperature from the boiler to the condenser, losing some part of itself in doing work on the way, while in another part heat goes up-hill in temperature from the cold body to the warmer body, gaining somewhat on the way by the work done on the conveying fluid.

The whole operation is somewhat like that of a waterfall driving machinery by means of which water is elsewhere pumped from a lower to a higher level. The analogy is not perfect, however, for no water is really turned into something else or turned back again in any part of the whole water-power affair.

Such *refrigerating machines*, as they are called, are much used for producing and maintaining a low temperature in cold-storage rooms. Brine, a strong solution of salt in water, which can be cooled many degrees below 0° C. without freezing, is pumped through a system of pipes, taking heat from the cold storage chamber in one place and giving heat to the evaporating ammonia in another place. Heat is taken away from the condensing ammonia by ordinary water, which also is carried to and from the machine in pipes.

In a different type of apparatus, called an *absorption* refrigerating machine, the parts of which are *indicated*, not *pictured*, in Fig. 147, the ammonia, after taking heat from the brine by evaporating near it in V, is absorbed in A by water, which can take up a large amount of it. The solution thus formed is driven by the pump P to the receiver, or "generator," G, where it is heated by steam and yields up ammonia vapor at a comparatively high pressure. This vapor is condensed in C by the action of cold water, and

the resulting liquid ammonia passes through the valve S to a place of lower pressure in V. At this lower pressure it evaporates again in V.

Different as this absorption machine is in its action from the engine and heat-pump combination previously described, the results are nearly the same in the two cases. For in the absorption machine as in the other we have one quantity of heat, that brought by the steam into G, conveyed from the high temperature body, the boiler (not shown in Fig. 147), to a low temperature body, the water

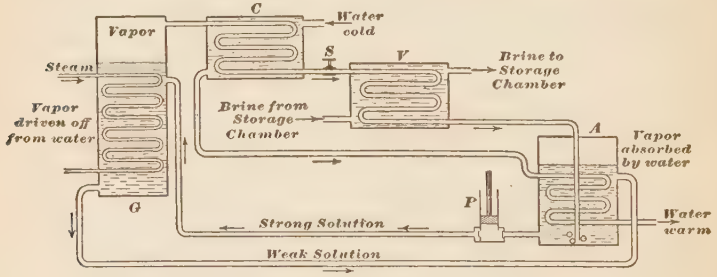


FIG. 147.—Illustrating an "absorption" refrigerating system.

in the jacket of C, while another quantity of heat is conveyed from a cold body, the brine in the jacket of V to a warmer body, the water which carries away from A the heat which is generated there by the absorption of the ammonia vapor in the solution water.

207. Condensation of the so-called Permanent Gases.—The English chemist and physicist Faraday was perhaps the first experimenter who succeeded in liquefying any of the gases which are permanent at ordinary temperatures. He was able by the joint use of cold and pressure to liquefy chlorine, and he afterward managed by these simple means to reduce many gases to the liquid state. His method is indicated in Fig. 148. A strong, closed, glass tube contains

in one end a solid or liquid which gives off at a high pressure, when heated, the gas that is to be condensed. The other end of the tube is surrounded by a freezing-mixture, and in this end condensation occurs.

Six gases, however, oxygen, nitrogen, hydrogen, marsh-gas, nitrogen dioxide, and carbon monoxide—had not been liquefied up to the year 1877. In that year a Genevese, M. Pictet, and a Frenchman, M. Cailletet, working independently and by somewhat different methods, succeeded in liquefying all of these gases.

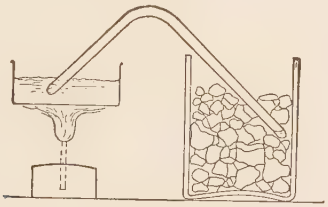


Fig. 148.—Faraday's liquefaction of chlorine.

Liquid air is now obtained by the process of "regenerative cooling," which may be described generally as follows: Air at a rather low temperature is allowed to escape through a valve from a very high pressure. Cooled, but not liquefied, by its expansion this air passes off through a channel surrounding the tube containing the still unexpanded air, which is thus cooled somewhat, so that, when *it* expands through the valve, it reaches a lower temperature than the preceding air reached. Continuation of this process finally cools the inner part of the apparatus so much that a part of the air within it is liquefied at a temperature not far from $-190°$ C. The heat taken from the apparatus and from the cooling air is carried off by running water.

To retard its absorption of heat from surrounding bodies

liquid air is kept in double-walled silvered glass vessels having a vacuum between the walls. Evaporation of the liquid air into the outer air must be permitted, lest the temperature rise and the pressure of the vapor become dangerously great.

Problem.

Explain as fully as you can the transformations of energy that take place when steam is generated in a locomotive engine, the engine employed to draw a train at a high rate of speed, and the motion finally checked by applying brakes to the train.

CHAPTER XX.

SOUND.

208. Definition.—The word *sound* refers sometimes to the sensations we receive through the auditory nerves, sometimes to the external cause of these sensations. Failure to recognize this distinction often leads to unprofitable argument. We shall use the word mainly in the latter sense.

209. Transmission of Sound.—Light comes to us perfectly well through a vacuum. Can sound do the same?

(1) Place a small alarm-bell, in operation, on a wad of cotton beneath the bell jar of an air pump, and exhaust the air as rapidly and as perfectly as you can.

Does the rarefaction of the air have any effect upon the loudness of the sound as perceived by the ear?

Let the air re-enter rapidly and note the effect.

If the alarm bell used in the preceding experiment had rested directly upon the firm pump-plate, the sound would have made its way to the outer air by way of this solid medium. The following experiment illustrates still further the capacity of solids for carrying sound:

(2) Hold one end of a slender rod of wood 3 m. or more in length firmly between the teeth while some one sounds a tuning fork and then holds the stem against the free end of the rod.

It is an interesting experiment to hold the head under water while a comrade at a distance raps two stones together under water.

210. Velocity of Sound.—A method of measuring roughly the velocity of sound in air is given in Exercise 26. When

the air is still and its temperature is 0° C., this velocity is about 331 m., or 1086 ft., per second. It varies but little with the pressure of the air, but is proportional to the square root of the absolute temperature and accordingly increases, under ordinary atmospheric conditions, about 60 cm., or 2 ft., per degree C. rise. In gases of different density, at the same pressure and temperature, it is inversely proportional to the square root of the density. It is greater in most solids and liquids than in gases. In water it is about 1440 m. per second. Through the interior of the earth, to judge from the records of earthquake disturbances, it may vary from 2500 m. to 16,000 m. per second, being greater far beneath than near the surface.

211. Sources of Sound.—If we trace back to its source a sound that comes to our ears, we can usually find it in the movements, often very minute, of some body which is said to *give out* the sound. Often this motion is apparent to the eye, as in the case of vibrations of sounding strings or long tuning forks. Frequently the sense of touch will show this state of motion in a sounding body, a short tuning fork for instance, when sight does not. Sometimes, as in the case of faint sounds of high pitch coming from short, thick metal bodies, we do not perceive the motion of the body directly in any way, and only by what we know of other sounding bodies convince ourselves that such motion exists.

212. Sound Waves.—But how is it that the air or any other medium brings the sound from its source to our ears? Do particles of the medium, propelled from the sounding body, move on until they reach our ears, as a baseball moves from the swinging bat to the hands of the player who catches it? Evidently this cannot be the case when the medium which transmits the sound is a solid, nor is it the case when the medium is a liquid or a gas. Particles of liquid or gas, if sent forth with the velocity of sound, about

340 m. per second, would soon be arrested by the resistance of their fellows, unless the latter were moving with similar velocity and direction. It is a *concussion,* or *jar,* sent through the air, that breaks glass miles away from an exploding powder mill or dynamite factory.

Now, a disturbance which travels through any medium without permanently displacing the parts of that medium, which is, in fact, merely passed on from one set of particles to the next, is called a *wave.* There are various kinds of waves. With waves at the surface of water all are familiar. Waves bring us energy from the sun. (See 154). In these two kinds of waves there is a motion of the particles of the medium to and fro *across* the direction in which the waves move forward. A sound wave consists merely of a *compression* traveling forward, followed by a *rarefaction.* Every particle of the medium affected by the wave is in turn crowded close to its neighbors and then withdrawn from them, making, on the whole, a slight[1] excursion forward and backward, *parallel to* the direction in which the sound is travelling.

This theory of the nature of a sound wave is sustained by our knowledge of the properties of bodies and by experimental evidence. We shall not enter upon the argument here, but the student will by himself probably find it easy to see, in a general way, how a wave of compression may be sent through a medium such as air. If we—as Professor Tyndall did in his lectures on Sound—explode a small balloon, at the moment of explosion the layer of air immediately about the balloon is violently[2] compressed. By

[1] Lord Rayleigh concluded from experiments that an audible sound may be carried by air particles whose wave-excursions from zero position are about a hundred millionth of a centimeter.

[2] The suddenness and violence of the impulse given to the air by the expanding or contracting source of the disturbance are important factors. The various *silencers, mufflers,* etc., used to reduce the noise of rifle-shots, engine-puffs, and the like, all oppose some obstacle to the sudden expansion of the highly com-

virtue of its elasticity this compressed air will instantly transmit the compression to another layer immediately outside of itself, and will at the same time itself become more rarefied than before the explosion. In this way a condensation and a rarefaction—in fact, a series of them—move outward in all directions from the exploded balloon. A single condensation and its accompanying rarefaction together constitute a sound wave.

The sound waves produced by a body in mid air giving off sonorous vibrations, as in the case of a bell rung in a

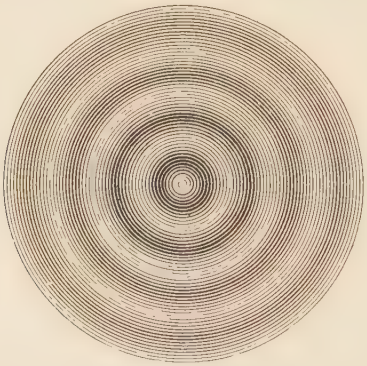


Fig. 149.—Illustrating the spread of sound-waves.

lofty steeple, have, at a distance from the source, nearly the form of hollow spheres having the source as their common centre. See Fig. 149.

It is rather hard at first to see why a condensation or rarefaction in air should travel in one direction rather than another, why it should not go backward as well as forward. The following diagram (Fig. 150) may make the matter a

pressed, combined, gases into the atmosphere. This obstruction, in the case of engines, increases somewhat the *back pressure* (§ 201) and thus lessens the effectiveness of the stroke.

little plainer: In the line AB the dots 1–16 represent particles of air which are equidistant from each other when undisturbed. A sound wave coming along the line AB in either direction will change the positions of these particles without moving them out of that line, but to avoid confusion the particles in their disturbed condition are here represented on another line, $A'B'$.

On this new line particle 1 is displaced a little to the right from its natural position, 2 is not displaced, 3 is displaced

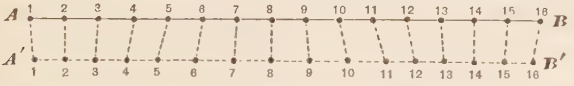


FIG. 150.—Illustrating the movements of air particles in the transmission of sound.

a little to the left, 4 a little more to the left, 5 still more and farthest of all, 6 being displaced no more than 4, 7 no more than 3, and 8, like 1, not at all. No. 9 is displaced to the right, 10 still more, etc. Looking along the whole line we see a condensation with centre at 2, a rarefaction with centre at 8, another condensation with centre at 14. At the centre of each condensation or rarefaction is a particle in its natural position.

Is it possible to tell from this diagram whether the wave is moving from A toward B or in the opposite direction? No. That depends upon the present *motions* of the particles, concerning which nothing has been said.

Consider the particles 2 and 3. If they are both moving toward the right, 2 will in a moment be displaced just as 1 now is, and 3 will be in its natural position, as 2 now is; in other words, the condensation now central at 2 will have moved to the right. The *wave* will be moving toward the right.

If we regard the other particles, we shall find everything

consistent with this direction of the wave, although we shall not find all the particles moving toward the right. No. 4 is moving to the right, 5 is at its turning-point, 6 is moving to the left, and also 7, 8, 9, and 10. No. 11 is at its turning-point, 12 is moving toward the right, and so are 14, 15, and 16, which are merely repetitions of 2, 3, and 4.

We can, therefore, say: *When a sound wave is moving toward the right, the particles at the centre of condensation are moving toward the right and those at the centre of rarefaction are moving toward the left.*

213. Inverse Square.—Since sound waves in a uniform medium travel outward from their centres with uniform velocity in ever-enlarging hollow spheres, and since the surfaces of spheres are proportional to the squares of their radii, it is plain that a given amount of energy in the shape of a sound wave must, as it recedes from its source, occupy—if none of it is lost on the way—successive portions of space which increase in proportion to the square of the distance from the source. Fig. 151 represents two of the positions occupied by a part of a wave front included between certain radii.

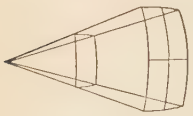


FIG. 151.—Illustrating the law of "inverse square."

Accordingly, the intensity of the wave at any given point must lessen as the point is taken farther and farther away from the source of the wave. *The intensity of a sound wave (and its loudness to the ear) varies inversely as the square of its distance from the vibrating body which produced it.*[1]

214. Reflection of Sound: Echoes.—Sound waves may rebound or be reflected from surfaces against which they

[1] This is true only when the distance is so large in comparison with the dimensions of the sounding body that the latter may be considered a *centre* from which the sound waves proceed along radii. A person 10 ft. distant from the nearest point of a moving railroad train does not hear four times as loud a sound as one 20 ft. distant from the train.

strike. They obey the same law of reflection as light waves; that is the angle of reflection is equal to the angle of incidence (see § 245).

Place two large spherical mirrors (No. LXXIV) facing each other at opposite sides of the room. Place a watch at the focus *A* of one mirror and listen for the tick with an ear-trumpet (No. LXXV) the mouth of which is held at *B*, the focus of the other mirror. See Fig. 152.

The usefulness of the ear trumpet and of the speaking trumpet is due to reflection of sound, one concentrating the sound as it approaches the ear, the other preventing it from spreading unnecessarily as it leaves the mouth.

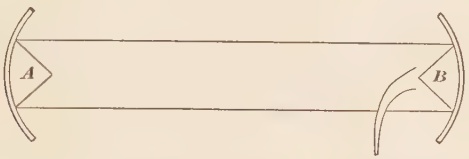


Fig. 152.—Reflection of sound; use of ear trumpet.

When a reflected sound, coming to the ear by some indirect course, arrives perceptibly later than the sound which has traveled directly, it is called an echo. An echo occurs when a sound traveling through one medium comes abruptly against another medium of different density. Echoes have much to do with the ease or difficulty which a speaker finds in making himself heard in a large hall. If the echo is nearly simultaneous with the direct sound, it reinforces the latter and may be a great benefit. If it is too much retarded or prolonged, it causes indistinctness. There is less echo from soft fabrics, like cushions or clothing, than from bare walls or benches. Hence, in a large hall, it is usually much easier to make one's self heard by a large audience than by a small and scattered one. The intro-

duction of a gallery into an audience room[1] sometimes reduces the echo.

The long, jarring rumble of a thunder peal is maintained by echoes from strata of air, from clouds, or from the earth.

215. Refraction of Sounds.—Sounds which *enter* one medium from another, or go from one place to another of different density in the same medium, suffer a bending, or *refraction*, somewhat as light waves do (§ 261). This sometimes occurs on a large scale in the open atmosphere, producing curious effects which are more or less troublesome in ordinary hearing and especially so in fog signals.

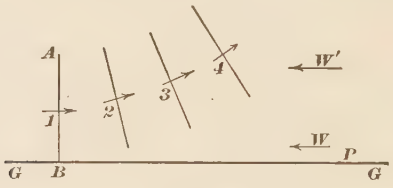


FIG. 153.—Refraction of sound.

Thus, if AB in Fig. 153 represents a wave front of sound traveling, in the direction of the arrow 1, over the ground GG, and if a wind is blowing in the opposite direction, the wave will not continue to move horizontally, but will mount in the air and perhaps pass unheard above the head of a person at P. The fact is that the wind current, indicated by the arrows W and W', has less velocity very near the ground, where it is impeded by various obstacles, than somewhat higher. Accordingly the upper part of the wave front of sound is retarded more than the lower part, the whole front becomes inclined and, traveling always in a direction at right angles with its own surface,

[1] For the best study of architectural acoustics ever made see the work of Professor W. C. Sabine, *American Architect* or *Engineering Record*, 1900, and *Proceedings of the American Academy of Arts and Sciences*, Vol. XLII, 1906.

shoots upward. This is the real reason why sounds appear to travel so badly against the wind. No atmospheric wind-velocity ever measured on the earth is one-fourth part as great as the velocity of sound, so that we must not, when we fail to hear a sound because of an opposing wind, think of the wind as actually carrying the sound away from us in the opposite direction to its natural course.

216. Musical and Unmusical Sounds.—All observe that some sounds have a musical quality, while others are merely noise.

(1) Upon the whirling apparatus fasten the disk apparatus (No. LXXVI).

Revolve the disk at a uniform rate several times a second, and hold the edge of a card lightly against the irregular row of pegs (see Fig. 154). Then hold the card against the other row of pegs. Is there any marked difference in the quality of the sound produced in these two cases?

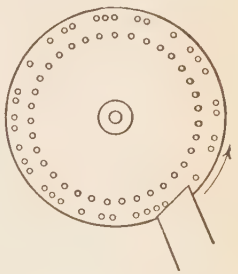


FIG. 154.—Musical and unmusical sound.

Although none of the effects obtained in this experiment are above criticism from a musical standpoint, they illustrate roughly the fact that air pulses striking the ear at regular intervals, and with a frequency greater than a certain limit, produce the sensation of continuous musical sound. All musical instruments are capable of giving out sound waves of regular intervals. The vibrating spring in the following experiment illustrates the action of a tuning-fork.

(2) Take a long, straight, piece of clock spring and fasten it in a vise or clamp, leaving about 40 or 50 cm. projecting horizontally. Set this part vibrating and note the regularity of its swings, observing that, like the beats of a pendulum, they take about the same length of time, whatever the length, or width, of the swing.

Shorten the vibrating part, and observe the effect upon the quickness of the swing. Shorten it to 2 or 3 cm., and observe that now it gives out a good musical note.

A long piece of rubber tubing, fastened at the ends and stretched, illustrating the action of the strings in a piano or violin, would be found to vibrate regularly with a quickness depending upon the degree of tension.

The vibrations of musical strings or wires will be considered at greater length farther on.

217. Measurement of the Length of a Sound Wave.— Since a musical sound consists of a series of waves following each other at regular intervals of time, we may call the distance between similar portions of two consecutive waves the *wave length* of the sound. We may take the distance from the point of greatest condensation in one wave to that of the point of greatest condensation in the next wave, or we may measure from rarefaction to rarefaction. The distance will be the same in both cases.

Suppose any kind of tuning fork, for instance a common "A" fork of 440 complete[1] vibrations per second, to be set vibrating, and to remain vibrating for one second. In that time we may suppose that, with the ordinary temperature, the sound will have travelled about 340 m. Then a sphere of 340 m. radius will have been filled with alternate shells, or layers, of condensed and rarefied air. Each complete vibration of the fork produced two layers, or one complete sound wave. So the length of the sound wave in this case is about $\frac{34{,}000}{440}$, or 77.3, cm.

Exercise 27 shows a simple and interesting method of measuring a wave length, or known fraction of a wave length, of sound in a tube. The principle of the Exercise can be discussed with the aid of Fig. 155, which represents

[1] A complete, or double, vibration is a movement in which the vibrating body goes over its whole path twice, once from left to right and once from right to left, that is, performs a *round trip*. The word *vibration* used alone will, in this book, mean such a complete vibration.

a glass jar containing a column of air of such length as to reinforce the sound of the tuning fork *A*.

Suppose the fork to be vibrating, the lower prong being at the position 1 and moving downward, sending a condensation into the jar. By the time this prong has reached the position 2, the condensation of the sound wave will, if the distance *BC* is properly chosen, have travelled from *B* to *C*. While the prong returns to 1 the condensation will travel back to 1. While the prong moves to 3 the condensation will continue to ascend, combining with and strengthening that sent directly from the prong, and so on.

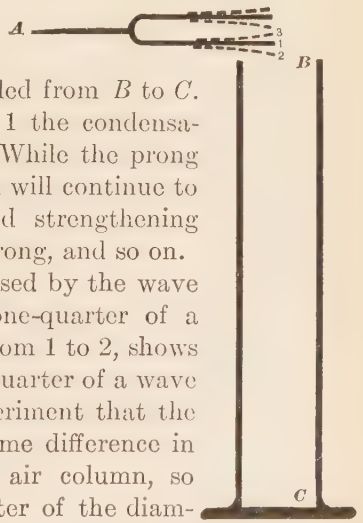


Fig. 155.—Resonant air-column.

The fact that *BC* is traversed by the wave while the fork is making one-quarter of a vibration, that is, swinging from 1 to 2, shows that the distance *BC* is one-quarter of a wave length. It is found by experiment that the width of the tube makes some difference in the effective length of the air column, so that in calculation one-quarter of the diameter should be added to the length of the cylinder.

If the jar were deep enough, the condensation might travel to the bottom and back just in time to coincide with the impulse sent out by the *second* up stroke of the prong. The depth of the jar in this case would be about three-quarters of a wave length. The same correction for width of jar would have to be made in this case as in the preceding one; but if both depths were determined by experiment, one being one-quarter of a wave length minus a certain quantity and the other three-quarters of a wave length minus the same quantity, the difference between the two

depths would be just one-half a wave length. This method is followed in Exercise 27.

Varieties of Musical Sounds.

Musical sounds differ among themselves in *loudness*, *pitch*, and *quality*.

218. **Loudness: Resonance.**—At the start the loudness of a musical sound depends upon the width, or *amplitude*, of the vibrations of the body giving out the sound waves, but the original loudness may be greatly reinforced by the vibrations of some other body, set into action by the sound itself. Increase of loudness produced in this way is called *resonance*.

In order that resonance may occur it is necessary that the natural time of vibration of the assisting body be the same, or nearly the same, as that of the original sounding body. The second body is gradually set into action by successive impulses from the first, each impulse coming just at the right time to add to the previous effect, very much as a heavily loaded swing is worked into wider and wider motion by properly timed pushes or pulls. If the two bodies did not naturally vibrate in unison, one impulse might destroy the effect of the previous one, so that the second body would be affected but little. Vibrations of the second body produced in this way are called *sympathetic* vibrations.

In Exercise 27 the reinforcing body was a column of air; but frequently solid objects serve a like purpose, as the following experiment shows:

(1) Take several unmounted tuning forks of different pitch. Put each in turn into vibration and then press the end of the handle against the table top, a door, a wooden box, etc., listening for resonant effects.

The fact that the same solid body may respond to several different notes is explained later, in § 225.

When a tuning fork is mounted upon a sounding-box, as in Fig. 156, the vibration is communicated through the wood to the air column within.

Musical instruments making use of vibrating strings or wires, like the piano and violin, are peculiarly dependent upon the resonance of the supporting wood. It is said that a coat of varnish may seriously affect the tone of a violin.

The transference of a given note from one sounding body through the air to another body of the same rate of vibration is strikingly shown by the following experiment:

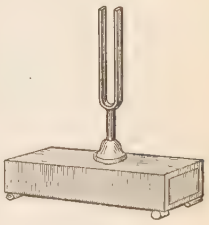


Fig. 156.—Tuning-fork on a resonator.

(2) Place two tuning forks of the same pitch, each mounted upon a resonance box, some feet apart on a table, the mouths of the boxes being turned toward each other. Set one fork into vigorous vibration by means of a bow, and, after some seconds, stop this fork and notice whether the sound is now given out by the other one.

219. Pitch.—The next characteristic of musical sounds to be considered is *pitch.*

It is easy to show, by turning with varying speed the rotating disk shown in Fig. 154, that the *sharpness* of sound changes as the vibrations producing the sound become more or less frequent. A like fact is noticed in the experiment with the vibrating clock spring, in § 216. An increase of sharpness in a musical sound is called a rise of *pitch.*

It is evident that an increase of frequency in the vibrations raises the pitch of a sound. We may naturally inquire how many vibrations per second are needed to yield a tone of given pitch, for instance some familiar note of a musical instrument. This question is taken up in Exercise 28.

The pitch of musical instruments has changed very

greatly in Europe from time to time and from place to place. A certain note, "A," has ranged all the way from 375 vibrations per second to 567. In Germany, during the seventeenth and eighteenth centuries the pitch of this particular note was about 425, but it has now risen considerably. The following quotations are from the History of Musical Pitch given by Ellis in the appendix to his Translation of Helmholtz's *Sensations of Tone:* "As this [the eighteenth century] was the period of the great musical masters, and as their music is still sung, and sung frequently, it is a great pity that the pitch should have been raised, and that Handel, Haydn, Mozart, Beethoven and Weber, for example, should be sung at a pitch more than a semitone higher than they intended. The high pitch strains the voices," etc. "The rise of pitch began at the great Congress of Vienna, 1814, when the Emperor of Russia presented new and sharper wind instruments to an Austrian regiment of which he was colonel. The band of this regiment became noted for the brilliancy of its tones."

The number of vibrations per second in a given note is called the *vibration-number*, or *vibration-frequency*, of the note.

220. Combinations of Sounds Differing Slightly in Pitch: Beats.—When one is within hearing of two sounding bodies differing a little in pitch, it will happen at certain instants that the condensations of the two sets of sound waves, and also the rarefactions, coincide at the ear, giving an effect which is the sum of the two effects produced by the bodies acting singly. At other instants the condensations of one set of waves will coincide with the rarefactions of the other set, giving an effect which is the difference of the two single effects. The result is a succession of changes in the combined sound, from strong to weak and back again. These fluctuations are called *beats*.

Take two tuning forks of the same pitch, mounted on sounding boxes, and set both into vibration by means of a bass viol bow drawn across the ends of the prongs. If the two forks are really of the same pitch, the two sounds will appear to be one, which will gradually die away.

Load both prongs of one fork, near the end, with a small quantity of wax, thus diminishing its rapidity of vibration, and then sound the two together, listening for the beats.

Nearly every musical instrument gives out sounds of different pitch at the same time, and different sounds from one instrument are sometimes related to each other in such a way as to cause beats. A gong or a bell nearly always gives perceptible beats when sounding undisturbed.

If sound waves are represented, as they may be for convenience, by a diagram resembling water waves, elevations

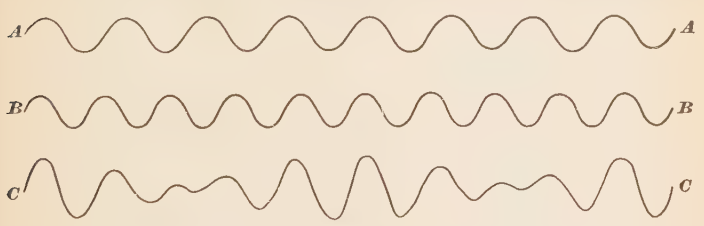


FIG. 157.—Illustrating the formation of "beats."

corresponding to condensations and depressions to rarefactions, the nature of beats is well illustrated by Fig. 157, in which *A* represents a sound of a certain pitch, *B*, a sound of somewhat lower pitch, *C*, the result of combining the two.

221. **Harmony and Discord; The Musical Scale.**—When two or more musical sounds are simultaneously produced, their effect upon the ear may or may not be agreeable. For example, if the tones are those produced by a "violin-A" fork, of 440 vibrations per second, and another fork of 220

vibrations per second, so that the ratio is 2:1, the interval between the two tones is called an *octave* and the effect is pleasing. Such a pleasing effect is called *harmony*. But if the fork of 440 vibrations is sounded at the same time with another of 420 or 460 vibrations, for example, the result is unpleasant. It is called *discord*.

Tones an octave apart are produced by any two sonorous bodies in vibration when their vibration numbers have to each other the ratio 2:1. An octave is the simplest possible musical *interval*, and this is subdivided, in what is called the natural *scale*, or *gamut*, into several smaller intervals. The vibration numbers of the whole series of tones, comprising a given tone, its octave above, and the intermediate tones of the so-called *natural* scale (see § 228), bear to each other relations indicated by the following representative numbers:

do		re		mi		fa		sol		la		si		do
C	:	D	:	E	:	F	:	G	:	A	:	B	:	C
24	:	27	:	30	:	32	:	36	:	40	:	45	:	48
1	:	$\frac{9}{8}$	:	$\frac{5}{4}$	:	$\frac{4}{3}$	:	$\frac{3}{2}$	:	$\frac{5}{3}$	:	$\frac{15}{8}$	:	2

No actually used octave has the vibration numbers 24, 27, etc., but these are the smallest whole numbers which are proportional to the vibration numbers of the tones in the natural scale. The number for "middle C" of the piano is 256. The lower line of numbers shows what the vibration frequencies of the various notes would be if the vibration frequency of the lowest note were called 1. Thus, the number of vibrations per second in *sol* is $\frac{3}{2}$ times as great as that in *do*.

Any person trained in music will see that the notes which go best with *do*, making the best harmony, are those for which the vibration ratio, with reference to *do*, can be expressed by the use of small whole numbers. The simplest

ratio, from this point of view, is that of the lower and the higher *do*, which go particularly well together. The next simplest ratio is $\frac{3}{2}$, that of *do* and *sol*, which notes also go very well together. Especially bad ratios are the $\frac{9}{8}$ and $\frac{15}{8}$.

The connection between simple ratios and harmony has been known for a long time. People used to try to explain it by saying that the human mind or soul delighted in simple ratios as such; but Helmholtz pointed out that many persons have excellent perception of musical harmony who never even heard of vibration numbers. He explained *discord* as due to rapid *beats*, and found, for certain tones, that beats are especially unpleasant when they occur about thirty times a second, too rapidly to be counted directly or even recognized by the ear as beats.

Keeping the above-given numerical relations in mind and knowing the vibration number of any particular note of the scale, we can calculate the number of vibrations that will be required to produce any given tone. Suppose, for example, that we are asked to find the number of vibrations per second that will produce the tone G next preceding that of the "violin-A" fork of 440 vibrations. Then we have the proportion $36 : 40 :: x : 440$, from which $x = 396$ vibrations.

The note *re* is called the *second* of the *do* below it; *mi* is called the *third* of the same *do*, *fa* the *fourth*, etc.

The intervals of the "tempered" piano-scale will be discussed in § 230.

222. **Quality.**—Aside from differences in the loudness of sounds and in their pitch, there is a well-known difference in *quality*. It is this, in part, which enables one to distinguish the voices of acquaintances in the dark, or to pick out a familiar voice singing among many others. Still more marked are the differences between the sounds of certain musical instruments, the violin and the flute for ex-

ample, even when both are producing sounds of the same pitch. The fact is that a musical sound is usually a combination of several tones differing in pitch, and what is called the pitch of the sound is merely the pitch of its loudest component.

Helmholtz invented a system of *resonators*, consisting usually of hollow brass bulbs or cylinders with an opening at one or at each end, each responding, by means of the air column within it, to some particular tone when any sound containing that tone is produced nearby. By the help of such instruments he analyzed the sounds of the human voice and explained, for example, the difference between the different vowel sounds when sung by the same voice at the same pitch. His explanation can be understood better after certain experiments on vibrating strings, etc. (§§ 223 and 224), have been given; but the student, in advance of all further discussion, will do well to try the following experiment, which is described in the great work of Helmholtz on *Sensations of Tone*:[1]

Lift the top of a piano so as to expose the strings to view, push down the loud pedal so as to leave the strings free to vibrate and give out their full volume of sound, then sing *into* the piano a prolonged *ä*, giving the sound as in *ah*. Keep the pedal down after the voice ceases, and listen to the response given by the strings. Then try the sound of *ō*, as in *note*, in the same way.

The piano returns the sound in each case with almost startling distinctness. With most of the other vowel sounds the effect is much less striking.

223. Fundamental Tone, Overtones, and Harmonics.— It has already been observed (§ 218) that a single body may give out several different notes. It is now time to study the behavior of vibrating bodies more closely. We may well begin with the transverse, sidewise, vibrations of strings or wires.

Stretch one of the sonometer wires (No. LXXXI) rather tight, until it gives a clear musical sound on being stroked at the middle with a bass-viol bow. While it is still sounding, drop a little $\wedge$-shaped rider, made of stiff paper, on the wire near the middle. Note the behavior of the rider.

Sound the wire again, and look at it in a strong light while it is sounding. Repeat the experiment with the rider, dropping it upon various portions of the sounding wire, until the trials have been carried quite to one end of the wire.

Then, holding the finger tip lightly against the exact middle of the wire, stroke the latter midway between the point where the finger is applied and either end. Then quickly remove the finger and put the rider in its place; then put it on some other portion of the wire.

Once more place the finger tip on the wire, twice as far from one end as from the other, sound it by stroking it half way between the finger and the nearer end, remove the finger and put the rider in its place; then put the rider on another portion of the wire.

Measure off the wire accurately into four equal parts, as in Fig. 158. Place a colored paper rider at point 2 and another at point 3. Place a white paper rider at each of the points *a*, *b*, and *c*. Hold the wire lightly at 4, and stroke it at *d*. Observe which riders are thrown off and which are not.

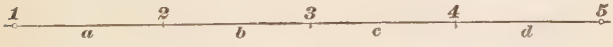


Fig. 158.—Riders on a sonometer wire.

These experiments will doubtless show that the vibrating wire has regions of greatest vibration and points of little or no vibration. The regions of greatest vibration are called *loops;* points of no vibration, *nodes*. In Fig. 159 the loops and nodes of some vibrating wires are shown. The centre of each loop is marked *l*, and each node is marked *n*.

Pluck one of the sonometer wires strongly at the centre, then at various intervals toward either end, until finally the end is almost reached. Note the clanging tones of high pitch that are produced as the end of the wire is approached, and observe how these quickly die away, leaving the lowest tone of the wire still sounding.

This lowest note is called the *fundamental note,* or simply the *fundamental,* of the wire. It is given by the form of vibration shown in Fig. 159, *A*. The higher tones produced by plucking the wire toward the end, in the way just described, are called *overtones.*

There are with some instruments two kinds of overtones, those which unite with the fundamental agreeably to the ear, and those which do not. The former are called

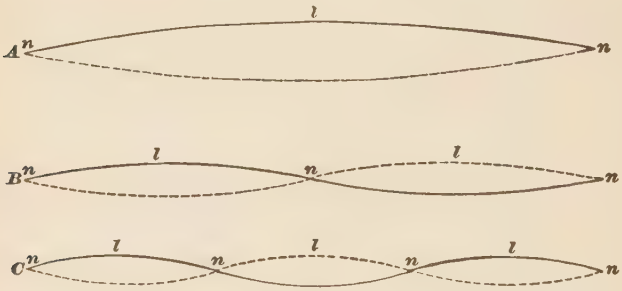


Fig. 159.—Nodes and loops of a vibrating string.

harmonic overtones, or *harmonics.* The overtones of freely vibrating wires are harmonics.

224. Laws of Vibrating Strings.—We can hardly undertake in this course of study to find by experiment the *laws of vibration* of strings or wires. They may be stated as follows: The number of vibrations per second is—

(1) *inversely proportional to the length of the string;*

(2) *inversely proportional to the square root of weight of the string per unit length;*

(3) *directly proportional to the square root of the stretching force.*

The application of the first of these laws to cases like those exhibited in Fig. 159 shows that the fundamental and the overtones of a string have vibration frequencies

bearing to each other the ratios of the numbers 1, 2, 3, 4, etc.

The fact has already been stated (§ 221) that the vibration frequencies of any two notes that unite pleasantly to the ear bear to each other some ratio that can be expressed by small whole numbers.

225. Vibrations of Plates: Chladni's Figures.—Strings are by no means the only objects that can vibrate in parts. Thin plates of metal or wood may give out a great variety of sounds corresponding to different modes of vibration. If this were not so, such instruments as the violin and piano would be very ineffective.

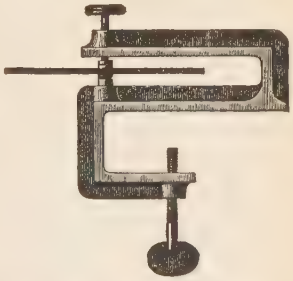


FIG. 160.—Support of "Chladni plate."

Plates of glass or of metal may be fastened at the middle by a clamp, as in Fig. 160, or in any other convenient way, and set vibrating by a violin bow. If sand is sprinkled on the surfaces of such plates while they are vibrating, it will arrange itself so as to mark

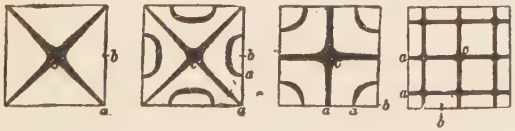


FIG. 161.—Sand figures on Chladin plates.

out the *nodal lines*, that is, the lines of least or of no vibration. The figures formed by the sand are known as *Chladni's figures*, in honor of the investigator[1] who first studied and described them.

[1] A German, 1756–1827.

Sprinkle fine sand and lycopodium powder over a square Chladni plate, and then, using a violin or bass-viol bow, produce as many as practicable of the sand figures shown in Fig. 161. In each case the letter *b* shows where the bow is applied to the edge of the plate and *a* indicates the position of the finger held in contact with the plate to produce nodes.

Observe the behavior of the lycopodium while the plate is vibrating, and the positions where it finally settles.

The *nodal lines*, marked by the sand figures, are those parts of the plate which have little or no up-and-down motion while the plate is vibrating. The sand particles are apparently tossed toward these lines by the blows they receive from the quivering parts of the plate.

The very light lycopodium is, however, not easily tossed through the air. It is controlled largely by the movements of the air above the plate; and these movements are such as to keep the powder at the points of greatest agitation of the surface. In a vacuum the lycopodium acts like the sand.

226. Vibrations in Organ Pipes.—A column of air in a pipe does not necessarily vibrate as a whole. This fact accounts for the variety of pitch obtainable from a fife, flute, or other similar wind instrument.

In every pipe giving out a musical sound there is at least one cross-section where the air is alternately compressed and rarefied without movement *through* the cross-section. This is called a *node*.

A in Fig. 162[1] indicates the condition of a pipe, closed at one end, giving out its lowest note. There is only one node, and this is at the closed end. The wave length (Exercise 27) of the sound is about four times the length of the pipe.

B of the same figure shows the nodes in the same pipe

[1] Figs. 162 and 163, and all statements here made about length, are only approximately correct. The length of the vibrating air column is practically a little greater than that of the pipe, the difference depending upon and increasing with the distance from the front to the back of the pipe.

giving its next higher note. The wave length of this note is four-thirds times the length of the pipe. In *C*, the next case, the wave length is four-fifths of the length of the pipe. In *D*, the next case, it is four-sevenths, etc.

All this shows that in a closed pipe the vibration numbers of the fundamental tone and the overtones (§ 223) are to each other as the numbers 1, 3, 5, 7, etc.

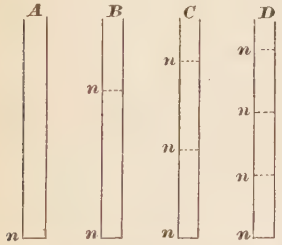


FIG. 162.—Nodes in a closed organ pipe.

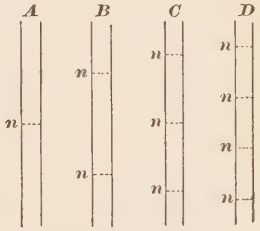


FIG. 163.—Nodes in an open organ pipe.

Fig. 163 shows the nodes in an open pipe giving out (*A*) its fundamental tone, (*B*) its first overtone, (*C*) its second overtone, etc.

The wave length of the fundamental tone is equal to twice the length of the pipe, that of the first overtone is the length of the pipe, that of the second overtone is two-thirds the pipe length, etc. The corresponding vibration numbers are to each other as the numbers 1, 2, 3, 4, etc.

If there are two, and only two, nodes in a pipe, there is condensation at one when there is rarefaction at the other. If there are three and only three, there is condensation at the middle node when there is rarefaction at the other two, etc.

Midway between the nodes the air moves alternately up and down a very slight distance, with very little, if any, change of density.

If a hole in the side of a vibrating pipe is opened halfway between two nodes, the sound is not changed. If a hole is opened at one of the nodes, condensation and rarefaction can no longer occur there; the place ceases to be a node, and the note given by the pipe suddenly changes pitch.

Many wind instruments have side openings which are closed at will by the player. Another device for changing the pitch of the note from such instruments is to blow with greater or less force at the mouthpiece.

These facts can be illustrated by the use of an organ pipe with properly placed openings and stops (No. LXXXIII).

227. **Quality of Sound due to Overtones.**—The quality (§222) of any musical sound is due to the number, kind, and relative strength, of the overtones which, together with the fundamental, constitute the sound. This has been proved both by analyzing sounds such as those of the human voice, of organ pipes, etc., into their component tones, and by uniting many different simple sounds to form a compound sound. Any sound which is caused almost entirely by vibrations of a single rate, that is, sound which is nearly destitute of harmonics, is comparatively thin and lacking in mellowness and richness.

Sound a tuning fork, for example the "middle C," or the "violin C," fork, alternately with the sonometer wire tuned to the same pitch. The wire should be plucked about one-eighth or one-ninth of its total length from one end, in order to give a full series of harmonics. Compare the rich sound of the wire with the thin sound of the fork.

228. **Other Musical Scales than the "Natural."**—The scale which has been described in § 221 as the *natural* one is the one which is approved by modern European and American musical taste, as having the most satisfactory intervals for voices and for those musical instruments in which all

gradations of pitch are possible. It has not, however, always been used by those most cultivated in music. The ancient Greeks, for example, who are supposed to have been far keener than modern audiences in the perception of pitch, used a different scale; and oriental peoples have not evolved the same "natural" musical scale that we have.

The fact seems to be that modern European practice makes far more use of *part*-music, in which different persons sing or play simultaneously notes of different pitch, than the ancients made or than the modern orientals make; and our "natural" scale is probably better adapted to such a use than any other scale.

229. Whole Tones and Semitones, Sharps and Flats.—If we turn back to the representative numbers of the natural scale, as given in § 221, and find the ratio of the vibration number of each note of the scale to the vibration number of the next lower note, we get the following result:

C	D	E	F	G	A	B	C
24	27	30	32	36	40	45	48
	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

The largest ratio is $\frac{9}{8}$; the next, almost as large, $\frac{10}{9}$; but the third, $\frac{16}{15}$, is decidedly smaller than either of the others. Where we find the ratio $\frac{9}{8}$ or $\frac{10}{9}$, the interval between the two notes concerned is called a *tone*, a *whole tone;* but where the ratio $\frac{16}{15}$ occurs the interval is called a *half tone.* We have, then, half-tone intervals between *E* and *F* and between *B* and *C*, but whole-tone intervals in all the other cases. It is, however, the practice of musicians to divide every whole-tone interval into half tones by inserting, about half way between its terminal notes, another note, which is called a *sharp* with regard to the lower terminal of

the interval and a *flat*[1] with regard to the upper terminal. For example, *C sharp*, written $C\sharp$, or *D flat*, written $D\flat$, is the intermediate note between *C* and *D*.

Introducing in this way five new notes, we have thirteen, including the upper and the lower *C*, in the scale, with twelve half-tone intervals.

230. Change of Key; Tempered Scale.—It has been stated in § 219 that the pitch, the actual vibration number, of a note of given name has not always been constant. In fact, when people start to sing without any instrument to give them the "key," or "key-note," they frequently *pitch* the tune high or low to suit the voices of the company; and, provided the *intervals* are correctly observed, all may go well. Thus, if the tune is "America," which begins, *do, do, re, si, do, re,* and if it is to be sung with a piano of standard pitch, the note taken for *do*, the *key note*, will commonly be the "middle *C*" of the piano key board, giving 256 vibrations per second. Apart from the piano, the *do* may have any number of vibrations in the neighborhood of 256 per second, or it may even be as much as one or two whole tones different from this, without making any great difference in the musical result.

But if the piano is used, and if the key note is to be changed, the *do* of the tune must be changed to some other key of the piano; that is, it must be changed by some definite amount. To make changes of this sort freely and still be able to find, on the piano, keys for the *re, si*, etc., making the strictly correct, natural-scale, *intervals* with the new *do*, we should need an enormous number of keys on the board.

Accordingly, since the piano must be so constructed as

[1] The sharp of a note and the flat of the next higher note are not, when the greatest refinement of notation is introduced, exactly the same, but practically they are the same.

to admit of change of key, its plan is to vary somewhat from the exact intervals of the natural scale, in such a way as to make possible several changes of key, with results which, though imperfect, are still fairly satisfactory to musical ears. The scale from any C to the next higher C on the piano, both C's included, contains thirteen notes, twelve successive half-tone intervals; and the vibration number of any note is obtained from that of the note next below by use of the multiplier $\sqrt[12]{2}$. The scale thus formed is called the *tempered* scale.

The five sharps or flats, introduced between C and D, D and E, F and G, G and A, A and B, are given by the black keys, the regular notes by the white keys, of the piano.

Questions and Problems.

(*Assume a temperature of* 0° *C. for the air in the following problems if nothing is said to the contrary.*)

(1) How does the loudness of a sound heard at a distance of 100 ft. from its source compare with that of the same sound heard at a distance of 200 ft. from the source? The source is supposed small.

(2) Find the temperature at which sound will travel in air 3400 meters in 10 seconds, the velocity at 0° C. being 331 m. per second? See § 210.

(3) How long after the flash of a gun is seen would its report be heard, if the temperature were 0° and the distance of the observer from the gun were 1.5 kilom.? Light will traverse this distance almost instantaneously.

(4) On a day when the temperature is 20° C. a man sets his watch by the striking of a clock 2000 meters away. Determine to the one-tenth of a second the error due to distance.

(5) A cannon 2 miles distant from an observer is fired and he sees the flash. The wind is blowing 10 miles an hour from the man to the cannon. How many seconds after the flash does the sound of the report reach him, the temperature of the air being 30° C.?

(6) In order to ascertain the distance of a cliff a gun was fired and the time taken until the echo was heard. The time was found to be 10 seconds. The thermometer stood at 10° C. Determine the distance of the cliff.

(7) Describe carefully any method which you have used or seen used for measuring the velocity of sound.

(8) Do sounds of high or low pitch travel faster? How does the sound of distant music illustrate your answer?

(9) What will be the length of the resonant air column that will respond most loudly to a fork of 220 vibrations per second? Neglect the influence of the diameter of the tube.

(10) If the velocity of sound in hydrogen is 4 times what it is in air, what must be the length of a tube filled with hydrogen which will respond to a fork making 480 vibrations per second?

(11) An open pipe l ft. long is found to respond to the note C. How much must be cut off to cause it to respond to the note G.?

(12) If the pipe in the previous example had been closed, how much must have been cut off?

(13) A person pumping water from a well sometimes detects among the various sounds that attend the operation one that changes in pitch as the water rises in the pipe. How can this be accounted for?

(14) Why does the sound of a circular saw, cutting through a board, grow lower in pitch as the saw enters the board?

(15) Just how could the sonometer wire be set vibrating in four loops? in five loops? How many nodes would there be in each case?

(16) In a sonometer wire 1 m. long, find the lengths of wire to give the first five harmonics of the fundamental tone of the whole wire.

(17) A certain musical note is caused by 256 vibrations per second. How many vibrations will be necessary to produce its *fourth?* its *fifth?* its *octave?*

CHAPTER XXI.

NATURE OF LIGHT; VISIBILITY OF OBJECTS.

231. **Light is Something that Travels.**—We say that a lamp *gives,* or *gives out,* light. This is true. Light is something that comes to our eyes from any object and enables us to *see* the object.

A substance through which light can travel is called a *medium* for light. We have ways of measuring the time required by light to travel a given distance in air and in many other media.

232. **Measurement of the Velocity of Light.**—One of the simplest methods for measuring the velocity of light is that devised by the French physicist Fizeau.

It consists essentially of a source of light, from which a bright beam comes, a toothed wheel which revolves in a plane at right angles to the course of the beam of light, and a plane mirror. Apparatus is provided for measuring the rate at which the wheel revolves.

The beam of light passes through the space between two adjacent teeth of the wheel, travels a distance of several kilometers, is then reflected by the mirror, and returns over the same path by which it passed out. If the wheel is at rest, the beam as it returns will repass the aperture between the teeth through which it passed out. But it is easy to see that, if the wheel could be revolved fast enough, a tooth might be brought into the path of the returning beam in time to intercept it. Still more rapid revolution would bring a new gap between teeth into the path of the returning beam, and so on. In fact, alternate eclipses

and appearances of the returning beam are produced when the wheel is revolved at a high and continually increasing velocity. From the rate of motion of the wheel, the width of its teeth, and the distance traversed by the beam, it is not difficult to calculate the velocity of light.

As a result of measurements made by means somewhat different from those just described, the velocity of light has been ascertained to be about 300,000 kilometers, or 186,000 miles, per second, in a vacuum. The velocity in air is a little less. In liquids and solids it is less than in gases.

233. Light is of Various Kinds.—Light as it comes from the sun, or from most lamps, is of many different kinds, all blended together so that the eye does not distinguish one kind from another; but when this mixture of light falls upon certain objects, pieces of glass called *prisms*, for instance, the mixture is broken up and we see the different *colors*.

Hold a glass prism in the direct sunlight in such a position that light after passing through the prism will fall upon a white surface not in the direct sunlight.

This breaking up of light is considered further in § 265.

234. Light a Wave Motion.—Before the nineteenth century light was believed to consist of particles of matter actually shot out in some way from the luminous body. These supposed particles were called corpuscles (that is, *little bodies*), and this theory as to the nature of light was called the *corpuscular theory*.

We now believe that light is not a substance, but a kind of wave motion, a shiver, which is sent along through bodies with great velocity and to very great distances, although the particles of the body, or *medium*, transmitting this wave motion travel very small distances on either side of their positions of rest.

The most direct evidence that light is a wave motion is found in the fact that two similar beams of light may interfere with each other so as to produce darkness, somewhat as two beams of sound may interfere with each other and cause silence in the phenomenon of beats (§ 220). Whenever we have two *like*[1] influences neutralizing each other, we must suppose that each operates by means of some alternating process, so that the negative part of one influence may occur at the same time as the positive part of the other. Such an alternation of state traveling through any medium constitutes a wave action.

Color and Wave Length.—The different kinds of light which produce in us the sensations of different *colors* are distinguished from each other by differences of wave length. Waves which produce the sensation of red, and which we often call red waves, are longer than the so-called blue waves, which produce the sensation of blue. One tint of red has a wave length of one thirty-thousandth part of an inch. One tint of blue has a wave length of one fifty-five-thousandth of an inch.

235. Light Travels in Straight Lines.[2]—When direct sunlight enters a darkened room through a small hole, one can usually trace its course and boundary in the room by means of the air-borne dust particles which are lighted up by it. It is easy to see that the boundary, the side, of the *beam* of light is straight. This is one of the familiar facts which show that light travels in straight lines. Applications of this property of light are found in the practice of *sighting* rifles, cannon, and other firearms; and in the method

[1] *Like* influences are such as, working *singly*, produce the *same* result.

[2] This statement holds good only in cases in which the light travels in a medium or substance of uniform composition and density throughout. Even under these circumstances there are certain exceptions to the general rule of rectilinear propagation. These occur where light passes close by the edges of objects, but the effects thus produced, although very interesting and beautiful, are not sufficiently prominent to make their study in this book necessary or advisable.

of glancing along the edge of a board, as the carpenter does, to see whether it is straight.

Light "Pencils and Rays."—If a beam of light is slender, it is called a *pencil* of light. If the pencil is very slender indeed, it is called a *ray* of light, and is represented in drawings by a single line.

236. Camera Obscura.—This name means *dark chamber.*

Push the small tube of No. LXXXVI, closed end foremost, into the larger, and then, pointing the apparatus toward a window, look into the smaller tube and move it back and forth in the other till the best image of the window or of objects outside is obtained.

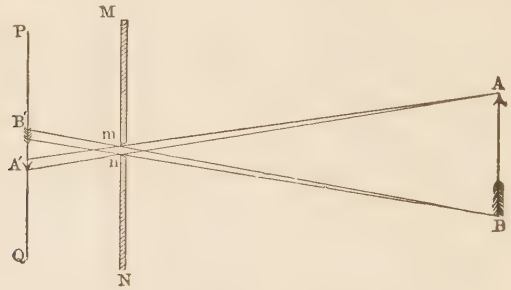


FIG. 164.—Action of *camera obscura.*

It is evident at once that the image is upside down; that is, that the bottom of the image represents the top of the object. This is due to the fact that the light-rays, coming from the object and traversing the very small aperture in the end of the tube, cross each other in their passage, as in Fig. 164, where the object is represented by the arrow AB. For instance, the cone of rays AA' from the tip of the arrow and the cone of rays BB' from the other end cross at mn and appear in the image at the spots A' and B' respectively.

If the aperture mn were gradually made larger, the spots A' and B', illuminated from A and B respectively, would

grow larger and larger. The same would be true of the spots illuminated from other points of the arrow; and at last the growing spots would so overlap each other that the image would be lost in a mere blur of light on the screen.

237. Shadows.—From the fact that light travels in straight lines, it is easy to see that it will be cut off from a portion of the space behind any illuminated opaque object just as waves of water are cut off by a breakwater, leaving a region of calm water behind it. The simplest case is that in which the light-giving object is as small as possible.

Light a bat-wing gas jet or a kerosene lamp with a broad, thin, flame, and cast the shadow of a lead-pencil, held vertical, on a sheet of white paper, having first the edge and then the broad side of the flame toward the pencil. Note the great difference in the sharpness of outline in the two cases.

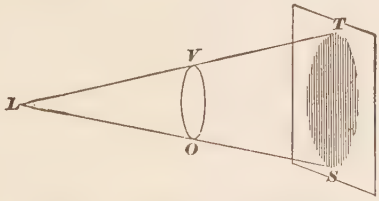


Fig. 165.—Formation of a shadow.

Umbra.—A shadow with a perfectly sharp outline could only be obtained by using as the source of light a mere point, as in Fig. 165.

Of the light rays proceeding from the point L, the circular opaque object OV intercepts all which strike its surface, thus forming a shadow the shape of which is in this case the frustum of a cone, $OSTV$. The black area ST on the screen is not the whole shadow, but a *section* of the entire shadow $OSTV$. A perfect shadow like this, equally dark at all points, is called an *umbra*.

Penumbra.—If now the source of light is of appreciable

size, a candle-flame for example, the opaque object O cuts off all illumination from some portions of the screen, and from other portions cuts off only a part of the light, as in Fig. 166.

That part of the screen which receives light from part of the flame AB, but not from all of it, will appear a partially shaded ring, $P'S'SP$, around the central area of total shadow. This ring forms what is known as the *penumbra*.

On account of the comparatively large size of many sources of light, shadows frequently are surrounded by a

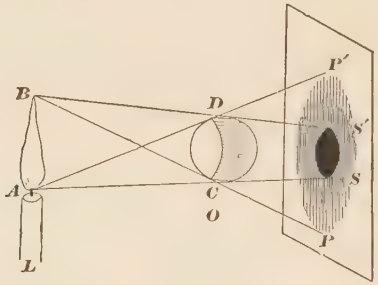


Fig. 166.—Umbra and perumbra.

wide margin of penumbra. The student will find the best examples of clear-cut shadows in those cast upon near surfaces by opaque bodies exposed to electric arc lights, and he may compare the dim and indistinct shadows of the leaves of shade trees exposed to the sun with those cast by the same objects exposed to the electric light at night, in which even the serrated margins of the leaves are sometimes clearly outlined.

238. How Light Weakens with Distance: Law of Inverse Square.—If two equally large surfaces are turned toward a very small flame, distant 1 m. from one and 2 m. from the other, the nearer surface will receive very nearly four times as much light from the flame as the more dis-

tant surface. It is easy to prove that this is true if light travels in straight lines diverging from a point. See § 213.

If one surface is three times as far away as the other, it will receive only one-ninth as much light as the nearer one, and so on.

The law, which holds when the diameter of the light-giving spot is very small compared with the distance from it to the receiving surface, may be stated thus: *The amount of light received on a surface of given area from a given source of light is, other things being equal, inversely proportional to the square of the distance from the source to the surface.* This is called the *law of inverse square.*

It follows from this law that, if a lamp L sends to a given surface at a distance D just as much light as another lamp L' sends to the same surface at a distance D', the light-*emitting* powers of the two lamps, which powers we will call P and P', must be such that

$$P \div D^2 = P' \div D'^2; \text{ or } P : P' :: D^2 : D'^2.$$

Questions.

A candle flame 2 m. from a white card and an incandescent electric lamp 8 m. from the same card light it up equally. What is the relative power of the two sources?

P_l (*for the lamp*): P_c (*for the candle*) :: $8^2 : 2^2$.

Hence $P_l = P_c \times 16$.

239. Photometry.—It is a matter of great practical importance to compare the illuminating power of different lamps. This operation is called *photometry*, or *light-measurement.* It cannot be done by merely observing the lamps directly, for the eye is unable in this way to distinguish slight differences of power.

Bunsen's Photometer.—The kind of photometer devised by the German chemist and physicist Bunsen makes use

of a translucent spot, a grease spot or a paraffined spot, in a screen of white paper. This screen is placed between the two lamps which are to be compared and then moved back and forth until a position is found in which the spot looks neither brighter nor darker than the rest of the screen. In this position the screen receives equal illumination from the two lamps. No light from other sources should be allowed to reach the screen.

Rumford's Photometer.—One of the simplest devices for measuring the relative power of two sources of light is Rumford's photometer, which compares the shadows cast by a rod placed in front of them. This is used in Exercise 29.

240. Effect on Light of the Body on which it Falls.—When light rays meet the surface of a body they may be:

a. Regularly reflected; that is, sent off from the surface in a direction which can be calculated or foretold, if we know the direction in which they are to strike the surface, as sunlight is reflected by a mirror.

b. Irregularly reflected or scattered; that is, sent back or off from the surface in many different directions, as sunlight is sent back from the surface of white cloth or white paper.

c. Transmitted; that is, allowed to pass through, as sunlight through clear window-glass.

d. Absorbed; that is, neither reflected nor transmitted, but swallowed up, as sunlight by a lampblack surface upon which it falls.

It usually happens that more than one of these effects is produced by the same body at the same time.

Visibility of Objects.[1]—Very few of the objects we see shine by their own light, as we can tell by testing them in

[1] For much interesting and valuable matter upon this subject see Rood's *Textbook of Color*, Appleton & Co.

the dark. They merely give off the light, or some part of the light, which has fallen upon them, directly or indirectly, from the sun or from some other light-giving body.

Let a beam of direct sunlight enter an otherwise dark room through a hole 4 or 5 cm. in diameter. Hold any, not very dark, body having an unpolished surface in the path of the beam near the hole, inclining the illuminated part of it so that it may be seen from some part of the room. Note the shadows cast in this part of the room, as evidence of the diffusion of light from the illuminated body.

241. Colors of Bodies; Transparent Bodies.—Colored pieces of glass, colored liquids, and other *transparent* bodies, generally owe their color to the fact that they are not transparent to all kinds of light. The light which enters them, sunlight for example, usually consists of many different colors blended together; and they rob this light of those colors which suit their own constitution, *transmitting* the rest. It is the transmitted, the rejected, light which we get from them that gives them their apparent color. The light which they *absorb* is turned to something else in the absorption, and is no longer light. It is usually turned into heat.

Bodies which absorb some kinds of light more readily than other kinds are said to have the power of *selective* absorption.

Opaque Bodies.—Most bodies with which we are familiar do not appear to transmit light. We cannot see through them, and we call them *opaque* bodies.

In fact, most so-called opaque bodies are not perfectly so. If they are made into very thin sheets, the sun can shine faintly through them. Even when they are in thick masses, the light penetrates a very little distance beneath the surface, where some of it is absorbed, and some, being reflected by interior particles, returns to the outside. This returning light is usually different in color

from the mixture of lights that entered, certain parts having been absorbed more than others.

242. Light from Surface of Colored Bodies.—The light reflected from the real external surface of *non-metallic* colored bodies receiving white light is usually not colored. The following experiment shows an illustration of this fact:

(1) Let a beam of direct sunlight, entering a window, fall very obliquely upon a sheet of colored glass in such a way that the reflected beam will fall upon a white surface. Observe the color of the reflected light.

(2) Try a like experiment with the reflection from a sheet of the thinnest gold leaf fastened by shellac to a glass plate.

Note also the color of the light that goes *through* the gold leaf.

Bodies which reflect from their outer surface some kinds of light more readily than other kinds are said to have the power of *selective* reflection.

Certain materials—silks, for example—may reflect white light from the outer surface, together with considerable colored light that has penetrated this surface and has been sent back from the interior. The white light gives the *sheen*, but in the spots where this is strong the *color* is not at the same time very evident, being made to look pale by the large amount of white light mixed with it.

The following experiment will show how the *color* coming from an object may be deepened by diminishing the amount of white light reflected from the external surfaces of its numerous particles.

(3) Grind a lump of sulphate of copper to a fine powder and observe how faint the blue color becomes; then wet the powder with water, which *adds* nothing but prevents some of the external reflection, and note the decided deepening of the blue.

In velvet the ends of the fibres, which reflect but little white light, are turned outward, and the light which penetrates the surface and then returns to the outside is deeply colored.

Questions and Problems.

(1) The distance of the planet Neptune from the sun being 2,800,000,000 miles, nearly, how long does it take a wave of light to go from the sun to Neptune?

(2) An upright stake, 8 ft. tall and 10 ft. from a window shutter, throws a *camera obscura* image on a screen 4 ft. beyond the shutter. The aperture in the shutter through which the light passes from the stake to the screen is very small. How great is the length of the image?

(3) The moon being smaller than either the sun or the earth, can a total eclipse of the sun occur for a whole hemisphere of the earth at once?

(4) At what distance from an arc lamp of 1600 c. p. will a surface be lighted by it as brightly as it would be by a standard candle at a distance of 10 ft.?

(5) Two lights, A and B, are placed 20 ft apart. The power of A is to that of B as 4 to 9. At what point between them must a screen be placed in order to be equally lighted up on both sides?

CHAPTER XXII.

REGULAR REFLECTION OF LIGHT.

243. Reflectors.—Smooth, even, surfaces, like the surface of still water, polished glass, or polished metal, reflect light regularly (§240).

Transparent reflectors are not convenient for ordinary use, partly because light which we do not want may come through them from behind, partly because they reflect really well only such light as falls upon them very obliquely.

Reflect, first almost perpendicularly, then very obliquely, a beam of sunlight from a sheet of unsilvered glass to a white surface, and compare the brightness of the illumination in the two cases.

The reflecting surface which we make use of in a common mirror is not the front surface of the glass, but the metallic surface at the back. The glass is merely a convenient transparent support for the metallic layer, keeping it in shape and protecting it from becoming tarnished, as it soon would be if exposed to the air.

REFLECTION FROM A PLANE MIRROR.

244. Where the Image Is.—A *plane* mirror is a *flat* mirror. We shall study curved mirrors later.

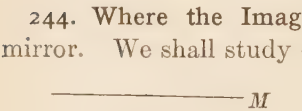


FIG. 167.—Points of view before a plane mirror.

When we place an object in front of a plane mirror and stand in a proper position, we see an image, or "reflection," of the object, and we say that we see the object, or its image, *in* the mirror. If *M*, Fig. 167, is the

mirror, O a point of the object, and P_1, P_2, P_3, and P_4, are the positions of four eyes, all may see at the same time an image of the point O in the mirror. Exercise 30 is intended to answer the question whether all these eyes see the *same* image; that is, whether all are looking toward the same point, and if so where this point is—in front of the mirror, or behind it, or at its surface.

245. The Law of Reflection.—In Fig. 168, MM is a mirror surface, CD a *normal* to this surface, OC a ray *incident* at the point C, and CP the same ray after reflection.

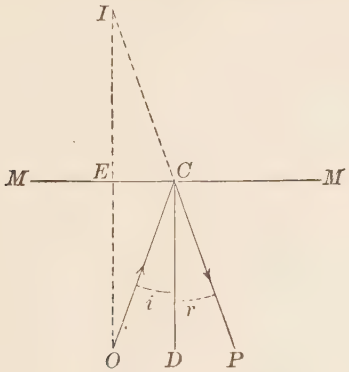


FIG. 168.—Law of reflection.

The angle i is called the *angle of incidence.*

The angle r is called the *angle of reflection.*

The "law of reflection" is, that *the angle of reflection is equal to the angle of incidence.*

This law is easily proved on the basis of what is learned in Exercise 30. The line of proof is this: *The image of O is at I. The angles at E are right angles; EI = EO; EC is common to the two triangles; hence the triangle CEI is similar to the triangle CEO. Then angle i = angle EOC = angle EIC = angle r.*

246. "Real" and "Unreal" Images.—If the rays of light proceeding from a point are by any means really brought together again at a different point, as in Fig. 179 (§ 255), the second point is called a *real* image of the first. A real image has an actual existence in space, and will show as a *picture* upon a properly placed white screen.

If the rays of light proceeding from a point are by any means made falsely to appear to diverge from a different

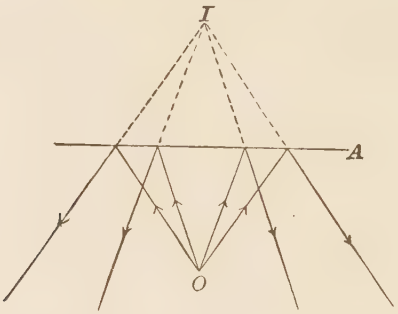


FIG. 169.—Action of a plane mirror.

point, as in Fig. 169, then the second point is called an unreal, or *virtual*, image of the first. A virtual image has no real existence in space, and would not show upon a screen.

Evidently the image formed by a plane mirror is an unreal image

247. Images of Images.—If any of the rays from O (Fig. 170) after reflection from the mirror A fall upon a second plane mirror B, they will be treated by this second mirror just as if they really came from I_1; that is, we shall, looking into the mirror B in the right direction, see an *image* of the *image* I_1, and this second image, I_2, will appear just as if it were the image of an actual object sending rays from I_1 directly to B.

The rays reflected first from A and next from B might then fall upon a third mirror, and give an image of the image I_2, and so on; but at each reflection there is some loss of light, and an image formed after many reflections might be dim.

Positions of the Various Images.—Let A and B in Fig. 171 represent the positions of two plane mirrors meeting at

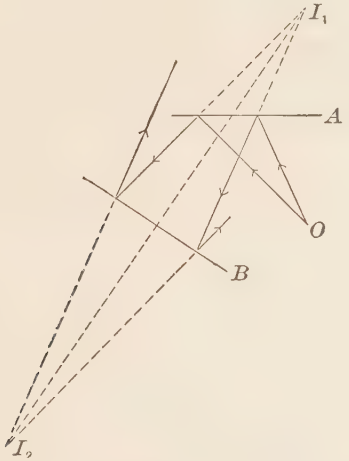


FIG. 170.—Image, I_2 of an image I_1.

right angles with each other at the point C. Let O be a small object placed between the mirror faces.

We shall have one image, I_1, formed by mirror A without any help from mirror B, and another, I_2, formed by B without help from A. There is also I_3, the image of I_1, seen in B, and there is I_4, the image of I_2, seen in A. I_3 and I_4 fall at the same spot.

We cannot with this arrangement of the mirrors get images of I_3 and I_4, because rays leaving mirror A as if diverging from I_4 would not strike the face of B, and rays

leaving mirror B as if diverging from I_3 would not strike the face of A.

Observe that O and its images fill the corners of a rectangle. If O were midway between the mirrors, the rectangle would be a square, with C at its centre.

If the angle between the mirrors were made a bit less than 90°, I_3 and I_4 would fall apart. If the angle were made 60°, *one-sixth* of a circle, O lying half way between them, O and its images would fill the corners of a regular hexagon having C at the centre.

If the angle were 30°, *one-twelfth* of a circle, O and its images would fill the corners of a twelve-sided figure.

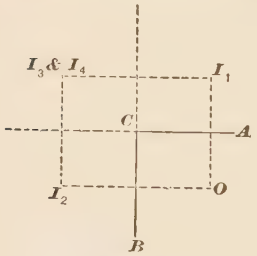


Fig. 171.—Action of two plane mirrors at right angles.

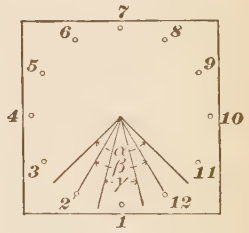


Fig. 172.—Principle of the kaleidoscope.

Place the hinged mirrors of No. LXXXVIII upon the board, with the reflecting surfaces making an angle of 90°, the point 1 (Fig. 172) being midway between them. Place a lighted candle, of such length that its flame will not be above the upper edge of the mirrors, exactly over the spot 1.

Note the positions of the images of the candle seen in the mirrors. Put pegs into the holes behind the mirrors in such positions that to an observer placed in front of the mirrors, so as to see the images in the mirrors and the pegs over the mirrors, the pegs will appear to coincide with the images.

Then make the angle between the mirrors 60° and place pegs to coincide with the images.

Finally, try an angle of 30°.

248. The Kaleidoscope.—The preceding passages give an explanation of the kaleidoscope (No. LXXXIX), with which most beautiful effects of endless variety can be obtained.

The kaleidoscope uses bits of colored glass instead of a candle flame, and sometimes has *three* mirrors put together at angles of 60°.

249. The Sextant.—If the ray 1 from point O in Fig. 173 strikes the mirror AA at right angles, it is reflected, as $1'$, back along its previous course. If the mirror is turned through the angle α, to the inclination shown by BB in Fig. 174, a ray, 2, from O will be reflected along the course $2'$, making an angle 2α with its previous line of direction. In

Fig. 173.—Perpendicular incidence and reflection.

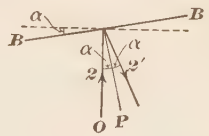


Fig. 174.—Effect of turning the mirror.

general, turning a plane mirror, from which a ray is reflected, through a given angle α swings the reflected ray through an angle 2α.

Fig. 175 illustrates the principle of operation of the sextant, an instrument used to find the angle of elevation of the sun above the horizon or for any like purpose. Let A represent the lowest point of the sun's disk; B, a point on the horizon at sea. M_1 is a small plane mirror, silvered at one end only; M_2 is another plane mirror. Both are carried on the frame of the sextant, M_1 being rigidly fixed thereto, while M_2 is mounted on a pivot and can be turned in the plane of the paper.

The ray b_1 from B passes through the unsilvered part of M_1 and goes on, parallel to its original course, into the

sighting tube T. The ray b_2 is reflected from M_2, goes as b'_2 to the silvered part of M_1, from which it is reflected as b''_2, alongside and parallel to b_1, through T. The ray a from A is reflected from M_2 along the course a'. Such is the first *setting* of the instrument.

The object of the procedure is to measure the angle α between the two rays a and b_2, which is the same as the angle between a' and b'_2. Evidently we can, by turning M_2 through the angle $\frac{1}{2}\alpha$, make a' take the present course

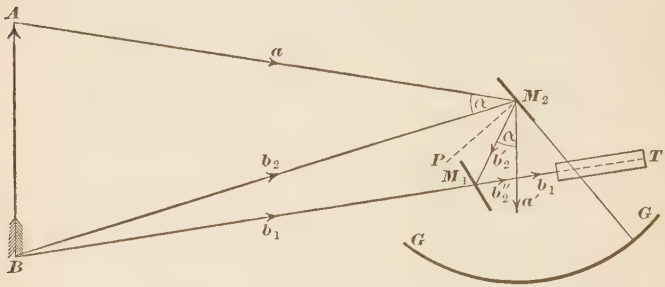


Fig. 175.—Illustrating the sextant.

of b'_2 and then a'', a' reflected from M_1, will take the present place of b''_2. Accordingly, if we measure the angle through which M_2 is turned to produce this result, we have only to multiply this angle by 2 in order to find the angle α. The arc GG on which the degrees are measured off is so constructed that we read off at once on it the angle α instead of $\frac{1}{2}\alpha$.

The instrument here described is called a *sextant* because the graduated arc is a sixth part of a circle. Observations made on the altitude of the sun at certain times of the day enable the sea captain to find the latitude and longitude of his ship.

Questions and Problems.

(1) In a lighted room at night the glass of a window will serve as a mirror. In daylight unsilvered glass with a black cloth behind it may be used in the same way. Can you explain this?

(2) Soon after the moons of Mars were discovered in 1877 someone announced in a newspaper that one of these moons could be seen near Mars by looking at the reflection of that planet in a common mirror. It is true that a faint bright speck appeared near the image of Mars as thus seen, which did not appear when the planet was looked at directly, but the true moons could be seen only by the aid of powerful telescopes. Can you, after trying the experiment with any bright star, explain the appearance seen in the mirror?

(3) Write some short word as it would appear in a mirror if the printed page containing it were reflected in the mirror.

(4) The clock on a wall indicates 9.30. What time will it appear to indicate if the observer sees the reflection of the clock in a mirror on the opposite wall but does not distinguish the numerals?

(5) A plane mirror lies upon a table and a pencil 6 in. tall stands upright on one edge of the mirror. How wide must the mirror be in order that a person whose eyes are 5 in. above its surface and 22 in. distant from the pencil may just see the whole length of the pencil reflected in the mirror? (To be solved by drawing and measuring. The thickness of the *glass* is to be neglected.)

(6) A person standing in the middle of a room 20 ft. wide looks with one eye into a mirror 2 ft. square set in the wall of one side of the room. How many square feet of the wall behind himself could he see reflected in the mirror if his own image did not obstruct the view? (*Suggestion:* Draw a diagram representing the position of the observer, the mirror, the reflected wall and its image, all on a horizontal plane.)

(7) Prove that, if an object is placed in front of a plane mirror and the mirror is moved either toward or from the object, without turning, the image will move twice as far as the mirror.

(8) Two plane mirrors, A and B, are placed 12 cm. apart, facing each other and parallel. A small object is placed between them 4 cm. distant from A. Calculate the distance from A to the first and second images seen in it. Do the same for B.

(9) Sunlight shines through a window in the south wall of a room, strikes on a plane mirror hanging on the east wall, and is thus reflected to the north wall, where it shows as a bright patch of light.

When a small object is placed near the mirror two shadows of this object fall on the bright patch. Explain, with diagram.

(10) Prove that, if a candle is placed in front of a vertical plane mirror and the mirror is turned 45° about a vertical axis, the image of the candle will move through an arc of 90° around the axis of the mirror.

(11) A candle-flame is placed half-way between two plane mirrors which meet at an angle of 40°. How many images appear, and how are they arranged?

(12) Does the fact that the images I_3 and I_4 (Fig. 171) fall at the same spot tend to make the image seen at the spot brighter than the images I_1 and I_3.

REFLECTIONS FROM CURVED MIRRORS.

250. Spherical and Cylindrical.—Most curved mirrors are parts of spherical surfaces. We shall, however, study mirrors which are parts of cylinders. They are more convenient for our present use than spherical mirrors, and they are less expensive.

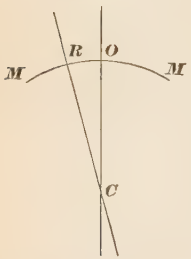


FIG. 176.—Center of curvature, etc., of a curved mirror.

We shall use both the *convex*, or bulging, and the *concave*, or hollowed, face of the mirror.

251. Centre of Curvature, etc.—*MM* (Fig. 176) represents a cut through a cylindrical mirror at right angles with the straight lines of its surface. This cut is of course a part of a circle.

C, the centre of the circle, is called the *centre of curvature* of the mirror.

The point *O* is called the *centre of the mirror*.

The line *CO*, extended indefinitely in either direction, is called the *principal axis* of the mirror.

Any straight line extending, like *CR*, through *C* and across the line *MM*, but not through the point *O*, is called a *secondary* axis of the mirror.

252. "Law of Reflection" Still Holds.—With curved mirrors, as with plane mirrors, the law (§ 245) *angle of incidence = angle of reflection* holds. With the help of this law we can see why the image of a point is nearer the mirror, when this is convex, than the point itself is.

Let O (Fig. 177) be the object-point in front of the convex mirror MM, the centre of curvature being at C. A line drawn from C to any point of the mirror is at right

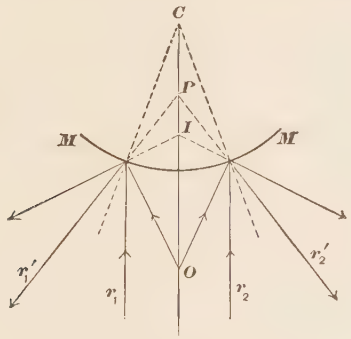


FIG. 177.—Action of a convex mirror.

angles with the mirror at the point of crossing. Two rays going from O to the mirror front appear after reflection to come from I, which is nearer the mirror than O is.

253. Principal Focus and Focal Length.—If rays come from some very distant point on the principal axis (§ 251), they are practically parallel to each other when they reach the mirror. Two such rays are represented by r_1, and r_2 in Fig. 177. Applying the law of reflection to them, we find that after reflection they appear, as r'_1, and r'_2, to diverge from a point P, which is very nearly midway between the reflecting surface and the centre of curvature.

The point P is called *the principal focus* of the convex

mirror. It may be defined as *the point which marks the image (virtual) of an object-point situated a long distance away from the mirror on the principal axis,* or as *the point from which rays coming to the mirror parallel to the principal axis appear to diverge after reflection.*

The distance, measured along the principal axis, from the principal focus to the reflecting face is called the *focal length* of the mirror.

The principal focus and the focal length play a very important part in the theory of curved mirrors and of lenses (§ 273). More will be said of this later.

254. Concave Mirrors.—If the *concave* side of the mirror were used, it is easy to see from Fig. 178 that rays from a point O near the mirror front would after reflection appear to come from a point I, which is *farther* from the mirror than O is. It is evident that the rays from O are more nearly parallel to each other after reflection than before.

Rays from a point O', somewhat farther from the mirror than O, appear after reflection to come from a still more distant point, I', and these rays are nearly parallel after reflection. It is easy to see that if the object-point were put somewhat farther still from the mirror, the rays proceeding from it might, after reflection, be strictly parallel to each other. They would appear to come from a point as far as possible behind the mirror.

If the object-point is placed still farther away from the mirror, as at O in Fig. 179, the rays may after reflection be actually *converging,* and cross at a point I in *front* of the mirror. This image I is a *real* image (§ 246), and, if O is bright enough, the image I may be seen, like a picture, on a piece of white paper or cloth placed in the right position.

We see that the centre of curvature C lies between the object-point O and the image-point I, in the case shown by Fig. 179. This is always so in the case of *real* images

formed by concave mirrors, unless the object-point is at C, in which case the image-point also falls at C.

If the object-point were placed where I now is, in Fig. 179, the image-point would fall where O now is.

The facts here brought out will be of service in Exercise 32, which without this preliminary discussion is somewhat puzzling.

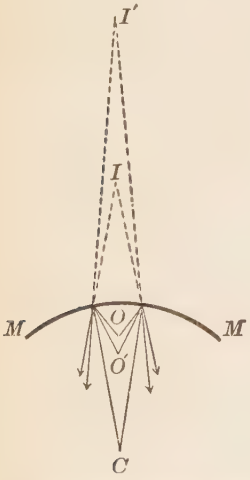


Fig. 178.—Virtual images formed by a concave mirror.

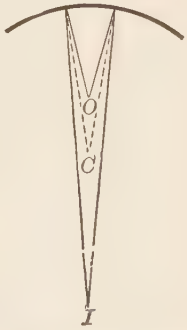


Fig. 179.—Real image formed by a concave mirror.

255. Principal Focus and Focal Length of Concave Mirror. —*The principal focus of a concave mirror is the point to which rays, coming to the mirror parallel to the principal axis, converge after reflection.* In other words, it is *the point which marks the image (real) of a very distant point on the principal axis.*

As in the case of a convex mirror, the principal focus lies very nearly midway between the reflecting surface and the centre of curvature.

The distance, measured along the principal axis, from the principal focus to the reflecting face is called the *focal length* of the mirror.

256. Rule for Placing Images.—Fig. 180 illustrates an easy rule for finding the position of an image in a convex mirror.

Let AB be the object. Draw one ray from A straight toward the centre of curvature. This ray will return on itself after reflection, coming as if from C. Draw another ray from A parallel to the principal axis. This will after

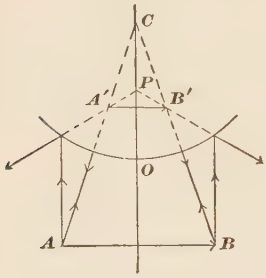


FIG. 180.—Convex mirror; diagram for placing the image.

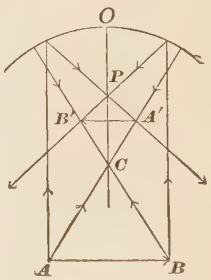


FIG. 181.—Concave mirror; diagram for placing the image.

reflection appear to come from P, the principal focus (§ 253). Both reflected rays appear to come from A', which is therefore the image of A.

The image of B is found in the same way.

If the object is a straight line, as in this figure, it is customary to represent the image by drawing a *straight* line from A' to B'. This is inaccurate, as Exercise 31 should show.

Fig. 181 shows the same method applied to a concave mirror.

257. Distorted Images.—In Exercises 31 and 32, and in all the figures that have been given representing cylindrical

mirrors, we have been dealing with rays, which are, both before and after reflection, parallel to the plane on which the mirror rests. If we make use of other rays, as we do when looking obliquely down at the mirror face, we see things much twisted, the effects thus obtained being too difficult for our profitable study in this course.

258. Relation of Cylindrical to Spherical Mirrors.—If we were to use a spherical mirror, placed with its principal axis horizontal, and employ only horizontal rays striking the mirror on a narrow horizontal strip through its middle, we should get effects quite like those we have already studied. All the figures from 176 to 181 would apply as well to a spherical mirror so used as to a cylindrical mirror. Indeed, these figures are like those commonly given to show the effects obtained with spherical mirrors.

For general use spherical mirrors are better than cylindrical mirrors, because they can be used from more points of view without giving badly distorted images.

A concave cylindrical mirror, in giving a real image, would change a figure like to or ; whereas a concave spherical mirror, in giving a real image, would change to . In one case there is rotation through 180° around one axis; in the other there is rotation through 180° around each of two axes at right angles with each other.

259. Principal Use of Spherical Mirrors.—Although spherical mirrors are sometimes used to form images, as in certain telescopes, probably their most common use is to concentrate light upon some object that cannot otherwise be well seen. Thus, the small objects which are to be looked at with a microscope need to be brightly illuminated, and a concave mirror is commonly used to throw light upon them.

The Ophthalmoscope.—Often a physician wishes to see what is wrong in the depths of a patient's eye. To

make this possible the interior of the eye must be especially lighted up. If this is done by holding a flame in front of it, the flame dazzles the eye of the observer and therefore is of little use. The difficulty is overcome by means of the ophthalmoscope (Fig. 182) where M is a curved mirror with a hole in the centre; L is some source of light, placed so that the rays proceeding from it to the mirror pass by reflection into the eye of the patient, represented by E; and O marks the position of the observer's eye.

FIG. 182.—Action of the ophthalmoscope.

This simple application of the concave mirror was made by the great physicist Helmholtz, and it has probably won for him more popular fame and gratitude than all his other work. The most remarkable thing about many inventions is the fact that they were not made earlier.

260. Formulas Relating to Curved Mirrors.—In the following formulas, which are here given without proof,

D_o = the distance of object-point from mirror;
D_i = the distance of image of object-point from mirror;
F = focal length of the mirror.

For a concave mirror we have

$$\frac{1}{D_o}+\frac{1}{D_i}=\frac{1}{F}, \qquad \text{(A)}$$

when the object-point is farther from the mirror than the principal focus is, and

$$\frac{1}{D_o}-\frac{1}{D_i}=\frac{1}{F}, \qquad \text{(B)}$$

when the object-point is between the principal focus and the mirror.

Hermann von Helmholtz

For a convex mirror we have

$$-\frac{1}{D_0}+\frac{1}{D_i}=\frac{1}{F}. \qquad \text{(C)}$$

It is doubtful whether work done with the cylindrical mirrors in Exercises 31 and 32 will be accurate enough to give results agreeing with these formulas. Similar formulas are used with respect to lenses. See § 276.

Questions and Problems.

(1) A small object is placed close to a convex mirror.

(*a*) Is the image real or virtual?

(*b*) If the object is moved farther and farther away from the mirror, will the image at any time change to the other kind?

(2) If one looks at the image of his own face in a convex mirror will the nose appear too prominent and the forehead and chin retreating, or will the opposite be true?

(3) If a small object is placed close to a concave mirror—

(*a*) Is the image real or virtual?

(*b*) If the object is moved farther and farther away from the mirror, will it reach such a position that its image will change to the other kind? If so, what is that position?

(4) (*a*) Have you seen any virtual image, formed by a single mirror, plane or curved, that was inverted as compared with the object?

(*b*) Have you seen any real image, formed by a single mirror, that was not inverted as compared with the object? (In applying this question to *cylindrical* mirrors, consider only objects which lie in a plane at right angles with the *straight* lines of the mirror face).

(5) Do you see anything wrong with the physics of the following statement, copied from a prominent newspaper?

There are times when the public sees things in a convex mirror, in which they appear broad, robust, and expanded. There are times when the public sees things in a concave mirror, in which they appear cramped, narrow, and contracted.

(6) If the radius of curvature of a concave spherical mirror is 50 cm., and if a candle is placed 40 cm. distant from the mirror,

(*a*) How far from the mirror will the image of the candle be?

(*b*) Will this image be real or virtual?

(*c*) Will it be erect or inverted as compared with the object?

(*d*) If the candle-flame is 2 cm. long, what will be the length of its image?

(7) If the candle mentioned in the preceding problem were 10 cm. from the mirror, what would be the answers to the questions there stated?

(8) What would be the answers in Problems (6) and (7) if the mirror were convex?

(9) Have you ever seen curved mirrors used except in a classroom or laboratory? If so, for what purposes were they used?

CHAPTER XXIII.

REFRACTION OF LIGHT.

261. Introductory.—In the experiment made with a prism in § 233 it may have been noticed that the light did not go in the same direction after leaving the prism as before entering it.

(1) Place a straight stick in an oblique position, partly in and partly out of water. Notice the apparent bending or disconnection of the stick at the surface of the water.

(2) Place on a table a pan (No. XCI), 15 cm. or more in diameter and with nearly vertical sides 4 or 5 cm. high. Place a small coin on the bottom of the pan, and adjust the head in such a position that the side of the pan will just hide the more distant portion of the coin from the eye at *E* (Fig. 183). Maintain the head in this position by resting it against any convenient support; keep one eye closed, and look with the other into the pan, just beyond the farther edge of the coin, while another person slowly pours in water. Have the pouring stopped as soon as the whole of the coin becomes visible.

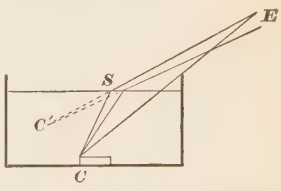


FIG. 183.—Apparent elevation, due to refraction of light (oblique vision).

(3) Repeat Experiment (2) with the eye held vertically above one edge of the coin, with a slender stick or a stout wire laid across the top of the pan, nearly in the line of vision, to serve as a point of departure from which to measure the apparent displacement of the coin, if any should be observed.

262. Interpretation of the Preceding Experiments.—Objects always appear to the eye to be in the direction *from which the rays are traveling at the moment of entering the eye.* Evidently, then, since the coin appeared to rise

when the water was poured into the jar, in Experiment (2), the light rays which proceeded from the coin must have been bent aside in some way by the water.

In Fig. 183 the straight line *CE*, which passes from the left-hand edge of the coin *C* to the pupil of the eye at *E*, represents the course of a light ray from that point before the water was poured into the pan. Any ray that passed farther to the right than *CE* would be intercepted by the side of the pan; any ray that passed farther to the left, or more nearly vertical than *CE*, would miss the eye; hence it is evident that, so long as the pan is filled with air only, and the eye kept in the position shown, the coin cannot be seen.

But as soon as water is poured into the pan, the rays no longer travel in straight lines from the object *C* to the eye. Each ray suffers an abrupt change of direction at the surface of the water, and from this it follows that such rays as those which take the general course *CS* in the figure are finally brought to meet the eye at *E*. As a result of the bending, *C* becomes visible, and its farther edge is seen apparently at *C′*, in a position somewhat raised above the bottom of the pan.

Experiment shows that the course *CSE* might be retraced by a ray. That is, a ray leaving *E* in the direction *ES* would reach *C* by the line *SC*.

Questions.

(1) If normals were drawn to the surface of the water, at the points about *S* where the rays emerge, would the bending of each ray be toward or from the normal (in the air)?

(2) If the rays were passing from E to *C*, would the bending at the surface of the water be toward or from the normal (in the water)?

When we look straight down into water at any small object it appears to be in its true direction from the eye,

but nearer than it really is. Fig. 184 indicates why this is so. O represents the object, E the eye, much magnified, and O' the apparent position of the object.

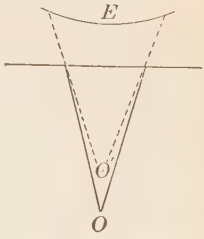


FIG. 184.—Apparent elevation, due to refraction with perpendicular vision.

263. Angles of Incidence and Refraction.—The change of direction which a ray of light undergoes when it passes obliquely from one medium into another is called *refraction.*

The amount of the bending, or refraction, which a ray of light suffers at any surface depends partly upon the two substances which meet at this surface, and partly upon the angle, i (Fig. 185), which the ray makes with a line NN, which is at right angles with the surface at the point C where the ray strikes the surface.

If the space above the line AB represents the air space, and that below this line the water, or glass, or whatever substance it may be that lies there, solid or liquid, the course of the ray is changed at the surface in such a way that the angle r, which it makes with NN inside the solid or liquid, is smaller than the angle i.

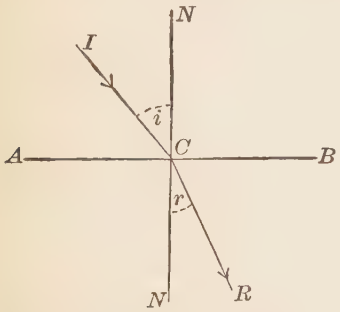


FIG. 185.—Angle of incidence, i, and angle of refraction, r.

The angle i in Fig. 185 is called the *angle of incidence.* The angle r is called the *angle of refraction.*

If the ray were represented as coming in the opposite direction, that is, first along R and then along I, r would be the angle of incidence and i would be the angle of refraction. The ray would be bent

just as much at the surface as it is when going first along I and then along R.

264. Index of Refraction.—When the direction of I is changed the direction of R is changed. The way in which the change of one depends upon the change of the other is easily shown by means of Fig. 186. I, I', and I'', show

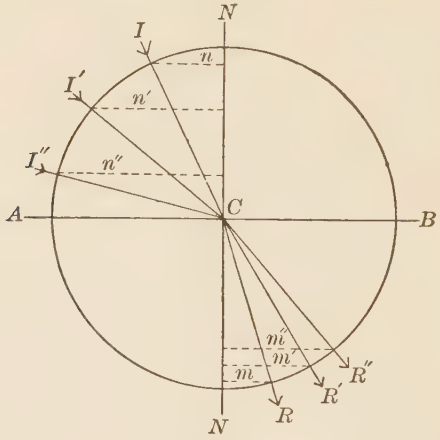


FIG. 186.—Index of refraction = (m ÷ n′) = (m ÷ n) = etc.

three rays all of which come to the point C and then separate, the first going along R, the second along R', the third along R''. The circle whose centre is at C is drawn with any convenient length of radius. The dotted lines, n, n', n'', and m, m', m'', are drawn from the points where the rays cut the circumference to the line NN, at right angles.

If this figure has been drawn so as to accord with the results of experiments on light rays, we shall have

$$\frac{n}{m} = \frac{n'}{m'} = \frac{n''}{m''},$$

and *any one of these equal ratios is called the Index of Refraction of the second medium, below AB, with reference to the first medium, above AB.*

If now we can measure $n \div m$ in any given case, we shall have a quantity which is very useful in physics, for by means of it we can *calculate* at once the value of a new m to go with any new n; that is, we can, if we know the index of refraction and the angle which any ray makes with NN in one medium, find without further experiment the angle which the same ray makes with NN in the second medium. Exercise 33 shows how to find the ratio $n \div m$ in the case of air and glass.

265. Index Different for Different Colors.—In Exercise 33 the observer may notice a tint of blue or of red at the edge of the image of the pin. The fact is that light of various colors comes from the pin, and that the rays are not all refracted alike, the blue being refracted more than the red. The index of refraction is therefore different for light of different colors; but for our present purpose we need not dwell upon that fact. We get a sort of *average* index by the method of Exercise 33.

266. Relation between Index of Refraction and Velocity of Light.—The velocity of light in any transparent substance depends on the nature of the substance. It is greatest in a so-called vacuum; it is least in the most highly refractive substances. Indeed, the *index of refraction from any given substance to another, as from air to glass, is equal to the velocity of light in the first substance divided by the velocity of light in the second substance.*

Refraction is often illustrated by an analogy suggested by the march of troops over ground of various kinds. Suppose a column of troops to be marching over smooth ground, represented by the space to the left of the line SS' in Fig. 187. The front of the column being at AB, let the line

SS' represent the border of a marsh or other difficult ground. Upon entering, the right of the column, B, first encounters the marsh, and the soldiers at B will fall behind those of the rest of the front. In consequence of this the

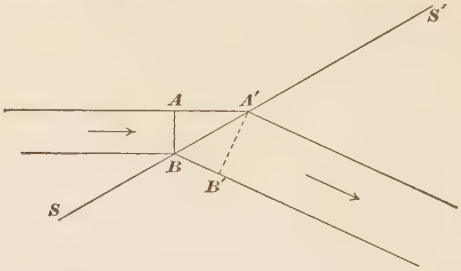


Fig. 187.—Relation of refraction to velocity before and after.

column will, one part after another, wheel to the right until, when the whole front has entered the marsh, it will have the new direction shown by the line $A'B'$. Substitute for the column of troops a beam of light, and for the marsh a highly refractive transparent substance, and one may

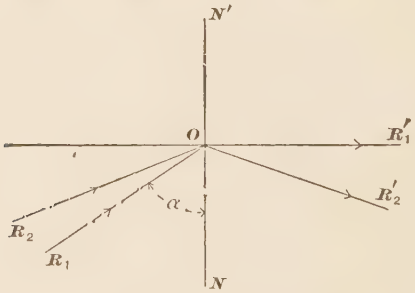


Fig. 188.—Internal reflection; critical angle.

get some notion as to how refraction depends upon the retarding effect of refractive substances upon light rays.

267. Total Internal Reflection; Critical Angle.—In Fig. 188 we have air above the horizontal line and water, glass,

or some other transparent medium, below the line. A ray of light, R_1, may come from beneath to the surface at such an angle with the normal that it will after refraction at O be parallel to the refracting surface. The ray R_2, coming up to O at a larger angle with the normal, will not pass out to the air, nor will it skim along the surface. It will be reflected at the point O, the surface acting as a perfect mirror, and will follow the course R_2', the angle of reflection being equal to the angle of incidence.

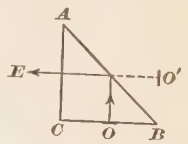


FIG. 189.—Use of prism as a reflector.

The angle α, which must not be exceeded if the ray is to pass out into the air, is called the *critical angle*. The reflection which takes place when this angle is exceeded is so good that it bears the especial name *total reflection*.

The critical angle for air and water is about 48.5°. For air and glass it is not very far from 40°, varying somewhat with the kind of glass.

(1) With the eye at E (Fig. 189), look at right angles into a glass prism shaped like ABC, at the same time holding a piece of white paper against the side CB at O. Note the position of the image O' and its remarkable distinctness.

(2) Try to see the paper by looking *directly* through AC and CB across the corner C.

(3) Wet the paper and stick it on at O by the adhesion of the water. Can you now see it *across the corner C*?

268. Effects of Atmospheric Refraction.—*False Elevation of the Sun and Stars.*—In Fig. 190 let the heavy circle indicate the earth, while the lighter circles indicate the atmosphere, which diminishes in density as its distance from the earth's surface increases. A ray of light, S, coming from a certain point of the sun, let us say, through the almost perfect vacuum of outer space obliquely into our atmosphere, is bent gradually until, when it grazes the

solid body of the earth and reaches the eye of the observer, it seems to come along the path S'. This effect makes sunrise come earlier and sunset later than it otherwise would, thus increasing the length of the day about four minutes at the least.

In high latitudes, where the course of the sun may be almost parallel to the horizon for the whole twenty-four hours, atmospheric refraction may increase the length of the day by hours. When, therefore, an explorer is in extreme polar regions and is attempting to find his latitude

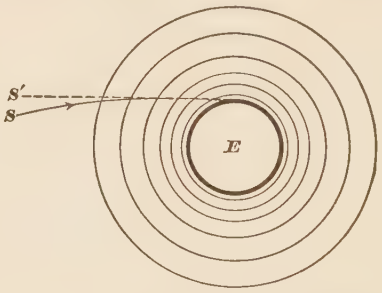


Fig. 190.—Illustrating atmospheric refraction.

by observation of the sun's altitude above the horizon or by the length of time it stays below the horizon, the effect in question becomes a very important one. There was much discussion of such matters during the North Pole controversy of 1909–10.

Mirage.—Light, like sound, being a progressive wave motion, effects analogous to those described in § 215 are observed with light traveling near the surface of water or level earth, if the atmospheric conditions are such as to make the lower part of a wave front (see Fig. 153) travel faster than the upper part. This happens if the air very close to the water or level earth is warmer than the air a little higher. The effect shown in Fig. 191 is a very com-

mon one. It can usually be distinguished from the somewhat similar normal view, of a boat and its *reflection*, shown in Fig. 192, by some one or by all of the following differences:

1. In Fig. 191, the under part repeats only a part, the lower part, of the visible hull of the floating boat, whereas Fig. 192, if the water is still enough to give a good reflection, repeats the whole of the visible hull.

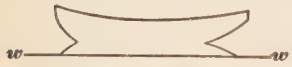


Fig. 191.—Duplication and inversion by mirage.

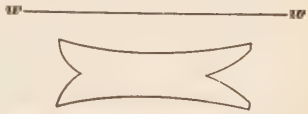


Fig. 192.—Duplication and inversion by reflection.

2. In Fig. 191 the horizon water-line, *ww*, lies below the boat figure, whereas in Fig. 192 it shows beyond and above the boat. There is frequently a blank strip between the horizon line and the mirage figure, as if one were looking *under* this figure and seeing the sky beyond it.

3. The aspect shown in Fig. 191 may be seen when the water is too much ruffled to give any well defined reflection like that of Fig. 192, though it is not likely to appear when the wind is strong and the water much disturbed.

Fig. 193.—Illustrating course of rays in mirage.

The explanation of this form of mirage is suggested in Fig. 193. Rays come to the eye at *E* from point 2 of the object by two paths, one, *A*, nearly straight, through the comparatively uniform air well above the surface of the water, the other, *B*, curved, through the unequally warmed air very near the water, the latter giving the appear-

ance of a repetition of 2 at 2′. A like statement may be made with regard to point 1. Rays inclining downward from point 3 may not be refracted enough to bring them to E. Rays starting from the water surface itself at any point to the left of P are refracted too much and pass above E, so that the horizon line appears to be at P, and the points 1′ and 2′ appear above the visible limit of the water surface.

Sometimes conditions are such as to make 2′ appear above 1′, in which case the whole effect, or the most striking part of it, is an apparent increase in the height of the object looked at.

It may happen that the air strata have such temperatures and densities as to show the inverted appearance of the object *above* the erect one.

Some of these mirage effects can be seen by looking along the heated top of a kitchen range at a boat-shaped piece of white paper, about 4 or 5 cm. long and 1 cm. wide, placed against a black background a meter distant from, and very slightly above, the range top.

EFFECT OF TRANSPARENT PLATES AND PRISMS.

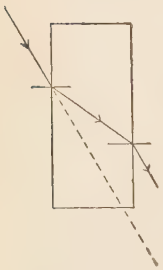


Fig. 194.—Action of a parallel-sided plate.

269. Transparent Plates.—A plate of glass, or other transparent material, with plane parallel sides, as in Fig. 194, refracts light which enters it obliquely, but refracts it equally and in the opposite direction when it comes out at the opposite side of the glass, so that the entering and emerging rays are parallel to each other, although, as Fig. 194 shows, they do not lie in one straight line.

Evidently a thick plate of glass will, other

things being equal, set the emergent ray farther to one side, from the line of the original ray, than a thin plate will.

270. Prisms.—A *prism*, in the study of light, is usually a piece of glass, or other transparent material, bounded by three rectangular and two triangular faces. *DEF* in Fig. 195 represents one end of such a prism.

It is evident that light entering the face *DE* from air will be refracted *toward* the normal *NM*. Going through the prism to the face *DF* it passes out into the air, being refracted again, this time *from* the normal *M'N'*, so that the two refractions have bent the ray far from its original direction.

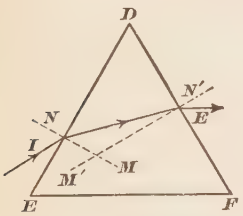


Fig. 195.—Action of a prism.

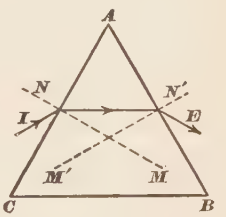


Fig. 196.—Action of prism: arrangement for minimum deviation.

The total bending or *deviation*, suffered by a ray in passing completely through a prism depends on a number of things:

1st. *On the angle which the two faces passed through make with each other.* This angle is called the *refracting angle;* see *D* in Fig. 195.

The greater this angle is, other things being equal, the greater the total deflection will be. We have seen in §269 that if the two faces are *parallel* the total deviation is zero.

2d. *On the color of the ray.*

This fact has already been noticed. Red light is deviated less than blue light.

3d. *On the angle which the ray makes with the first surface.* The total deviation is least when the ray strikes in such a way as to follow, within the prism, a course parallel to that shown in Fig. 196, which makes the points of entering and leaving equally distant from A, and makes the refraction equally great at both surfaces.

Repeat the experiment of § 233, varying the angle at which the sunlight strikes the first face, in order to show that there is one inclination which gives a less total deflection of the light than any other position.

4th. *On the material of the prism.*

For example, all kinds of glass do not refract equally.

271. Dispersion: The Spectrum.—The separation of rays of different colors by a prism is called *dispersion.*

The spot or band of colored light produced by the dispersion of a sunbeam is called the *solar spectrum.*

It is customary to divide the spectrum into seven regions called *red, orange, yellow, green, blue, indigo, violet,* and to call the general colors of these the *primary colors,* to distinguish them from those formed by compounding two or more of them. This division of the spectrum is a mere matter of convenience. We might name a hundred colors of the spectrum if we chose to do so.

So long as we keep to any one refracting material the dispersion is, in general, greater when the average deviation of all the rays is greater. Thus, with a given prism, the dispersion is least when all the rays go through the prism nearly as the ray goes in Fig. 196.

When prisms of different material are used—two kinds of glass, for example—one may disperse the rays more than the other, while producing no greater *average* deviation of all the rays; or one may disperse the rays about as much as the other while deviating them, as a whole, much less.

272. "Achromatic" Prisms.—Two prisms of nearly equal

dispersive power but of unequal deviating power may be combined, by turning their refracting angles in opposite directions so that one tends to counteract the other, as in Fig. 197, making a compound prism which produces considerable deviation with very little final dispersion. Such a combination is called *achromatic*, that is, *colorless*. In Fig. 197, C is of "crown glass" and F of "flint" glass. The *refraction* of C prevails, but not its *dispersion*.

FIG. 197. Compound prism, for refraction without separation of colors.

Achromatic combinations of *lenses* (§ 279) are used in many optical instruments.

LENSES.

273. Shapes of Lenses.—A *lens* is, usually, a piece of glass whose two faces are parts of spherical surfaces.

Sometimes there is a cylindrical surface between the two spherical faces.

Fig. 198 shows various lenses as they would look if cut through the middle.

FIG. 198.—Various forms of lenses.

Lenses are classed as *convex*, or *converging*, and *concave*, or *diverging*. Convex lenses are all thicker in the middle than at the margin, and cause parallel light-rays to converge, as in Fig. 199. Concave lenses are thinner in the middle than at the margin, and cause parallel light-rays to diverge, as in Fig. 200.

Some of the lenses used in the most accurate optical instruments have convex or concave surfaces which are not strictly parts of spherical surfaces. Such lenses possess certain advantages over spherical-surfaces lenses (see § 279).

274. Definitions Relating to Lenses.—The lenses used

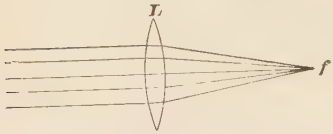


Fig. 199.—Principal focus of a convex lens.

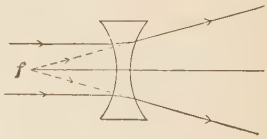


Fig. 200.—Principal focus of a concave lens.

in the laboratory Exercises of this book are much like No. 1 in Fig. 198. The two sides are supposed to be just alike.

To understand such a lens better we will make use of Fig. 201.

C is the centre of the spherical surface of which ASB is a part. It is called the *centre of curvature* of the face ASB. C' is the centre of curvature of the face ARB.

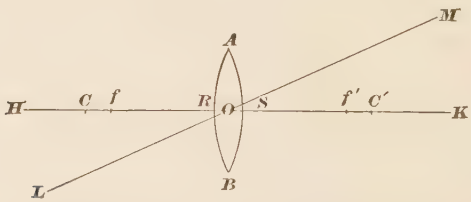


Fig. 201.—Axes, etc., of a double convex lens.

The straight line $HCOC'K$, continued to any distance in each direction, is called the *principal axis* of the lens.

Any straight line going, like LM, obliquely through the centre of the lens is called a *secondary* axis of the lens.

If the two faces of a lens are exactly alike, as we suppose them to be here, any ray of light going through the centre

of the lens, the point O, will have the same direction after leaving the lens as before entering it, because the two little spots of surface at which it enters and leaves the lens are parallel to each other, so that the ray is affected just as if it were going through a plate with parallel faces.[1] O is called the *optical centre* of the lens.

Rays entering a convex lens *parallel to its principal axis*, as in Fig. 199, are refracted in such a way that after leaving the lens they will cross this axis. They do not all cross at one point, but if the faces are near together, and are *very small* parts of spherical surfaces, as in our lenses, such rays will cross at or near a certain point, f, on the principal axis, and this point is called the *principal focus* of the lens. There are two principal foci, one on each side of the lens. See points f and f' in Fig. 201.

The distance from the principal focus to the nearer face of the lens is called the *focal length* of the lens.

Focal length is a quantity of very great importance in dealing with lenses, and Exercise 34 will show how to find it by experiment. For this purpose we need to have the light come to the lens in rays nearly parallel to each other and to the principal axis. This we can do by taking the light from any small spot of any distant but distinct object; for instance, a chimney or a church spire outlined against the sky.

275. Discussion of Exercise 34.—It is common to speak of the rays coming to a lens from a distant object as *parallel* rays. This does not mean that rays coming from different parts of the object to the lens are parallel to each other. It means merely that rays coming from any one spot of the object to the lens are parallel, or very nearly parallel, to each other. In fact, if rays from the *different*

[1] The direction of the ray *within* the lens is, of course, not quite the same as its direction before entering. This fact is not shown in Fig. 201. See §269.

parts of a luminous body could be converged to the same point, the result would not be an *image* repeating the features of the original object. It would be a mere point, or very small patch of light.

The image seen in the *Second Method* is, like that of the *First Method*, a real image (§246), but it is *in the air*.

As there is an image in the air, we may well inquire why this image cannot be seen by a whole class at once without the use of a screen. It is because the light forming the image in the air goes straight on *through* this image, and can be received only by placing one's self behind the image. The light which forms an image upon a *screen* is by the threads of the screen reflected back in all directions and therefore some part of it reaches every eye that sees the screen.

Question.

If a bright point were placed at the principal focus of a lens, what direction would the rays going from this point to the lens have after passing through the lens?

276. Object-Distance and Image-Distance; Conjugate Foci.—*Two points so placed with respect to a lens that an*

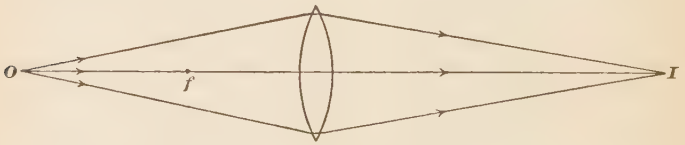


Fig. 202.—Conjugate foci, with real image.

object placed at either of them will have an image at the other are called Conjugate Foci of the lens.

If we take an object-point on the principal axis (§274) of the lens, the following formula holds, at least approxi-

mately, when the image is *real*, as in Fig. 202, where f is the principal focus:

(A) $\frac{1}{F}=\frac{1}{D_o}+\frac{1}{D_i}$; that is, $D_o \times D_i = F\ (D_o + D_i)$,

F being the focal length of the lens.

D_o the distance from the object-point to the nearer face,
D_i " " " " image- " " " " "

Evidently the object-point can be placed in such a position, even on the principal axis, that the lens cannot give a real image of it, for, if the object-point is at the principal focus, the rays going from it through the lens will issue parallel to each other and will therefore not come to a point beyond the lens; and if the object-point is still nearer the

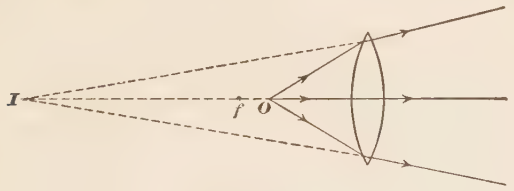


FIG. 203.—Virtual image with a convex lens.

lens, so that D_o is less than F, as in Fig. 203, the rays, though bent somewhat toward each other by the lens, will issue from it still diverging and will therefore appear to come directly from some point I behind the lens and farther away from it than O is. I is in this case a *virtual* image, and of course it cannot be shown upon a screen. To see it the observer must put himself in the path of the rays issuing from the lens and then look back *toward*, and, in a sense, *through*, the lens.

The formula which holds in the case of virtual images given by convex lenses is the following:

(B) $\frac{1}{F}=\frac{1}{D_o}-\frac{1}{D_i}$; that is, $D_o \times D_i = F\ (D_i - D_o)$.

The minimum distance, D_{oi}, from an object to its real image, as given by a lens, is about four times the focal length, F.

Exercise 35 illustrates much that is stated in this Article, especially in regard to the case of real images.

It should be noted that formulas (A) and (B), holding for real and virtual images respectively, in the case of a *convex* lens, are exactly the same as formulas (A) and (B), holding for real and virtual images respectively, in the case of a *concave* mirror.

Moreover the formula (C) of § 260, which holds for a *convex* mirror, holds also for a *concave* lens (Fig. 200). No real images are formed in either of these cases.

Questions and Problems.

(1) A luminous point is placed 50 cm. distant from a convex lens of 10 cm. focal length and on its principal axis.

(*a*) How far from the lens will the image of this point be? (*b*) Will it be real or virtual? (*c*) Will it be on the same side of the lens as the object-point?

(2) Let the image-point placed 12.5 cm. from the same lens, and take the same questions as before.

(3) Let the image-point placed 4 cm. distant from the lens, and take the same questions as before.

(4) Let the image-point placed $6\frac{2}{3}$ cm. from the lens, and take the same questions as before.

277. Real Image Formed by a Lens.—In Exercises 34 and 35 the object presented to the lens has been small, or has been at such a distance as to give a rather small image. It is now desirable to study larger images, and to study them with especial reference to their *shape* and *size*, rather than their distance from the lens. We shall in Exercise 36 find the shape and size of an image of an arrow placed at right angles with the principal axis of the lens and not far from the lens. We shall not attempt to find

the whole image at once, but shall find separately the images of several points of the arrow, and then make an approximate image of the arrow by connecting these points.

The formation of the image-points in this Exercise is illustrated by Fig. 204. One ray from the object-point A follows a secondary axis (§ 274) passing through the centre of the lens, and its direction after leaving the lens is the same as before entering it. Its direction *inside* the lens is not quite the same, but the figure does not show this.

Another ray from A runs parallel to the principal axis before entering the lens, and will therefore pass through

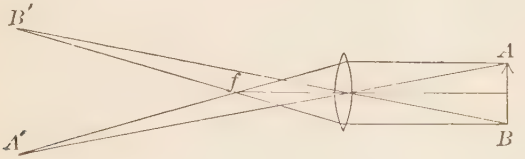


Fig. 204.—Formation of real image, point by point, by a convex lens.

the principal focus, f, on the farther side of the lens. The crossing of these two rays at A' shows the position of the image of A.

In a similar way B', the image of B, is located.

Size and Shape of Image.—If a straight line is drawn from A' to B' in Fig. 204, we may call this the length of the image, although the images of points between A and B will not lie on this line. It is evident from Fig. 204, and also from the figure obtained in Exercise 36, that the distance $A'B'$, the length of the image, is to the distance AB, the length of the object, as the distance of the image $A'B'$ from the lens is to the distance of the object AB from the lens.

The curved shape of the image obtained in Exercise 36, if the work has been correctly done, is due partly to the fact that the ends of the object-arrow are farther from the lens than the centre of the arrow, and partly to the fact that the focal length along a secondary axis is less than the focal length along the principal axis. This latter fact can easily be shown by direct experiment with either method of Exercise 34.

278. **Virtual Image Formed by a Lens.**—The formation of virtual image-points for object-points on the principal axis has been discussed in § 276 (Fig. 203). We have

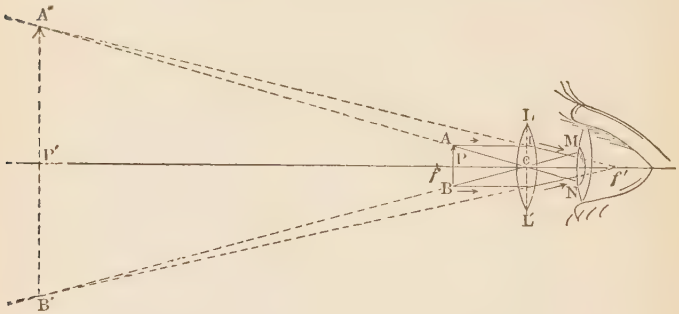


Fig. 205.—Action of a convex lens as a magnifying glass.

next to see how the virtual image of an object of considerable extent can be found point by point experimentally, and how this image compares in size and shape with the object, which for simplicity we shall take as an arrow lying across the principal axis at right angles. The experimental details are given in Exercise 37.

Let AB (Fig. 205) be the object, placed between the lens LL' and the principal focus f. To find the position of the virtual image of the point A, draw $A1$ parallel to the principal axis of the lens. This ray will, after leaving the lens,

pass toward f', the principal focus[1] on the farther side, and so will appear to have come along the path If'.

Draw another ray, Ac, passing through the *centre* of the lens. This ray will, after leaving the lens, have the same direction as before entering it (see § 274) and will be represented by the line cN. If, then, we carry back the line cN till it crosses the line If', also carried backward, the point A', where the crossing occurs, is a point from which both of the rays appear to come. A', then, is the virtual image of A.

By a similar process B' is found to be the virtual image of B.

P', the image of the point P, is here represented as lying in the straight line between A' and B'. It is usually so represented in books. Exercise 37 shows that it does not lie there, in the use of common double-convex lenses.

The image $A'B'$ is evidently larger than the object AB. A virtual image formed by a short-focus convex lens, usually appears, to an eye placed near the lens on the other side, as in Fig. 205, larger than the real object would look if held at a comfortable seeing-distance from the eye. Hence the name *magnifying-glass*, so commonly given to a lens used as in Fig. 205. It is called also a *simple microscope*.

Spectacles for far-sighted eyes are lenses used in this way. See § 293.

279. Faults of Lenses; Lenses of Special Form.—*Curvature of Field.*—The fact, illustrated in Exercises 36 and 37, that an ordinary double-convex lens does not give a straight-line image of a straight-line object, makes such a lens unsatisfactory for certain kinds of work requiring what is called *flatness* of the field of view. For example, if one were

[1] The dotted lines drawn from M and N to f in Fig. 205 are not intended to show the actual course of the rays *within the eye*.

to undertake with such a lens to photograph the side of a near building, from a point in front of its centre, it would be possible to get on the photographic plate a tolerable image of the middle part or of the marginal parts, but not of all parts at once.

Spherical Aberration.—Not *all* the rays going from a point O (Fig. 206) to an ordinary lens, $A\ B$, converge, after passing through the lens, to a single point I. Those which

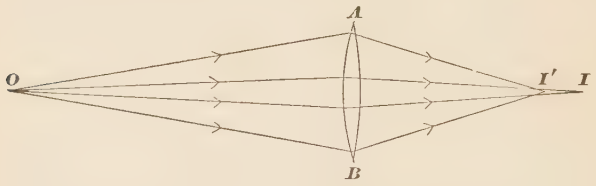


Fig. 206.—Illustrating "spherical aberration."

go through the lens near its margin converge to a nearer point I'. This imperfection of a lens is called *spherical aberration.*

When a very clear-cut image is needed, it is customary to put a *stop* in front of the lens; that is, a thin metal plate, with a hole, which permits only those rays to pass which are near the principal axis of the lens.

Lenses can be so constructed, with surfaces not quite spherical, as to do away with this defect in great part, for light of any one color, but such lenses are difficult to make and are uncommon except in large telescopes.

In photographic cameras the evils of field curvature and spherical aberration are lessened by the use of two lenses of peculiar shape, each called a *meniscus*, combined as in Fig. 207, with a stop, S, between them. Any one of several stops, with holes of various sizes, may be used with the

same combination of lenses, which is called an *object-glass*, or, more commonly, an *objective*. See also § 281.

Chromatic Aberration.—Ordinary lenses, made of a single piece of glass, give rise to colored fringes or borders about the images which they produce. The cause for this defect, which is called *chromatic aberration*, is this, that the objects looked at send more than one kind of light to the lens and that rays of different colors are not refracted equally by the lens, and so do not come to a focus equally near the lens.

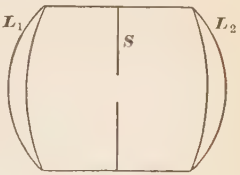


FIG. 207.—Illustrating the combination of "concavo-convex" lenses with a stop, *S*, between them in a photographic "objective."

But little trouble from this source is found in the use of lenses of slight convexity, the images from which are not to be further magnified; as, for instance, in spectacles and ordinary magnifying-glasses.

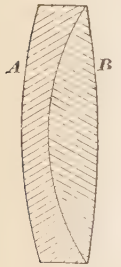


FIG. 208.—Combination of "crown" glass, B, with "flint" glass, A, to form an "achromatic objective."

The use of a *stop* with a simple lens improves its chromatic performance. How much help such diaphragms give by reducing spherical and chromatic aberration, may be learned by taking out some or all of the stops of an ordinary cheap spy-glass, and then looking with it at distant objects in bright sunlight.

Achromatic Lenses.—Fortunately for the manufacturers and users of optical instruments, it is possible to make an *achromatic* lens; that is, a lens nearly free from chromatic aberration. This is usually accomplished by uniting into one lens two separate lenses,[1] one

[1] Sometimes more than two pieces are employed in making an achromatic lens.

convex, or "converging" (§273), of crown glass, the other concave, or "diverging," of flint glass, like the B and A of Fig. 208, the whole making a convex combination. In such a compound lens the part A has a tendency to undo, or counteract, the work of B. The superior *dispersive* power (§ 271) of flint glass enables A, though of less general refractive power than B, to counteract the dispersive tendency of the latter, while leaving to the combination as a whole a considerable refracting and converging power.

Many of the lenses used in optical instruments of the best quality are achromatic. Eye-pieces (§ 285), however, of the ordinary pattern do not require achromatic lenses.

A large lens practically free from spherical and chromatic aberration is a marvel of skilful and patient work. Glass suitable for making a large lens of the best quality is very difficult to procure, as a very slight flaw or unevenness of quality may spoil a large block. The shaping and polishing and testing of the largest lenses, after the proper kind of glass is obtained, is a work of years, and men who are skilful and patient enough to do it become known throughout the world.

Questions and Problems.

(1) Define the term *index of refraction.*

(2) For which of the colors here named, red, green, yellow, blue, is the index of refraction the greatest? For which of them is it least?

(3) An object is placed at a great distance from a coverging lens and on its principal axis.

(*a*) What changes of position will the image of this object undergo while the object is moved along the principal axis up to the surface of the lens?

(*b*) In what part of this operation will the image be erect and in what part inverted?

(*c*) In what part will it be real and in what part virtual?

(4) In Exercise 36 the virtual image of a straight line was found to be a curve. How should a line be curved with respect to the lens in order to make its virtual image a straight line?

(5) The focal length of a certain convex lens is 15 cm.

(*a*) How far from the lens will the image be if the object, on the principal axis, is 30 cm. from the lens?

(*b*) How far if the object is 10 cm. from the lens?

(6) An object is 40 cm. from a convex lens, on the principal axis, and the image equally far from the lens. What is the focal length of the lens?

(7) If the object mentioned in Problem (5) is 5 cm. long, at right angles with the axis of the lens, how long will each of the images there mentioned be? (In answering this question disregard the curvature of the images.)

(8) A bright point, which is more distant from a converging lens than its principal focus is, sends white light to the lens. Which falls nearer the lens, the red image of the point or the blue image? Why?

(9) What would be the answer to the questions in (8) if the point were between the lens and its principal focus?

CHAPTER XXIV.

OPTICAL INSTRUMENTS.

280. Importance of Optical Instruments.—Much of the modern progress of science has been due to improvements in the construction of optical instruments and their more general use in scientific investigations.

Improvements in telescopes and the invention and perfection of the spectroscope have enabled the astronomer to discover, and even to measure, objects and motions whose existence was unsuspected by the observers of a few generations ago. The chemist is today able by means of the spectroscope to ascertain in a few minutes the presence, in a substance of unknown composition, of elements which it would have taken him days to detect or which he could not detect by purely chemical means.

To the physician, the food-analyst, the manufacturing druggist, and to those engaged in many other professional or technical occupations, the microscope is a necessary piece of apparatus, a tool of daily, almost hourly, use.

Optical instruments comprise a great variety of combinations of mirrors, lenses, and prisms. Only some of the simpler ones can be described in an elementary book on physics.

281. The Photographer's Camera.—This instrument consists essentially of a box in the front of which is fastened a convex lens or a combination of lenses, L (Fig. 209), the distance of which from a ground-glass screen, P, at the other end of the box, can be varied at will. An inverted, usually diminished, real image of an outside object

within a certain range can be formed on P. When this adjustment has been precisely made, the lenses are covered with an opaque cap; a plate of ordinary glass, coated with a film of gelatine made sensitive to light by the presence in it of certain compounds, usually of silver, is substituted for P; the cap is then removed, and the light is allowed to act for a sufficient time upon the sensitive plate, after which the cap is replaced and the plate removed and "developed" into a photographic "negative."

FIG. 209.—Photographic camera.

Those who are interested in practical photography will find in Exercise 36 some explanation of the difficulty experienced in making all parts of the ground-glass screen show clear images at the same time; and in § 279 there is a suggestion as to the effect of "diaphragms," or "stops," and lenses of peculiar shape.

The subject of color-photography, reproduction by photographic process of the natural colors of objects photographed, will be discussed very briefly in the next chapter.

282. The Projecting Lantern.—This instrument, known also by various other names—*stereopticon*, or *magic lantern*, for instance—requires a powerful source of light, such as a large kerosene flame, or some form of "calcium light," in which a cylinder of quicklime is heated by a flame formed by burning together oxygen and coal gas, or, best of all, the electric arc-light. See L (Fig. 210). By means of a large two-piece lens, C, called the *condenser*, a powerful beam of light from this source is thrown upon the painted or photographed "slide," at S, the image of which is to be exhibited. By means of the two-lens objective, O, a greatly enlarged real image of the picture is thrown upon the screen. By

means of a simple device not here shown O can be moved back and forth for the purpose of "focusing."

The throwing of large images upon a screen is called *projection* of these images, and apparatus used for this purpose is called *projecting-apparatus.*

There are contrivances by means of which the image of an opaque object, a picture on a printed page, for example, can be projected, the necessary illumination of the object coming obliquely from the side.

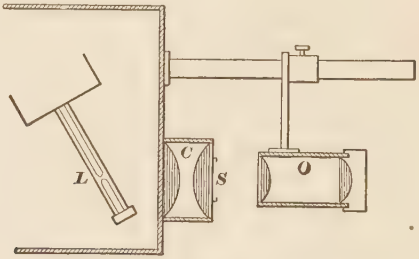


Fig. 210.—Projecting lantern.

283. The Spectroscope.—When a very careful study of the spectrum is to be made, when, for example, it is necessary to measure the exact amount of deflection which a given prism effects with light of any given color, a *spectroscope,* or *spectrometer,* is used. The construction and action of this instrument is indicated by Fig. 211. At s is a narrow slit, between two edges of metal, illuminated by the flame F, or perhaps by the sun's rays reflected upon it. This slit is at the principal focus of the lens L, so that all the rays going from *any one point* of s to L issue from L parallel to each other. Falling on the first face of the prism P, these parallel rays, if they are all of just the same wave-length—that is, of just the same color—are all deflected equally on entering the prism, as they are also on leaving

it by the second face, and so they come to the lens L' parallel, as if they had come from some very distant point, and are brought to a focus, or image-point, at the principal focus of L'. This image-point is looked at through the magnifying lens E.

The angle between the directions of the two tubes, C, called the collimator, and T, the telescope, is measured on the graduated circle G. It is called the angle of deflection, or deviation, of the light in question.

If from the same point of s there comes also light of a somewhat different wave-length from that which is followed

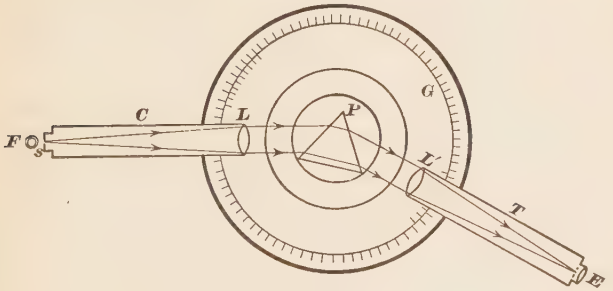


Fig. 211.—Spectrometer.

in Fig. 211, this second beam will be deflected somewhat more or somewhat less by the prism than the first beam and will come to an image-point not quite coincident with the first image-point, but somewhat to one side or the other, as the dots near the lens E indicate. By turning the tube T a little one way or the other from its present position either of these new points can be brought into the centre of the field of view of E, and the deviation of the corresponding beam of light can then be found.

Some of the objects of studying the spectrum so carefully are indicated in Chapter XXVI.

284. The Simple Microscope.—In its least complicated form the simple microscope, or magnifying glass, consists of a convex lens used, as explained in §278, to form an upright, magnified, virtual image of any small object. When much magnifying power is required, two or even three convex lenses, mounted one behind the other with their surfaces only a few millimeters apart, are often used. Such combinations are called *doublets* or *triplets*, according to the number of lenses composing them.

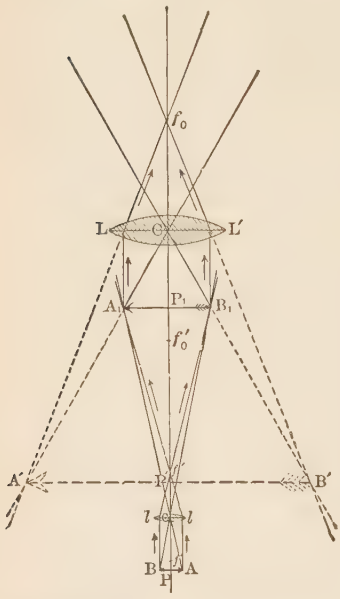


FIG. 212.—Action of a compound microscope.

The discussion in §278 will help the student to see that the magnifying power of a simple microscope is the greater the less its focal length.

285. The Compound Microscope.—For viewing objects under any but the lowest magnifying powers, that is, in all cases where the apparent diameter of the image is to be anywhere from 50 to 5000 times the actual diameter of the object, the *compound microscope* is employed. The essential optical parts of this instrument, as usually constructed, are (Fig. 212) an *eye piece, LL′*, here represented as single, but generally consisting of two convex lenses, and an *objective, ll*, frequently consisting of from two to six pieces. These lenses are fixed in a brass tube so arranged that the distance between the eye piece and the objective can be

varied at will, within certain limits. A mirror, not here shown, which is adjustable to any desired angle, is usually employed for throwing light upon the object from beneath.

The object to be viewed, AB, is placed on a platform beneath the objective, and is strongly illuminated by light reflected from the mirror. A real, inverted, magnified image, A_1B_1, of the object is formed within the tube of the instrument at a position somewhat nearer to the eye piece than its principal focus f_0. This real image is therefore magnified by the eye piece, which forms an enlarged virtual image, $A'B'$, of it at a position not far from the object.

The foci of the object glass are at f and f', those of the eye piece at f_0 and f_0.

The total *magnifying power* of the instrument is that of the objective multiplied by that of the eye piece. In general, the shorter the focal length of a microscope objective the greater its magnifying power.

Exercise 38 gives some practice in putting together the essentials of a compound microscope.

286. The Astronomical Refracting Telescope.—This instrument consists essentially of the long-focus *object-glass*, or *objective*, L (Fig. 213), mounted in one end of a tube, at the other end of which is placed an *eye piece*, L', precisely similar to that of the compound microscope. The eye-piece can be moved toward or away from the object glass in order to make the image appear as distinct as possible.

The real image, A_1B_1, of any distant object is, of course, always formed by the objective very near its principal focus. The foci of the eye piece are at f and f'. $A'B'$ is the magnified virtual image.

Astronomical telescopes are always furnished with achromatic object-glasses (§ 279).

Efficiency of the Telescope.—The usefulness of the telescope as an aid to vision depends upon the following

points: (*a*) the clearness and sharpness of the image, or what is called the *definition* of the instrument; (*b*) the brilliancy of the image; (*c*) the amount of allowable magnification.

Good definition depends upon the accuracy with which the lens is shaped and finished, and upon the quality of the glass, which should be free from flaws.

Brightness depends upon the amount of light which can be concentrated in the different parts of the image. Hence a large objective will, other things being equal, give the best illumination. In some telescopes the objective has a diameter of 3 feet or more.

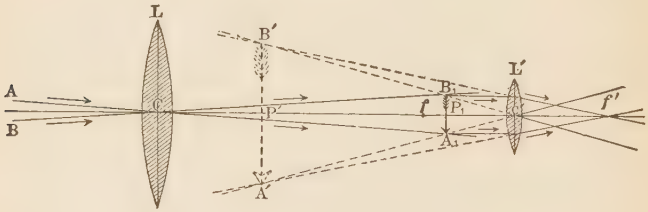


FIG. 213.—Action of a refracting telescope.

The magnification due to the objective alone is evidently very nearly proportional to the focal length of the objective, as we see in Fig. 213, where the length of A_1B_1 is evidently proportional to its distance from C. But unless the objective is large, and therefore gathers in much light from the object, it is useless to give it great focal length, for the reason that the much-magnified image would be too faint to be seen to advantage.

Exercise 38 gives some practice in putting together the essentials of a refracting telescope.

287. Reflecting Telescopes.—In some telescopes a concave mirror is used as the objective instead of a lens. It is

easier to make a satisfactory large reflecting objective than to make an equally large and equally good refracting objective. Some of the reasons for this are readily seen: There is only one surface to be formed in a reflector and the necessary shape, not strictly spherical but parabolic, like *PP* in Fig. 214, is well known; all wave lengths are reflected at the same angle, that is, there is no chromatic aberration (§ 279) from a mirror; the action of a reflector depends on its surface only, so that interior homogeneity, provided the shape of the surface remains constant, is not in itself important, as it is in a refractor.

On the other hand, a polished surface of "speculum metal," as the metal alloy sometimes used for large mirrors

Fig. 214.—Parabolic mirror, *PP;* spherical, *SS*.

Fig. 215.—Action of a reflecting telescope.

is called, though far less easily tarnished than silver, may become dulled by long exposure to the atmosphere; moreover, the mirror must be thick in order to retain its shape perfectly, and so the weight of a very large objective becomes troublesome. Indeed, the device of floating the massive mirror of a large reflector in mercury, instead of mounting it rigidly in a tube, has been adopted.

As a reflecting objective naturally forms an image between itself and the object, the question arises how the observer is to see this image without putting himself in front of the mirror and so cutting off much of the light

which is approaching it. The problem is solved in several ways, one of which is illustrated in Fig. 215, where M is the objective, forming at I the real image of a distant point, m is a small concave mirror forming a real image of I at I', and E is an eye piece of the ordinary kind.

The largest reflecting objective now (1911) in use is the famous speculum-mirror, 6 feet in diameter, established by Lord Rosse at Parsonstown, Ireland, in 1845. The astronomical observatory on Mt. Wilson in California has a silver-coated glass reflector 5 feet in diameter, and a still larger one is planned for use there.

288. Telescope of Galileo; Opera Glass.—The telescope invented by Galileo, one of the first to be used for astron-

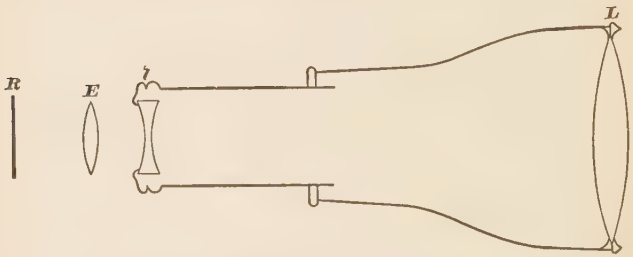


FIG. 216.—Illustrating the opera glass.

omical purposes, is represented by the modern opera-glass. It consists essentially of a concave lens used next to the eye and a convex lens at the outer end of the tube, as in Fig. 216. The concave lens, l, may be regarded[1] as intended to neuutralize or nearly neutralize the convex lens, E, of the eye (see § 291) thus practically leaving the convex lens, L, of the instrument free to throw the real image of distant objects directly upon the retina, R, of the eye as upon any other screen. The net result, then, is to substitute for the

[1] See Hastings and Beach's *General Physics*.

lens of the eye a much larger lens of much greater focal length, so that the images formed on the retina are much larger than they would be if formed by the unassisted eye. Unfortunately, however, the iris, or *pupil* of the eye, is not enlarged by the use of the telescope, and it is evident from Fig. 217 that many rays like r_3, coming from the same point as r_1 and aiming at p_1 on the retina, and r_4, coming from the same point as r_2 and aiming at p_2, are cut off by the iris diaphragm I. Accordingly objects appearing near the

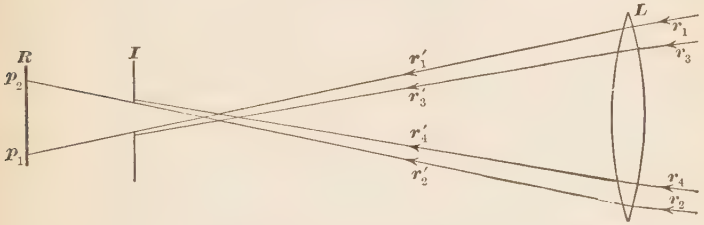


FIG. 217.—Illustrating the narrowness of field of the opera glass.

boundary of the field of view in the telescope of Galileo or the opera glass are comparatively obscure, as they are seen by means of light coming through the edge only of the lens. In the opera glass this defect does little harm.

One telescope with which Galileo made exceedingly important discoveries magnified about thirty times. Opera glasses, which of course must be short, usually magnify only three or four times. As a very rough rule we may take the magnifying power of a Galilean telescope or opera glass as equal to its length, in inches, when it is extended for use.

289. Terrestrial Telescope; Field Glasses.—The astronomical refracting telescope, as described in § 286, leaves the final image inverted as compared with the object, a matter of no consequence to the astronomer, who uses this

instrument often and become accustomed to the inversion. But for popular use in viewing earthly objects a telescope must show things right side up. This result can be and frequently is reached by inserting between the eye piece, E (Fig. 218), and the objective, O, a convex lens, I, or a combination of lenses, which takes the inverted real image $A'B'$, formed by the objective, and gives an inverted image, $A''B''$, of it, but an erect image of the *object*, AB, in the proper position for the action of the eye piece. The final image is $A'''B'''$. Such is the plan of the ordinary cheap but very useful spy *glass*.

One disadvantage of this arrangement is the added length which it requires in the instrument; another is the

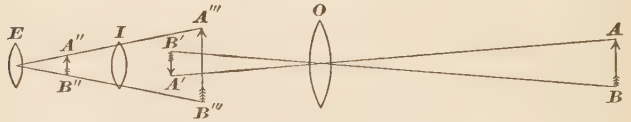


Fig. 218.—Illustrating an erect-image telescope.

distortion or blurring of the image and the loss of light which may result from the use of the additional lens or lenses. Accordingly an ingenious device, called[1] the Porro prism, is sometimes used in place of the inverting lens. This consists of two right-angled prisms combined and used in such a way that one of them, by an internal reflection (§ 267) turns the image $A'B'$ around, without inverting it, while the other, acting also by internal reflection, inverts the rotated image. The resulting image, which corresponds to $A''B''$ in Fig. 218, is actually thrown back so as to be nearer the objective than $A'B'$ is. That is, the Porro prism makes the terrestrial telescope, showing objects as right side up, shorter than it would be if the

[1] From its inventor, an Italian military officer.

objects were shown wrong side up, as they are in the astronomical telescope.

This arrangement, which came into frequent use near the beginning of this century, is much employed in "field glasses," portable, binocular, instruments for observing objects one or two miles distant.

290. Special Window-Glasses.—For sending light from the sky through low rooms reaching far back from the windows ingenious devices are sometimes used. One of these is illustrated by Figures 219 and 220. In place of

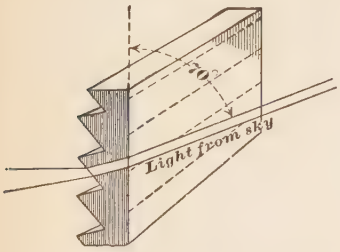


FIG. 219.—Action of special window glass: light from sky nearly horizontal.

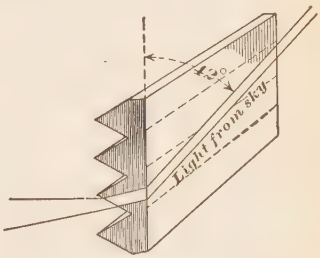


FIG. 220.—Action of special window glass: light from the sky very oblique.

ordinary window-glass transparent slabs, plane on the outer face but shaped on the inner surface into a series of prisms with horizontal edges, are set into the window frames. The shape of the prism best suited for any particular case depends on the angle at which the light from the sky reaches the window.

Questions and Problems.

(1) If a simple camera should be made out of a box and a double convex lens of 12 inches focal length,

(*a*) How deep must the box be in order to give an image of the sun?

(*b*) What should be its depth in order to give an image of a small object which is 10 feet distant from the front of the camera?

(*c*) If the centre of the image of a flat object is in good focus, how must the distance from lens to rear of box be changed in order to focus the edges of the image?

(2) Why are powerful telescopes made very long? Why are they made very wide?

(3) Why is it advisable, in projecting a spectrum, to use the light from a very narrow slit?

(4) Can you suggest some arrangement by which phenomena occurring on the surface of a layer of water, surface tension phenomena, for example, could be shown upon a vertical screen?

CHAPTER XXV.

THE EYE: SIGHT AND COLOR.

291. Parts of the Eye.—The eye as an optical instrument consists of a liquid lens, *A* (Fig. 221), called the *aqueous humor*, a solid lens, *B*, called the *crystalline lens*, a transparent jelly-like mass, *C*, called the *vitreous humor*, and a screen, *rr*, called the *retina*, upon which the image of the object looked at falls.

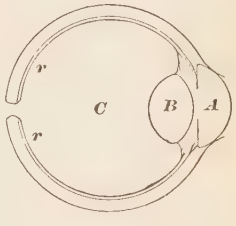


Fig. 221.—Illustrating the eye.

The aperture at the back of the eye is occupied by the optic nerve leading from the retina to the brain.

292. Accommodation.—Muscles attached to the lens *B* have power to change its form to some extent, thus adapting the eye to see distinctly near or distant objects at will. This is called the power of *accommodation*.

A *normal* eye, that is, a healthy, undistorted eye, has such shape as to give upon the retina, without effort, distinct images of very distant objects. In accommodating itself to see nearer objects such an eye has to make an effort, which grows greater as the distance lessens, but which, in young persons, does not become painful until the object looked at is less than eight or ten inches from the eye.

293. Far-Sight and Near-Sight.—Some eyes lack the power of accommodation for near objects, and are called *far-sighted*, or *long-sighted*, although they cannot see distant objects any better than normal eyes can.

Some eyes are slightly egg-shaped, the retina being farther back than in normal eyes. These eyes are called *near-sighted,* or *short-sighted,* because they are well adapted for seeing near objects, while they cannot see distant objects distinctly.

For some purposes near-sighted eyes have a certain advantage over normal eyes, for they enable their possessor to hold an object very near, when there is need, and so make it look larger than it would look to the normal eye.

Astigmatic eyes[1] are those which, because of some lack of symmetry, give a line instead of a point as the image of a point.

Eye-Glasses.—Far-sighted eyes must wear convex lenses to help them converge the rays from a near object to an image upon the retina. Near-sighted eyes must wear concave lenses to prevent the rays sent by a distant object from coming to an image in front of the retina. Astigmatic eyes need glasses of a peculiar shape, the surfaces of which are not parts of spheres.

294. Sight and the Perception of Color.—*Sight.*—The images formed on the retina are of course inverted, and the question is often asked why the corresponding objects do not therefore appear inverted. This question seems to imply that we *see* the retina and the images on it, as we might see a screen with images on it. But we do not *see* the retina or the images it bears. What could we see them with? *Seeing* is a complicated act which involves the whole depth of the eye as well as the optic nerves and some part of the brain beyond.

Perception of Color.—We do not know just why or just how light of a particular wave length produces in us a particular color sensation. As with sound (see §208) or with taste so with sight, we must learn to distinguish our

[1] From *στίγμα*, *point* and *ἀ*, *not.*

own sensations from the external cause of these sensations. We say that sugar is sweet, meaning that sugar has the power of producing the sensation of sweetness when it is dissolved in the mouth. So we say that a rose is red, meaning that the light coming from it has the power of producing through the action of sight the sensation of redness in a person of ordinary, normal, vision. We can say there is sweetness in the sugar, and redness in the rose, very much as we can say there is amusement in a printed joke or pleasure in a printed song. Full discussion of this subject would take us into the field of physiology and psychology. We must not go far in that direction, only far enough to see what some of the problems and difficulties of the inquiry are.

Various attempts have been made to determine a set of not more than three or four *primary*, or elementary, colors which, taken in combination with each other, would give all the other colors. These attempts have been made from different points of view, and the results, accordingly, have been various.

Artists, finding that they can by the skilful use of three colored *pigments*, or paints, red, yellow, and blue, respectively, get all or nearly all other hues or tints, naturally consider red, yellow, and blue as the primary colors. This does not satisfy the physicist, who knows that when we put two pigments together, yellow and blue, for example, we do not get from the mixture all the light which would come from the pigments taken separately; for the grains of one pigment absorb some of the light that comes from, or through, the grains of the other. Taking red, yellow, and blue as primaries, then, is a good practical rule for the *art* of mixing pigments to produce color effects; but it cannot be accepted as satisfactory in the *science* of color.

The physicist has his own ways of combining lights of

different colors. One of these is to let one kind of light shine through a piece of glass, as y does in Fig. 222, while another, b, is reflected from one surface of the glass in such a way as to join the transmitted beam in its further course.

A second method, which easily combines several beams at once, is that of Maxwell's color disk. Thus Fig. 223 represents a disk having four sectors, colored red, yellow, green, and blue, respectively. If this disk is revolved rapidly about an axis through its centre, the four beams of colored light fall in rapid succession on the same parts of the retina, and, as any impression on the retina lasts for a little time after the light producing it is withdrawn, the various impressions become blended into one. The colored *beams* are practically mixed without that loss of certain parts of the light which occurs in the mixing of pigments. Under the leadership of two great physicists, Young (§ 98) and Helmholtz (§ 182), other physicists found that nearly all color effects, in normal eyes, could be produced by the use of red, green, and violet lights, and their combinations, and accordingly they declared a certain red, a certain green, and a certain violet to be the three primary color-sensations.

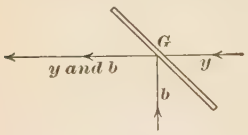


FIG. 222.—Device for blending two beams of light.

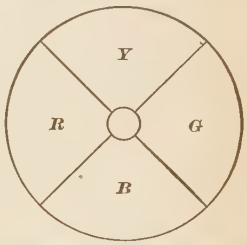


FIG. 223.—Illustrating Maxwell's color disks.

But physiologists and psychologists are not satisfied with this, the Young-Helmholtz theory of color vision. They declare, 1st, that this theory is contrary to the plain evidence of ordinary color perception, which, if unprejudiced by scientific doctrine or by artistic practice, usually selects red, yellow, green, and blue as primary color

sensations, feeling others to be producible by mixtures of some or all of these four; 2d, that the theory in question does not account satisfactorily for all the kinds of "color-blindness," deficient or abnormal perception of color, which continued investigation has made known; 3d, that the theory needs, at least, some extension, in order to account for the facts that in growing darkness all colored objects lose their appearance of color before their forms become indistinguishable, and that objects seen on the edge of the field of vision do not show their color.

The theory of Mrs. Franklin[1] accordingly takes red, yellow, green, and blue as primary color sensations, and accounts for the non-perception of color in dim light or in very oblique vision by assigning different capacities to the "rods" and the "cones," respectively, of the retina, these being minute bodies which constitute or are contained in one of its layers. The rods are supposed, according to this theory, to give only the sensation of light, without color, whatever rays may fall upon them, and to be more easily stimulated by a dim light than the cones, which are supposed to give the color sensations. The rods are known to be more numerous near the boundary of the retina, the part used in very oblique vision. This theory gives, moreover, an ingenious and interesting, if not entirely convincing, account of the known phenomena of color-blindness.

295. Production of the Sensation of White; Complementary Colors.—It has already been shown that ordinary white light is composed of many different colors, ranging from red to violet, but it is not necessary to put together all of these colors in order to get the *sensation* of white. There are many pairs of colors any one of which pairs will give the sensation of white when its elements are mixed in

[1] An American logician and psychologist.

the right proportions. The two colors making such a pair are called *complementary* to each other. Thus, according to Rood's *Text-book of Color,*

red is complementary to green-blue,
orange " " " cyan-blue (between blue-green and blue),
yellow " " " ultramarine-blue,
greenish-yellow " " " violet,
green " " " purple.

(1) Place ultramarine-blue and yellow disks upon the whirling apparatus, and so proportion the visible parts that when revolving rapidly they will produce the effect of gray. (In the study of *color* all shades of gray, from brilliant white to dead black, must be classed together as white, the difference between them being merely a difference of brightness.)

Try the same experiment with other pairs of complementary colors.

The sensation of white can be produced by a combination of three colors.

(2) Place a red paper, a green paper and a violet paper upon a whirling apparatus, and so vary the proportions of the visible parts of these papers that, when rapidly whirled before the eye, they will produce the effect of gray.

296. Fatigue of the Retina.—If one looks steadily for a short time at some strongly colored object held against a background of gray or white, that spot of the retina upon which the image of the colored object falls loses in part, for the time being, the power of giving the particular color sensation which it is furnishing, while its power of giving other color sensations may remain as great as ever. This temporary loss of power is called *fatigue* of the retina, and it may give rise to curious effects.

Hold a piece of bright-green paper against a white background and look very steadily *at one spot* on this paper for thirty seconds. Then look steadily at some one spot of the white surface for a few

seconds and note any peculiar color effect that is observed. The color complementary to green will probably appear as a patch upon the white, the shape of this patch being exactly like that of the green paper.

Try the same experiment with other colors.

297. After-Images.—The effects observed in the following experiment are still more curious than those of § 296.

Look steadily for half a minute *at some particular spot* on a window having the sky as a background. Then close the eyes and wait a few seconds for the figure of the window to show out against the darkness. Watch the changes of color the figure undergoes. Observe that details appear in this persisting image which were not noticed while the eyes were open.

"After-images" like the one here mentioned cannot be the work of memory. They must be due to some change of state in the retina, some real *impression* made there, which lasts for a considerable time but gradually passes away.

298. Other Ways of Producing Color.—We have noted that colored light can be obtained from white light by selective absorption or transmission (§ 241), by selective reflection (§ 242), by dispersion in refraction. There are certain other ways of breaking up white light.

Opalescence.—This occurs when a beam of white light enters a transparent medium containing very small diffused particles of any substance optically different from the medium itself. Some of the light passes through a medium in this condition, and some is turned back or aside by a process resembling selective reflection, though the particles which cause the phenoemnon are too small, in comparison with the wave length of light, to act by the regular laws of reflection. The light which is most largely sent back or deflected is the short-wave blue or violet; the light which is transmitted most largely is the comparatively long-wave red or yellow.

Opalescence gives the blue of the ordinary daylight clear sky, seen when we do not look very near the sun, and the yellow or red seen when we do look near it. We are especially conscious of the color of the transmitted light at sunset or sunrise; for the long stretch of dust-laden or water-laden lower strata of the atmosphere, through which the transmitted light then comes, colors it highly and at the same time makes it dim enough for the eyes to bear and observe.

Place a jar of clear water in direct sunshine, or in the path of an arc-light beam, and then stir into it little by little a strong soap solution, noting the changes of appearance of the water, as viewed from various points relative to the course of the incident light.

The blue of the ocean, of lakes, and certain streams, is due in part to opalescence, the water containing a certain amount of undissolved matter in the form of small particles, not necessarily enough to make it visibly turbid. The natural color of water, as seen where the daylight shines through it in waterfalls or in waves about to break, is green. Of course the apparent color of great bodies of water is much affected by the color of the sky or clouds above it, the light of which is reflected from the liquid surface.

Interference.—In certain cases some parts of a beam of white light are neutralized or suppressed by the like parts of another beam, when things are so arranged that the hollows of one set of waves are filled by the crests of a corresponding set. This is called *interference.* It often happens that some part, the blue let us say, is thus suppressed in certain places, the green in certain other places, and so on, the total result being a sort of spectrum or rain-bow effect. This effect may be produced in any one of several ways; for example, by reflection from a surface marked in certain ways, as in mother-of-pearl, or by a sort of bending, *diffraction,* suffered by light in passing through

a fine grating or mesh work, as when one looks at a small brilliant light through the eyelashes or through a handkerchief.

Phosphorescence and *fluorescence*, which also give color effects, will be described and discussed briefly in the next chapter.

299. Color Photography.—Many attempts have been made to reproduce in pictures by photographic processes the natural colors of the objects represented. One of the resulting methods has proved fairly successful in various special forms and will here be described in general terms.

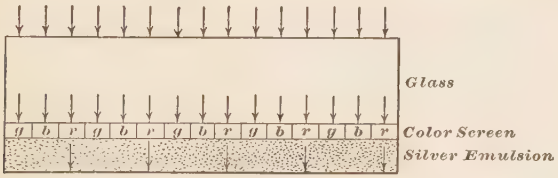


FIG. 224.—The exposure of a color-photograph plate.

This method proceeds, as do most if not all of the others, on the principle, or assumption, that, if we can reproduce the red, the green and the blue of a scene, the whole color-scheme will be reproduced thereby with tolerable accuracy.

The glass for the photographic dry plate is coated on one side with a layer of very small particles or spots, some red, some green, and some blue, distributed and mixed as well as may be, with no two particles or spots overlapping. See the layer marked *color-screen* in Fig. 224.

Outside this color-layer is the layer of sensitive compounds of silver, held in a gelatine film.

In use the plate is "exposed" in such a way that light falls upon it in the direction of the arrows. If, as we will for the present suppose, this light is all red, the green and the blue spots of the color screen will shield the silver

salts just beneath them from this light, and only those portions of the sensitive layer which can be reached through the red spots will be affected chemically. Accordingly, when the plate after this exposure is "developed" by immersion in the proper chemical bath, a "negative" is obtained, the character of which is indicated by Fig. 225,

FIG. 225.—Negative from a color-photograph plate.

the dark patches being parts of the silver layer which have been "reduced" to the metallic state, while the parts between are the still unchanged silver salt.[1] This negative is subjected to the action of another bath which dissolves the metallic silver of the dark spots, making these spots transparent, while leaving the silver-salt spots unchanged and still opaque. The result is a "positive," shown in Fig. 226.

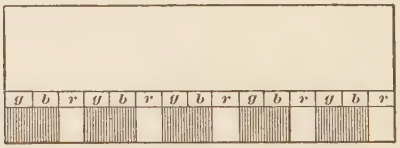


FIG. 226.—Positive (final form) from a color-photograph plate.

If now one looks through the plate at a white surface or toward a white light, one gets red light, and red only, through that part of the plate which we have been showing, the

[1] The plate, after development, is not put into the ordinary "fixing bath," to remove the patches of silver salt which have not been reached by the light; so the state shown by Fig. 225 is not that of a finished, "fixed," negative. The silver-salt spots shown as white in this figure are really opaque.

part on which the image of a red spot of the scene photographed fell during the exposure. The red spots which we now see on the plate, or the holes through which we now get red light, are so small and so near together that they give the effect of a continuous, though perhaps rather dim, patch of red.

Other parts of the plate, upon which during the exposure lay the images of green or of red spots of the scene photographed, will similarly show these colors now in their proper places; and thus we get a fairly good reproduction, in color, of the original, provided the white light toward which we look through the plate in its final condition is sufficiently strong.

CHAPTER XXVI.

RADIANT ENERGY.

300. The "Luminiferous Ether."—That medium (see §§ 154 and 231) by which light is transmitted through what we commonly called a vacuum is named by physicists the *luminiferous* (*light-bearing*) *ether*. This medium is a good deal of a mystery. We have overwhelming proof that it transmits energy by means of a wave motion or, at least, some progressive periodic change of state. It would seem, then, to have inertia, like ordinary matter, but it is not, so far as we know, subject to gravitation like ordinary matter. Its wave motion has been compared to that of an elastic *solid*, yet it does not, so far as we have discovered, impede the heavenly bodies in their motion through it. These various properties, positive and negative, do not harmonize very readily, and we have still much to learn about the ether.

All ether waves are called *radiant waves*, or waves of radiant energy.

301. Nature and Length of Radiant Waves.—It is now well known that radiant waves are electrical and magnetic oscillations in the ether. We do not believe that the sun sends *electricity* to us, but we do believe that the heat vibrations of the sun's particles disturb the electricity of the ether, and that this disturbance is transmitted to us somewhat as a shock at one point of the earth sends a quiver to another point.

The length of radiant waves, the distance from the centre of one wave to the centre of the next (C to C_1 in Fig. 227) is known to range from a few ten-thousandths of a millimeter to many meters. The shortest waves known to us

are revealed by their photographic or fluorescent effects; those a trifle longer give us directly the sensation of light, enabling us to see things; the longest waves are at present best shown by their electrical or magnetic effects.

All of these waves produce heat when they fall upon bodies incapable of reflecting or transmitting them as *waves*.

One surface may give off many different kinds (lengths) of vibration all at the same time. With most bodies the proportion of short waves given off increases as the temperature of the radiating surface rises.

FIG. 227.—Illustrating a wave of radiant energy.

302. Radiating Power, Absorbing Power, etc.—*Radiating Power.*—Different surfaces at the same temperature do not radiate equally well. The best radiating surface known, for all temperatures, is the lusterless black of soot, called lamp-black; though some other dull surfaces, that of white lead for instance, may perhaps radiate equally well at low temperatures, such as that of boiling water. Polished metal surfaces are bad radiators.

Absorbing Power.—The *absorbing power* of a substance is its power to take in radiant energy which falls upon its surface and turn it into heat. Absorbing power and radiating power go together; that is, any surface at a given temperature absorbs particularly well the kinds (lengths) of vibration which are most freely given off from it at the same temperature.

Though a dull white surface may radiate nearly as well as a lampblack surface when both are at the same low temperature, the white surface does not radiate so well as

the black at high temperatures, and the following experiment shows that white absorbs less readily than black the radiations from a very hot body, the sun.

Take a lens 4 or 5 cm. in diameter, and let the bright sunlight pass through it and fall upon a piece of white paper. Find by experiment at what distance from the lens the paper must be held in order that it may become charred very slightly without being consumed. Then blacken a spot upon the paper by rubbing it with a lead-pencil, and again present it to the lens, letting the light fall upon the blackened spot, the distance being the same as before. Is the paper now consumed?

Reflection of Radiant Energy.—The student is already familiar with the reflection of the light; that is, of radiations giving the sensation of light. It is easy to show by experiment that other radiations, the so-called *obscure* radiations, coming from a body that is not hot enough to be luminous, are reflected in the same way:

Exchange of Radiations.—The rate at which one body loses heat to another by radiation depends upon the difference in temperature of the two bodies. If the two are at the same temperature, there is an interchange of radiation between the two; but it is an equal interchange, neither, on the whole, gaining at the expense of the other. If the two are at different temperatures, the hotter loses to the cooler, and loses more and more rapidly if the difference in temperature is increased. When the difference in temperature is not large, the rate of loss from the warmer body to the other is nearly proportional to this difference; but when the difference of temperature rises to hundreds or thousands of degrees the rate of loss by radiation increases much more rapidly than the difference of temperature.

Attempts have been made to estimate the temperature of the sun by finding the rate at which we receive radiant energy from it, and calculating, by means of the law of

radiation, how hot the sun should be to radiate to us as it does. But the law of radiation and the nature of the sun's surface are so uncertain that estimates so made have varied enormously. It now seems likely that if the sun's surface were of lampblack, or something radiating equally well, its temperature would have to be in the neighborhood of 6,000° C., to enable it to radiate as freely as it now does. The highest temperature obtained by human contrivance, in the electric arc, is probably between 3500° and 4000° C.

303. Characteristic Radiations: "Spectra" of Different Substances.—When a substance is in the gaseous condition, so that its particles are free to vibrate in their own peculiar way, it gives off radiations or a combination of radiations different from those of other substances.

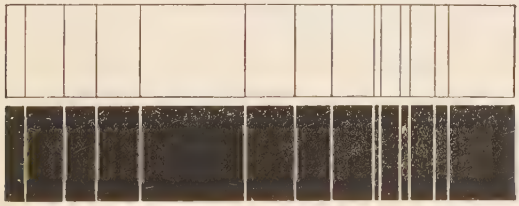


FIG. 228.—Comparison of spectral lines.

When, therefore, the light from any self-luminous, or *incandescent*, gas is allowed to pass through a prism (§ 270), under proper conditions, it forms beyond the prism a spectrum (§ 271), which is different from the spectrum of the sun.

The spectrum of any gas appears upon a screen as one or more bright lines parallel to the edges of the prism. In some gas spectra there are scores or hundreds of such lines. The upper part of Fig. 228 shows a few of the lines in the spectrum of iron vapor.

When a gas is very much compressed, its spectrum shows more lines. The explanation probably is that the more frequent collisions which its particles now suffer keep these particles vibrating in certain ways which would not be maintained if the particles were less disturbed.

Suspend a stick by a long string tied to one end. Strike the stick smartly at the lower end. Observe that the *quiver* of the stick dies out long before the *pendulum* motion, both of which are set up by the blow.

When compression goes so far that condensation occurs, giving either a liquid or a solid, the bright lines of the spectrum become so numerous that they meet each other, making an unbroken band of light from one end of the spectrum to the other. All *incandescent*[1] liquids or solids give such *continuous* spectra, that is, become "white-hot," when raised to the proper temperature.

304. Absorption Spectra.—Every substance absorbs especially well the same kinds (wave lengths) of radiation which it gives out especially well.[2] Accordingly, if the light from an incandescent solid or liquid shines through a vapor which is not incandescent, the spectrum of the light that has passed through the vapor is not perfectly continuous but shows certain comparatively *dark* lines. These lines correspond in position in the spectrum to the *bright* lines which the vapor in question would give if it were incandescent. A spectrum showing such dark lines is

[1] By *incandescent* liquids or solids we mean those which have become luminous through being greatly heated, giving off long-wave, non-luminous, rays only, when slightly heated, continuing to give off such rays but adding also short-wave, red, rays, when heated further, continuing to give all these but adding rays of shorter and shorter wave length, yellow, green, blue, etc., with still greater heating. The exceptional cases (see § 306) in which liquid or solid bodies are luminous without being hot, in the ordinary sense, are called cases of *luminescence*, and the bodies are said to be *luminescent* in this condition.

[2] A similar fact is observed and discussed in the case of sound waves. See experiment 2, § 218.

called an *absorption* spectrum. It is just as good a witness to the presence of the vapor as the bright-line spectrum of that vapor would be.

Absorption spectra can be obtained with *incandescent* vapors, if the light shining through them from behind is so powerful that the vapors get more than they give of their peculiar radiation.

305. Spectrum Analysis.—The spectrum, bright-line or absorption, of the vapor of a substance is often made use of to detect the presence of this substance in cases where chemical tests or other physical tests would be powerless. Certain of the chemical elements have been made known to man for the first time by means of their spectra, appearing under such conditions that substances already known could not account for them. The examination of a substance by means of the spectrum of its vapor, so as to find out what it is made of, is called *spectrum analysis*.

Spectrum analysis finds magnificent application in the study of the sun and the stars. The spectrum of the sun, examined under proper conditions, is found to be crossed by a vast number of dark lines. These lines, which are called Fraunhofer's lines, because Fraunhofer studied and described many of them very carefully, are now known to be absorption spectra. They indicate the presence of certain vapors in the atmosphere of the sun, which is cooler than the inner part that radiates through it. Among these absorption spectra many are recognized as corresponding to the bright-line spectra of substances on the earth. The lower part of Fig. 228 shows some of the absorption lines of iron in the sun's spectrum.

On such evidence as this we are persuaded that the sun and the stars are made up of just such elements as those which make our earth.

306. Luminescence: Fluorescence, Phosphorescence,

etc.—As a rule, bodies which are giving off light of their own, not simply reflecting or transmitting light that has fallen upon them, are hot, far too hot to be handled directly. But there are numerous cases in which certain bodies are self-luminous, giving off certain kinds of light, without being hot in the usual sense. This phenomenon is called (see p. 380, foot note) *luminescence*. There are several classes or cases of luminescence.

Fluorescence.—When a body which is receiving light of one color gives off light of a different color, not a constituent of the first light, without being hot in the ordinary sense, the phenomenon is called *fluorescence*, the name being taken from *fluor spar*, a mineral which shows the effect.

The fluorescent screen used in applications of X-rays, or Röntgen rays, consists essentially of a piece of pasteboard, one surface of which is covered with a substance that glows with greenish-yellow light when and where the X-rays fall upon it. These rays are not luminous in the ordinary sense. They might fall directly upon the eye without giving the sensation of light.[1]

A part of the color shown by kerosene contained in a glass vessel is due to fluorescence.

The full theory of fluorescence has not yet been worked out, though much study is given to the subject.

Phosphorescence.—Some bodies, without being hot in the ordinary sense, continue for a sensible time to give off light after light has ceased to fall upon them. This phenomenon is called *phosphorescence*, after the elementary substance phosphorus, which, if the surface is fresh, glows in the dark in air. This luminosity of phosphorus, however, is not *phosphorescence* within the strict use of the term; for it is due to a chemical action, a slow combination of the phosphorus with the oxygen of the air, and not to a previous

[1] They produce photographic effects, however, as well as fluorescent effects.

illumination of the substance by light from some other source. It is like the genuine phosphorescence, however, in the most interesting feature of the two phenomena, the giving off of luminous rays by a body which is not hot and is not, at the time, illuminated from without. This is the mystery of the fire-fly and of the glow-worm; but the beauty and marvel of the action is perhaps greatest in the luminosity of sea water at night under certain conditions; for it will sometimes happen that every drop falling from an oar upon the still surface gives rise to spreading rings of light, and darting fish leave gleaming trails[1] behind them.

Still another form of luminescence is produced by friction or percussion. This may be seen when two hard lumps of white sugar are struck against each other with a glancing blow in the dark. It is something very different from the sharply localized *incandescence* observed when the edge of a flint and the edge of a piece of steel "strike fire." We cannot set any thing on fire by means of the glow from the sugar, or, indeed, by means of any form of *luminescence* known to us.

[1] The glow is due to microscopic living creatures in the water. They can be strained out by the use of filter-paper. They glow for a time after still water is disturbed. If the disturbance is continued, they glow less or cease to glow, apparently becoming exhausted.

CHAPTER XXVII.

MAGNETISM.

307. Magnets, Natural and Manufactured.—There is a certain iron ore of which lumps are occasionally found having the power of attracting particles of iron. Such lumps of ore are called *load*stones, or *lode*stones, a word equivalent to *leading*stones. They are called also natural magnets.

A piece of iron acted upon by a loadstone becomes in turn a magnet, and will attract particles of iron as the loadstone does. Magnets can be made in other and better ways and are familiar objects to most students. Cobalt and nickel have strong magnetic properties, but are inferior to iron in this respect, and magnets made of them are curiosities rather than articles for real use. Hard steel magnets retain their power better than those made of soft iron, and therefore most magnets are made of steel *tempered* very hard. Cast iron, after being treated by a certain heat process, is very retentive of magnetism.

308. Induced Magnetization; Temporary and Permanent Magnetization.—Magnetization maintained in a piece of metal by the action of magnetic force is called *induced* magnetization. Such magnetization disappears when the maintaining force is withdrawn, and it is therefore called *temporary* magnetization. That which remains is called *permanent* magnetization.

Take several small pieces of very soft wrought iron, horseshoe-nails for instance, and test them among themselves to see whether they will exhibit any magnetic power. Then lift one of them by means of a strong bar-magnet and apply one end of a second nail to the lower end of the one so lifted. If the second nail remains sus-

pended by magnetic action, attach a third to it, and so on until the chain so formed breaks. Finally, after removing all the nails from the neighborhood of the magnet, test them again among themselves for evidences of magnetization

309. Magnetic Needle; Magnetic Compass.—A slender magnet suspended by a flexible fibre or balanced upon a sharp point, so as to be free to turn in a horizontal plane, is called a magnetic *needle*.

It has been known to Europeans for about seven centuries, and possibly to the Chinese for some thousands of years, that a magnetic needle, in coming to rest after any disturbance, tends to a position in which its length will be in a general north and south direction. At some parts of the earth's surface the needle points somewhat to the west of north, in others somewhat to the east of north; and its exact direction of pointing at any one place on the earth's surface varies from century to century.

The angle between "true north" and the magnetic north is called the *declination*, or sometimes the *variation*, of the needle. Charts for the use of mariners indicate its magnitude for various places and times. For example, the 1910 U. S. Coast Survey chart for Gloucester Harbor, Massachusetts, gives the "variation" as 13° 45′ W. in 1903 and the rate of its increase as 2′ a year.

An instrument in which a magnetic needle is placed within or over a graduated horizontal circle, so that the turning of the needle from its normal position may be at once read off in convenient divisions of the circle, is called a *magnetic compass*. The *mariner's compass*, by which the sailor steers his course in the open ocean, is such an instrument, having usually several small magnets fastened to a cardboard support balanced upon a point.

310. Magnetic "Dip"; the Earth's Magnetic Poles.—If a symmetrical magnetic needle is supported at its middle

so as to be free to turn in a vertical plane on an east and west horizontal axis, one end of the needle will, at most parts of the earth's surface, hang lower than the other end. See Fig. 229. At most places in the Northern Hemisphere the north-seeking end of the needle will hang low; at most places in the Southern Hemisphere the south-seeking end will hang low. In either case the needle is said to *incline,* or *dip.*

In general, as one goes north the north-seeking end of the needle will dip more and more; and at some place in

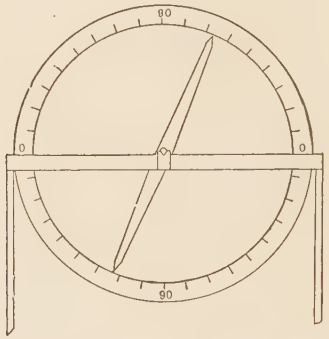


Fig. 229.—"Dipping "magnetic needle.

the far north it will point straight down. This place, which is called the north *magnetic pole* of the earth, is not at the geographic north pole. Somewhere in the far south is a corresponding south magnetic pole, where the south-seeking end of the magnet would point straight downward.

It is easy to show by experiment that the earth has the power of magnetizing iron. The earth is, in fact, a magnet, and the "natural" magnetic power with which the loadstone is found endowed has, no doubt, been imparted to it by the magnetic action of the earth as a whole.

What makes the earth magnetic is not certainly known, although various theories to account for its condition have been proposed.

311. Poles of Ordinary Magnets; Magnetic Attraction and Repulsion.—If one takes an ordinary bar-magnet and presents different parts of it in turn to one end of a magnetic needle, it will be found that the ends have much more effect than the middle. As in the case of the earth we speak of two magnetic *poles* from which the magnetic influences appear to come, so in the case of the bar magnet we call the two regions, usually near the ends, in which the peculiar power of the magnet seems to lie, the *poles* of the magnet.

If we present the two ends of a bar magnet in turn to the magnetic needle, we shall find that one end attracts the north-seeking point of the needle and repels the south-seeking point, while the other end of the magnet repels the north-seeking point of the needle and attracts the south-seeking point.

If we now float the magnet on water, on a board just large enough to carry it safely, or suspend it properly, we shall find that the pole which repelled the north-seeking pole of the needle will itself point north. We conclude, then, that *poles which repel each other are alike* and that *poles which attract each other are unlike.*

Law of Inverse Square.—By means of careful and delicate experiments upon the attractions and repulsions exerted by the poles of magnets, it has been found that these attractions and repulsions are, other things being equal, *inversely proportional to the square of the distance between the mutually acting parts.*

For example, if two poles one foot apart attract each other with a certain force F, the same poles when two feet apart will attract each other with a force $F \times \frac{1}{2 \times 2} = \frac{1}{4} F$.

312. **Opposite Kinds of Magnetism.**—The property by virtue of which a magnet attracts or repels is called *magnetism*. To give it a name does not *explain* the property, but the name is a convenience.

As there are two kinds of magnetic poles, so we must recognize two kinds of magnetism.

Does a magnet have equal amounts of the two kinds of magnetism? The following experiment would help to answer this question:

Returning to the floating magnet (§311), note whether it tends *as a whole* to *drift* either toward the north or toward the south.

Try this experiment with a variety of magnets, and note whether any of them move as a whole in either direction, taking care in all cases to have the magnet at rest when the experiment begins, and having the surface of water as large as practicable.

The float should be the lightest that will bear the magnet with security.

If, on experimenting carefully in this way, we should find a magnet which always floats toward the north, we should have to explain it by the supposition that this particular magnet has more north-seeking magnetism than south-seeking magnetism. But in fact no such magnets are known.

This is an interesting and rather surprising fact. One would naturally expect, after breaking at the middle a long magnet, with a well defined pole near each end, to find each half possessed of only one kind of magnetism and one pole, but this does not happen.

Not all poles are near the ends of magnets. A saw blade that has been touched by one end of a strong magnet near the middle will have three poles, one at the middle and one at each end, the end poles being like each other and unlike the middle pole. The strength of the one middle pole will be equal to that of the two end poles.

313. Magnetic Field; Lines of Magnetic Force.—Any portion of space in which magnetic force is found is called a *magnetic field.*

If, starting at any point of a magnetic field, we note the direction in which the north-seeking end of small magnetic needle points, and move the needle bodily in this direction, and if we continue this process, changing the direction of motion, if need be, continually so as to make it at all times agree with the direction of pointing, the centre of the needle will trace out what is called a *line of magnetic force.*

The following experiment shows in a striking way some of the general features of the lines of force around a magnet.

Place a bar magnet (No. 77) under a sheet of paper, and then slowly sprinkle iron filings on the paper.

Exercise 39 makes a more particular study of lines of magnetic force.

314. Theory of Magnetism.—Putting together what we have learned in the preceding sections, we can form some theory of the nature of magnetism. The fact that the lines of force of an ordinary bar magnet come from or return to its ends, the middle having little or no effect upon the needle, seems at first to show that the ends only are magnetized; but when we find that such a magnet if broken at the middle at once shows opposite poles at the break, and when we find that every magnet has equal quantities of the two kinds of magnetism, we are led to conclude that a magnet, instead of having all its magnetism confined to the poles, is magnetized with opposite poles in its smallest particles—is made up, in fact, of particles, or molecules, each one of which is endowed with equal quantities of the two opposite kinds of magnetism.

If we think of a bar magnet as made up of such particles

(see Fig. 230), with their *north* poles all pointing in the same general direction, we can see that in the middle of the bar the north pole of any particle is sure to be very near the south pole of some other, so that this part of the bar will have little or no action upon outside bodies. At either end of the magnet, however, there is a surface made up of poles all of the same kind, and so from the ends comes the power which affects the needle.

Breaking the bar at the middle merely separates two surfaces, which, one being made up of north poles and the

Fig. 230.—Illustrating the theory of magnetism.

other of south poles, neutralize each other while in contact, but have each the power of attraction or repulsion when separated.

As to the magnetization of the molecules themselves, it seems likely that it is natural or permanent in them, so that the act of magnetizing a bar is merely a process of *arranging* the molecular magnets of which it is composed.

It is thought that this magnetism of the molecules may be due to electric currents *within* them, not passing from molecule to molecule. This theory cannot well be discussed until after the student has become somewhat familiar with the common properties of electric currents.

315. Strength of Magnetic Pole and of Magnetic Field.—The *unit magnetic pole* is defined as *a pole that would act upon an equal magnetic pole at unit distance with unit force.*

We will take the centimeter as the unit of distance and the *dyne* as the unit of force in this definition.

A magnetic field (§ 313) is said to be of *strength*, or *intensity*, H, if a unit magnetic pole placed in it requires a force of H units (dynes) to prevent it from moving in the direction of the lines of force of the field.

Questions and Problems.

(1) How great is the repulsion of a unit pole upon a like unit pole at a distance of 5 cm.?

(2) How great is the attraction of a unit pole upon an unlike 5-unit pole at a distance of 10 cm.? How great is the attraction of the 5-unit pole upon the unit pole in this case?

(3) How great is the action of a 5-unit pole upon a 10-unit pole at a distance of 20 cm.?

(4) Four like 5-unit poles are placed at the corners of a square 10 cm. on a side, the diagonals of the square extending east and west and north and south. How great is the resultant repulsion of the east and west poles upon the north pole, and what is its direction? (Refer to the Parallelogram of Forces.)

(5) With the arrangement just described, how great is the resultant repulsion of the east, the west, and the south pole upon the north pole?

(6) If a magnet having two poles 20 cm. apart, each pole having a strength of 100 units, is suspended in a horizontal magnetic field of intensity 0.2, describe some combination of forces that will hold the magnet at right angles with the lines of the field.

(7) If the magnet in the preceding problem were allowed to point in its natural direction in the field, how great a force would be required to prevent it from moving, as a whole, in the direction of the lines of force of the field?

(8) Can a bar magnet have its two ends magnetized alike, so that, for example, both of them will be repelled by a north-seeking pole? If so how can this state of magnetization be produced?

CHAPTER XXVIII.

ELECTRICITY.

316. Historical.—It was known to the ancient Greeks and Romans that certain substances acquire, when rubbed, the power of attracting light objects. Amber was one of these, and from the Greek name of this substance, ἤλεκτρον, the name *electricity* was formed by the English philosopher Gilbert, who was born in 1540 and died in 1603.

It appears that Gilbert was the first to make systematic and extensive observations of electrical phenomena. From his time the subject has grown in interest and importance through the invention of frictional electrical machines and the Leyden jar, Franklin's discovery of the identity between lightning and electricity, the invention of the *galvanic*, or *voltaic cell* (§ 330) about the beginning of the last century, the work of Oersted, Ampère, Faraday, and a host of others more recent, until the name *electricity* is now the most popular and fascinating in the whole vocabulary of physical science.

317. Electrification by Friction.—Experiments with electricity produced by friction are very beautiful, and of great theoretical interest, but many of them are troublesome to perform, and their practical importance is comparatively small. Those which follow are selected partly for the light which they throw on the phenomena of electric currents.

(1) On a cold, dry, day rub a rod of gutta percha or hard rubber with a catskin, and then present the rubbed part to small pieces of paper or bits of thread lying on a table.

(2) Fasten two small pith balls to the ends of a dry silk thread about 15 cm. long and suspend them by the middle of the thread from any convenient support.

Touch these balls with the freshly-rubbed rod of gutta percha. Note the behavior of the balls with respect to the rod just before they are touched and just after. Note also their behavior with respect to each other after they are touched by the rod. If they act in an unusual manner, it is because they have become electrified, or "charged with electricity," by the rod.

(3) Rub a smooth glass rod vigorously with a piece of silk, and present the rubbed part to the suspended pith balls still charged from the gutta percha. Note their behavior before and after being touched by the glass.

What evidence do we find in these experiments that there are two kinds of electrification? Is there attraction or repulsion between bodies similarly electrified? between bodies oppositely electrified?

318. Nature and Kinds of Electricity.—There has been a difference of opinion as to whether electricity is or is not a *substance*. A century ago, when heat and light were believed to be weightless fluids, electricity was classed with them as a substance. Later, when it was shown that heat and light were not substances, but mere "modes of motion," in which particles of matter are involved, the notion gained currency that electricity was a mode of motion, rather than a substance by itself. During recent years belief in the existence of electric substance, or substances, has been growing again.

It is shown by the experiments preceding that there are two kinds, or states, of *electrification*, and that bodies may be oppositely electrified, but this does not prove that there are two kinds of *electricity*.

It has been held by some physicists that there is only one kind of electricity. According to their theory all bodies in their normal, apparently unelectrified, state are endowed

with a certain quantity of electricity, any addition to which produces one kind of electrification, and any subtraction from which produces what we call the opposite kind of electrification. This is the so-called *one-fluid theory.*

Other authorities have held that there are two kinds of electricity, and that in the normal state a body is endowed with equal quantities of the two kinds, which neutralize each other. According to this theory a state of electrification is produced by making either kind of electricity on or in the body exceed the other in amount, the nature of the electrification being this or that according as one kind or the other of the two electricities is in excess. This is called the *two-fluid theory.*

The nature of electricity, and the question whether there are two kinds, is still in debate, but in describing electrical phenomena there is great convenience in using the terms of the two-fluid theory, and these terms will be freely used in this book.

Positive, or *Vitreous,* and *Negative,* or *Resinous.*— We shall, following custom, speak of the state into which sealing wax or any other resinous substance is brought by rubbing with catskin as being due to a charge of *resinous,* or *negative,* electricity, and the state of a glass rod rubbed with silk as due to a charge of *vitreous,* or *positive,* electricity.

It is customary to arrange a considerable number of articles in a list and state that any one of them becomes electrified positively when rubbed with another farther down the list and negatively when rubbed with one farther up the list. Such a list is given below.

If the lists given by different books are compared they will probably be found not to agree exactly. The fact is that the surface condition of a body of given material has much to do with its electrical behavior when rubbed. For example, as the following table indicates, silk is negative with

respect to smooth glass, but is sometimes positive with respect to roughened glass. The two sides of a glass plate, ground on one side and smooth on the other, may be used to show this fact.

1. Fur of cat.	4. Feathers.	7. Silk.
2. Polished glass.	5. Wood.	8. Shellac.
3. Woolen stuffs.	6. Paper.	9. Rough glass.

319. Conductors and Insulators.—If one attempts to repeat Experiments (2) and (3) of § 317, using a very thin wire or a cotton thread or a wet silk thread for suspending the pith balls on a metal support, in metallic connection with the ground, it will be found that the balls do not retain their charge as they did before. The explanation is, that the charge escapes along the thread or wire to the support, and so is lost. *Materials which, without motion of their own, can serve as avenues of escape for an electric charge, are called* CONDUCTORS *of electricity. Materials which cannot do this are called* NON-CONDUCTORS, *or* INSULATORS.

Metals are the best conductors, and resinous and vitreous substances are among the best insulators. No perfect insulator is known, and, on the other hand, there is, apparently, no perfect conductor—no conductor which is not somewhat heated by the passage of electricity through it, thus showing that it offers a certain amount of resistance[1] to the movement.

320. The Gold-Leaf Electroscope.—The pair of pith balls suspended by a silk thread used in §317 is sometimes called an *electroscope*, for the reason that it enables us to detect, or make evident, an electric charge; but for more delicate experiments a more sensitive instrument is needed. This is found in the *gold-leaf electroscope* shown in Fig. 231.

[1] It is thought, however, that molecular currents of electricity, not passing from one molecule to another, may exist without friction and so continue indefinitely after they are once set up. See § 314.

G may be an open-topped glass *receiver*, such as is frequently used with air pumps, fitted at the top with a cork, *c c*. *B* is a disk of metal, usually brass, perhaps 12 or 15 cm. in diameter, from which a metal rod, *r*, reaches downward through the cork. The upper ends of the strips of thin gold foil, *l l* (the thinnest used by dentists), are crowded into a narrow slot sawed in the lower end of *r*, so that there is good metallic contact from *B* to *l l*.

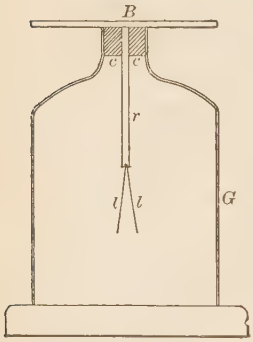


Fig. 231.—Gold-leaf electroscope.

Hold a charged rod of gutta percha or glass over the electroscope, and lower it gradually, stopping an inch or two above *B*. Then raise and lower the rod alternately several times. Are the leaves *l l* affected when the rod approaches the plate? Do they continue to show any peculiar action after the rod is removed?

(If the leaves become permanently charged, they can be restored to their normal condition by touching the edge of the plate *B* with the finger. One must take great care in dry cold weather not to rub the glass of the electroscope, even with the hand, lest it acquire a charge which will be difficult to remove, and which will greatly disturb the proper course of the experiments. Moreover, one must guard carefully against the action upon the electroscope of possible charges on the clothing. The experimenter should frequently test for a charge upon his sleeve by holding his arm, *without the charged rod*, over the plate *B*, the *electroscope being previously discharged*.)

321. Electric Induction.—In §317 pith balls were charged from an electrified rod by direct communication.

In §320 the electroscope was charged, temporarily, without contact with the electrified rod; but the fact that its charge disappeared when the rod was removed indicated that it really received nothing from the latter save a certain *influence* which threw it, for the time being, into a peculiar

state of electrification without really changing the amount of electricity upon it. This operation is called electrification by *induction.*

One can readily understand that, if every body is, in its normal condition, endowed with a certain quantity of positive electricity and a certain quantity of negative electricity (see § 318), the approach of a charged body may produce in the electroscope, previously uncharged, a redistribution of its electricities, one kind being drawn toward the approaching charged body, while the other is repelled into the leaves. If, on the other hand, one holds that there is only one kind of electricity, one may suppose that the approach of a body having more than its normal amount will repel the electricity of the electroscope into the leaves and that the appoach of a body having less than its normal amount will attract the electricity of the instrument into the plate *B*, either event putting the leaves into condition to repel each other. Whichever theory one adopts, experiment shows that a redistribution of electricity, and nothing more, does take place at the appoach of the charged body.

But can we now find some means of charging a body permanently by the aid of such induction?

Put the plate of the electroscope, *B* (Fig. 231), into good electrical connection with any large conductor, the body of the experimenter, for example, or the earth itself by means of water pipes or gas pipes, and then bring the charged rod toward *B*. Break the connection at *B* while the charged rod is still held near. Do the leaves now show any evidence of a charge after the rod is removed?

If any charge remains, is it of the same kind as that on the charged rod? To answer this question, bring the charged rod[1] again toward

[1] If there is a charge on the electroscope, the approach of an *unelectrified* conductor —the hand of the experimenter, for instance— toward the plate *B* may cause the leaves to approach each other, for the charge already on the electroscope induces a charge upon the approaching conductor, and is itself somewhat changed in consequence. To avoid error from this cause the charged rod should be a long one, so that the hand need not come near *B*.

B, and notice whether the *first effect* of its approach is an increase or a decrease of the divergence of the gold leaves. If it is an increase, the charge on the leaves is like that on the rod; if a decrease, the charge on the leaves is unlike that on the rod.

The explanation of the permanent charge obtained in this experiment is that the charge on the approaching rod drives electricity of its own kind from the metal of the electroscope to the larger conductor or attracts electricity of the opposite kind from the larger conductor to the electroscope.

322. The Electrophorus.—This instrument, the action of which depends upon induction, is very convenient for

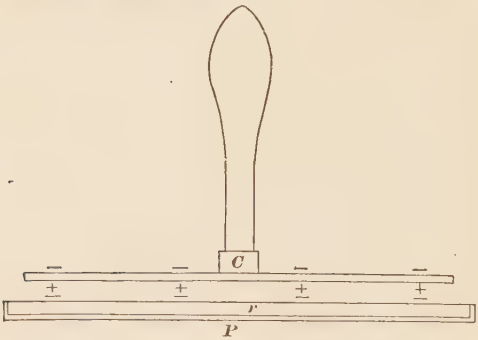


FIG. 232.—Action of the electrophorus.

supplying electricity with which to charge conductors of moderate capacity. It consists usually of a shallow metal pan (*P*, Fig. 232), which may be about 25 cm. in diameter, containing a quantity of resin *r*, or other similar material[1] and a flat circular plate of metal, *C*, somewhat less in diameter than the pan, furnished with an *insulating* handle of glass or hard rubber.

To prepare this apparatus for use, the resinous surface

[1] This material has been poured in while hot and has cooled in place, forming a smooth hard surface.

is rubbed with a dry catskin, and thus acquires a charge of negative electricity. The metal disk is then placed upon this surface. If the two fitted each other perfectly, the metal plate would become charged with the same kind of electricity as the surface upon which it rests; but the two surfaces really touch at certain points only, and so the negative charge upon the resinous plate *induces* a positive charge on the lower surface of the metal plate and repels some of the negative electricity to the upper surface.

If this upper surface is now touched for an instant by the hand of the experimenter, negative electricity escapes, and when the plate is lifted it carries the positive charge, which may be used to charge other conductors. The resinous plate meanwhile has suffered little loss, and the operation may be repeated many times without recharging it.

323. Self-Repulsion of Electrical Charge; Discharging Action of Points.—The self-repulsion of an electrical charge spreads it over the *outer* surface of the conductor upon which it rests.

Charge an insulated hollow metal sphere (see Fig. 233) by means of the metal plate of the electrophorus, making several contacts if necessary.

Take a metal ball, *b*, about 1.5 cm. in diameter, mounted on an insulating handle about 15 cm. long, touch it to the outer surface of the hollow charged sphere, and then to the plate of the gold-leaf electroscope, repeating the operation a number of times if the effect upon the electroscope is not at once evident.

Discharge the electroscope, by touching it with the hand, and then try to charge it by touching it with the little ball after this has been in contact with the *inner* surface of the metal sphere, repeating the operation as many times as before.

Does the ball appear to get any electricity from the inner surface of the sphere?

The self-repulsion of the charge tends to drive it off from the conductor, and great care must be taken to enable a

conducting body to retain its charge for any great length of time. The escape takes place with peculiar readiness from sharp points, especially when they are directed toward other neighboring conductors.

On the other hand, a conductor provided with such points receives electricity from neighboring bodies more readily than one not so provided.

The familiar disruptive effect of lightning, which frequently strips the bark from trees or projects abroad

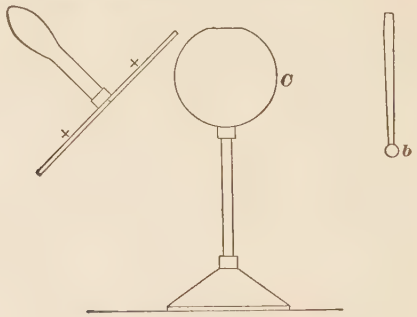


Fig. 233.—Charge on a hollow metal sphere.

splinters from a wooden steeple, is probably due to the self-repulsion of the very high electric charge produced for an instant on the body "struck."

324. Electrical Potential; Electrical Capacity.—There is a limit to the electrical charge which a given electrical agent can produce upon a given body.

If the upper plate of an electrophorus, after being charged as in the preceding article, is brought into contact with a piece of metal or any other conductor, *C*, supported on a glass or other *insulating* stand, the insulated hollow sphere already used, for example, the self-repulsion of the electricity on the plate will cause some of it to flow over upon *C;* but presently the self-repulsion of the charge thus imparted to

C prevents further flow. The plate and C are now at the same electrical *potential*, as it is called.

Two conductors are said to be at the same electrical POTENTIAL *when the potential energy of a quantity of electricity on one is just as great as the potential energy of an equal quantity of electricity on the other, so that there is no flow of electricity from one to the other when they are connected by a conductor.*

The difference of electrical potential between two bodies may be measured by the amount of work required to convey a definite small quantity of positive electricity from the body of lower potential to the body of higher potential.

If the plate is repeatedly charged and as often touched against C, the spark at contact of the two will become less and less noisy until it is almost or quite imperceptible. C is now charged about as highly as it is possible to charge it from the electrophorus in its present condition.

If a larger piece of metal than C, of the same shape, were used, more repetitions of the charging operation would be necessary in order to make the spark equally small. That is, the larger body would take more electricity than C in being charged to the same potential.

The larger body is said to have a greater *capacity* for electricity than the smaller.

The ELECTRICAL CAPACITY *of a body is measured by the quantity of electricity which must be given to the body in order to raise its potential a certain amount.* But see § 325.

Twice as much electricity would raise its potential twice as much, and so on.

325. Condensers: the Leyden Jar.—The electrical capacity of a conducting body does not depend upon its size and shape alone. It is much affected by the nearness or distance of other conductors, as the following experiment will show:

Take the inner coating, *I* (Fig. 234), from a dissectible Leyden jar and place this coating upon a glass support. Charge it by means of the electrophorus plate, counting the number of repetitions necessary to make the charge complete, so that the spark at the last contact shall be very slight.

Discharge the coating, by touching it with the finger or otherwise, and replace it in the jar. Connect the *outer* coating of the jar with the gas pipes or water pipes, or have some person keep his hand against it, and repeat the operation of charging the inner coating from the electrophorus plate; count, as before, the number of contacts necessary to make the charge practically complete, if this can be done.

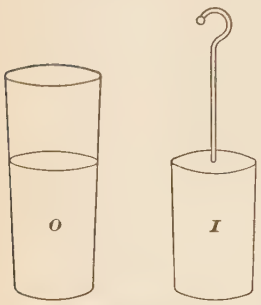


Fig. 234.—"Dissected" Leyden jar.

Compare the number of contacts in the two cases, and decide whether the capacity of the inner coating is greater or less when it is in the jar.

Discharge the jar by means of a discharging-rod (*D*, Fig. 235), touching first one knob to the *outer* coating, and then, without breaking this contact, bringing the other knob to touch the knob of the inner coating.

The Leyden jar is the most familiar form of electrical *condenser*. Its invention, so the story goes, was suggested by an accident. In 1746, Cuneus, a student of physics at Leyden, undertook to electrify some water contained in a glass vessel which he held in one hand. He used an iron wire to carry the electricity to the water, and was quite unconscious that the charge communicated to the water was inducing upon his hand which held the vessel an equal charge of the opposite kind, and that the mutual attraction of these two charges was making the water take up a much greater quantity of electricity than it would have done by itself. When he considered the water sufficiently charged, still holding the vessel in one hand, he attempted to remove the iron wire with the other hand. The positive and nega-

tive charges united through his arms and body, giving him a shock that frightened him greatly. He kept his wits, however, sufficiently to know how the accident had happened, and soon all the leading scientific men of Europe were repeating the experiment, with precautions.

Before long, coatings of tin were substituted for the hand and the water, and in this form the *Leyden jar* has been used ever since.

There are many other forms of electrical condensers, but they are all essentially alike: *two conductors separated by a non-conductor, the positive charge upon one conductor and the negative charge on the other mutually attracting each other, so that the capacity of each conductor is increased by the presence of the other.*

The definition of electrical capacity given in § 324 does not hold for the whole condenser, used as such. *The* CAPACITY OF A CONDENSER *is measured by the quantity of electricity, positive or negative, which goes to either half of the condenser while the difference of potential between the halves is increased by a certain amount.*

326. Specific Inductive Capacity.—The capacity of a condenser depends upon the nature of the non-conductor, or insulator, as well as upon the dimensions and shape of its parts. Thus, two metal plates separated by paraffin have a capacity about twice as great as that of two similar plates separated an equal distance by air.

The ratio of the capacity of a condenser using any given insulator to the capacity of a like condenser using air is called the *specific inductive capacity* of that insulator.

The specific inductive capacity of most, if not of all, solid or liquid insulators is greater than that of air.

327. Machines for Production of Electrical Charge.—We have seen that electrical charges can be produced by friction and by induction. The apparatus we have used thus

far in either method is very simple, and very slow in operation if any large charges are to be given. But "machines" have been invented and are in common use which make much more convenient and effective application of both friction and induction.

Friction Machine.—This usually consists of a glass plate, mounted upon a horizontal axis and made to revolve between two soft pads which press it on both sides, covering only a small part of it at once. Electricity of one kind appears upon the pads and electricity of the other kind upon the glass, as a result of this rubbing.

The pads are covered with a conducting preparation by which their electricity is steadily carried away. The part of the glass which has just been rubbed by the pads is by the revolution brought near a number of sharp metal points which take off its electricity. Thus the action is continuous and comparatively rapid.

Induction Machine.—This machine appears in several forms, but each of them may be described as a continuously acting electrophorus, or a combination of such instruments. For further description see Appendix VI, with Figs. 22a–26a.

328. Charge on Glass of Leyden Jar.—The induction machine makes it easy to charge Leyden jars heavily. A common method of doing this is to connect one coating of the jar with the rod r (Fig. 24a), and the other coating with the rod r', while the disk is kept in rotation.

The following experiment with a dissectible Leyden jar is interesting and instructive, as it shows that the coatings are merely a convenient means of *charging* and *discharging*, but are not essential to the action of the jar as a reservoir or *condenser* of electricity.

Charge the dissectible Leyden jar heavily and then lift out the inner coating by means of a glass rod. Place this coating on the table, then lift the jar by its top, and push off the outer coating.

Touch the two coatings together, noting whether any spark occurs.

Then put the jar back into the outer coating, drop the inner coating into place, *taking care now not to touch both coatings with the hands at the same time,* then connect the two coatings in the usual way, by means of the discharging-rod, to see whether any charge remains.

Repeat the experiment, rubbing the glass inside and out with the hand after the coatings have been removed.

329. Protection from Lightning.—Since the time of Franklin it has been customary to use certain arrangements of metal rods for the purpose of protecting buildings from lightning strokes. The most common device is an iron or copper rod having one end buried in the ground and the other attached to a chimney and extending some distance above the chimney top.

A common theory of such rods is: *first,* that they tend to *prevent* lightning strokes, by assisting the *quiet* passage of electricity between the earth and the clouds; *second,* that, if the stroke occurs, it will pass along the rod rather than through the building over which the top of the rod is raised. The first of these uses is probably very slight. The second may be very effective, *if the rod runs down several feet into water, or is attached to a large sheet of metal buried in damp earth.* Otherwise the rod may be useless, or worse than useless.

Under some conditions the discharge current is not wholly confined to the rod, a part of it passing in a spark between the rod and some neighboring piece of metal—the gas pipes or water pipes of the building, for example. See the discussion of discharge of outer coatings of the Leyden jars at the end of Appendix VI.

Sometimes the lightning pays little attention to the rod, but strikes some other elevated object connected with the building.

In short, one or two rods on a large building do not afford sure protection from lightning. The only certain means of

protection is to enclose the building in a network of rods or heavy wires, some vertical and some horizontal, like the bars of a cage. The following experiment will illustrate the efficiency of such an arrangement:

Place the gold-leaf electroscope on a sheet of metal near the induction machine. Cover the electroscope with a wire cage, the bottom of which shall rest upon the metal sheet (see Fig. 235). Connect the metal sheet with one of the rods of the machine. Lead a metal rod from the other rod of the machine almost to the top of the wire cage, leaving a gap of 1 cm. or more.

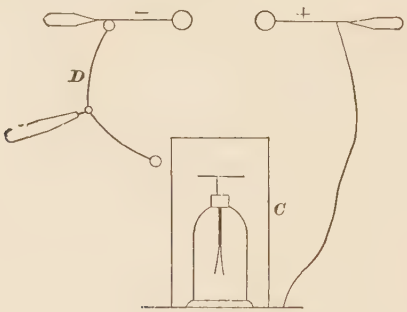


Fig. 235.—Illustrating protection from lightning.

Work the machine so as to allow sparks to pass to the cage and watch the behavior of the gold leaves.

Take away the cage and the metal sheet, and note the behavior of the leaves when a rod connected with one of the poles of the machine is brought near (not too near) the top of the electroscope.

The interior of an iron ship, having the iron of its masts and rigging connected with the outer shell, is a peculiarly safe place, so far as danger from lightning goes.

It is hardly practicable to cover a building with a close network of wires for protection, and such a network is not necessary. The meshes of the net may be many feet square and yet give effectual protection. The net should

be connected at several points with heavy wires soldered to large sheets of copper buried deep in damp earth.

The electric currents of lightning strokes last for an exceedingly short time—a millionth of a second, let us say. It is now known that such transient currents flow along the outer layer of a conductor rather than along the interior, and it is considered well to use flattened strips of copper, rather than round rods, for lightning conductors. Convenience, however, is in favor of the round rod, and this is more often used.

It is a false proverb which declares that lightning never strikes twice in the same place. Some localities are peculiarly liable to lightning strokes.

A forest probably affords very good protection from lightning; but it is well to keep away from the tallest trees. The shelter of a single tree standing alone in a field is a dubious protection at best and may be dangerous.

A person compelled to stay in a large open field during a violent thunder storm had better lie flat on the ground, since, other things being equal, the lower a body is the less likely it is to be struck by lightning.

Indoors, one would do well to avoid the immediate neighborhood of water pipes and gas pipes. Very few people indoors are killed by lightning.

Question.

Do you know from observation what precautions are taken to prevent lightning from entering buildings by way of telephone wires or other similar conductors?

CHAPTER XXIX.

THE GALVANIC CELL AND ELECTRIC CIRCUIT.

330. Electricity Developed by Chemical Action: the Voltaic, or Galvanic, Battery.—"In 1780, Galvani, professor of anatomy at Bologna, studying the influence of electricity upon nerves, observed by chance, in the skinned legs of a frog recently killed, convulsions which occurred at the moment when an electric machine was discharged near-by. . . . He set himself at once to study the circumstances of the phenomenon with various animals, warm-blooded and cold-blooded, with the desire of proving the identity of the nervous fluid with electricity. He devoted six years to this work.

"In the course of these researches, in 1786, wishing to see what effects the discharge of thunder clouds would produce, he suspended on a balcony of the terrace of the Zambeccari palace the hind legs of a frog, by means of a copper hook which passed through the spinal cord, and saw, with keen surprise, these members convulsively agitated, though no thunder cloud was near. He observed soon that this took place at the moment when the limbs accidentally touched the iron of the balcony. He could thenceforth repeat the experiment as often as he wished, and he found himself in possession of a new and unexpected fact, which has become the point of departure of a long series of brilliant discoveries, and the origin of one of the most extended and most important parts of physics."[1]

In the year 1800, Volta, another Italian, following the

[1] Daguin's *Traité Elémentaire de Physique*, tome troisième, vol. 3.

hint given by Galvani's discovery, showed how to maintain a continuous supply of electricity by chemical action. The most famous arrangement which he employed for this purpose consisted of a column made up of disks of copper, zinc, and cloth moistened with acidulated or salt water, the order of arrangement being copper, zinc, cloth, copper, zinc, cloth, etc., the column beginning with one metal and ending with the other. This is called *Volta's pile.* It has the advantage of great simplicity of construction and portability, but loses its power as the liquid evaporates from the cloth disks, and is not so easily cared for as another arrangement (see Fig. 236), also devised by Volta,

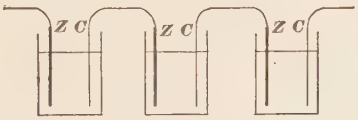


FIG. 236.—Galvanic "pile," or "battery."

in which the liquid is contained in vessels, each vessel containing also a copper and a zinc strip. The copper of one vessel is joined to the zinc of the next, and so on through the whole series of *cells,* which was for a long time called, like its predecessor, a *pile.*

By means of such a pile, or *battery,* as it is now generally called, it is easy to produce effects resembling in kind those shown in the preceding experiments upon electricity, thus showing that the electricity obtained from chemical action is just like that obtained by rubbing bodies together. The intensity of charge obtained from a battery of any moderate number of cells is, however, much less than that obtained by friction, and some pains must be taken to make the electrification produced by the battery evident.

(1) Take a battery of 50 small cells, each containing water, a strip of zinc, and a strip of copper, connected according to the de-

scription given above, the one free zinc at one end of the series and the one free copper at the other end of the series being each provided with a copper wire about 50 cm. long.

Now take the electroscope and cover the plate with a sheet of thin paper, *S* (Fig. 237), which has been soaked in melted paraffin. Place on this paper the metal cover of the electrophorus, taking care that the paper shall prevent all contact between the two plates of metal. Then touch the lower plate with one wire leading from the battery and at the same time touch the upper plate with the other wire. Remove the wires, and lift the upper plate and the paper. Do the leaves show any evidence of an electric charge?

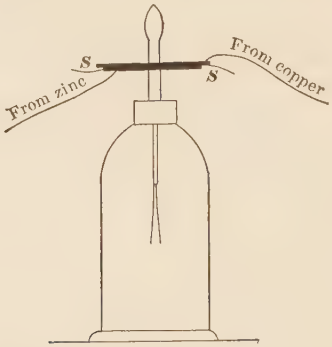


Fig. 237.—Charging a "condenser" by means of a galvanic battery.

(The two metal plates with the paper between them serve as a *condenser* (§ 325), thus enabling the electroscope to take a larger amount of electricity from the battery than it otherwise could. When the plates are separated, the *capacity* of the electroscope is diminished and the same quantity of electricity produces a higher state of *charge*.)

(2) Charge the electroscope again, touching the lower plate with the wire leading from the copper end of the battery and the upper plate with that leading from the zinc, and then, after removing the upper plate and the paper, test the charge upon the electroscope, in the way already described (§321), to find whether it has received positive or negative electricity from the battery.

Then repeat, touching the lower plate with the wire leading from the zinc end of the battery and the upper plate with that leading from the copper.

Which end of the battery furnishes positive electricity?

THE ELECTRIC CIRCUIT WITH A GALVANIC CELL.

331. Open Circuit and Closed Circuit; Current.—A galvanic cell with the external wires, or other conductors, connecting its plates, or strips, of metal, is one form of *electric circuit.* When the plates of the cell are not connected by an unbroken line of conducting material, the circuit is said to be *open,* or the cell is said to be *in open circuit.*

If the conducting line is unbroken, as when the wires from the cell are connected with the galvanoscope in Exercise 40, the circuit is said to be *closed.*

It is probable that during the greater part of the nineteenth century most physicists would have described an *electric current* as made up of two streams of equal strength, one of positive and the other of negative electricity, moving simultaneously in opposite directions through every cross-section of the circuit, that is, every cut across the streams either inside or outside the cell.

It is now well known that *within the cell* the two oppositely directed streams are not usually equal, the positive stream being the stronger in some cells and the negative stream the stronger in other cells. In every cell through which an electric current is passing there is a movement, invisible, of some constituent of the cell in the direction of the positive stream and a movement, also invisible, of some other constituent in the direction of the negative stream.

These movements of matter are, apparently, inseparably connected with the accompanying movements of electricity; and the difference in strength of the streams of electricity is believed to be due to a difference in the facility of movement of the positive-carrying and the negative-carrying streams of matter.

The *sum* of the positive and negative streams of electricity within the cell is believed to be the same as the

sum of the positive and negative streams through any cross-section of the circuit outside the cell, and when the phrase *strength of the current* is used without explanation it means this *sum*.

The phrase *direction of the current* means in common use the direction of the *positive* stream.

332. Terms Relating to the Cell.—Michæl Faraday (1791-1867), who was one of the greatest investigators in electricity that the world has known, invented, with the help of a friend learned in the ancient tongues, several terms from the Greek, which he applied to various parts of the galvanic cell. These terms are now in very common use.

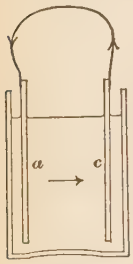


FIG. 238.—Anode, *a* and cathode, *c*.

Thus, the conductor, usually a solid (*c*, Fig. 236), by which the current (positive) flows out from the cell is called the *cathode*, that is, the *way down* out of the cell.

The conductor, usually a solid (*a*, Fig. 238), by which the current flows back into the cell is called the *anode*, that is, the *way up* into the cell.

These terms are apt to be confusing, because it may seem that the "*way down*" should lead *into* the cell, but Faraday fixed the use once for all.

The cathode is frequently called the positive pole, or plate, of the cell, and the anode the negative pole, or plate.

In a storage cell (§ 334) the set of plates by which the current enters during the "charging" process is called the positive pole, because during the real use of the cell the current leaves the cell by this pole.

The anode and the cathode together are called *electrodes*.

The passage of electricity through a cell is accompanied, as we shall see in the next Article, by decomposition of some chemical compound in the cell. This kind of decomposi-

Michael Faraday

tion is called *electrolysis*, and the compound so broken up is called an *electrolyte*.

The parts into which the compound is broken up are called *ions*. Some of these make their appearance at the anode, and are called *anions*. Others appear at the cathode and are called *cathions*, or *cations*.

333. Chemical Action of a Single-Fluid Cell.—A *molecule*[1] of sulphuric acid, we are taught by chemists, consists of two *atoms* of hydrogen (H_2), an atom of sulphur (S), and four atoms of oxygen (O_4). The symbol showing the composition of the molecule is H_2SO_4.

When sulphuric acid comes in contact, under proper conditions, with zinc, chemical action takes place, and the two atoms of hydrogen are replaced by one atom of zinc (Zn). The hydrogen is thus set free as a gas, while the compound that remains is $ZnSo_4$, called zinc sulphate, or sulphate of zinc. This change, or *reaction*, as it is called, may be represented briefly by the following equation:

$$Zn + H_2SO_4 = H_2 + ZnSO_4,$$

in which the sign = is equivalent to the word *produces*.

[1] **Atoms and Molecules.**—*Chemistry and Physics.*—Of all the substances known to us, about eighty are called *elementary substances*, or *elements*.

The smallest particles of an elementary substance that science has, until recent years, had any knowledge of are called *atoms*, the name, which means *undivided*, or *indivisible*, implying something that science does not know how to divide. It now appears that, under certain conditions, atoms can be broken. The atoms of any one substance are generally supposed to be exactly alike.

There is reason for believing that in most elementary substances each atom is, in its ordinary condition, joined in some definite manner to one or more other atoms. These definite combinations of atoms are called *molecules*.

The atoms of one element may unite in definite combinations with those of one or more other elements. A group of atoms so formed is also called a molecule; but it is a molecule of some new substance, different from any of the elements that go to make it up. Such a substance is called a *compound substance*. It has molecules, but does not have atoms of its own.

The science of *chemistry* deals with atoms and the ways in which they combine to form molecules. *Physics* takes molecules ready-made, and studies their behavior and the phenomena which bodies can show without suffering a break-up of their molecules.

If this reaction takes place while a battery is not in use, it is evidently wasteful of zinc, which is a rather expensive metal. Chemically pure zinc, not in contact with other metals, is but little affected by sulphuric acid, but its high cost forbids its use in batteries. Impure zinc when amalgamated acts much like chemically pure zinc. See Exercise 40.

Copper is, under ordinary circumstances, but little affected by sulphuric acid. Any action seen at the surface of the copper strip in Exercise 40, even when the circuit is closed, is mainly due to hydrogen bubbles, freed by chemical action which leaves the copper chemically unaffected.

In the cell of Exercise 40, SO_4, called *sulphion*, is the *anion* and H is the *cation*.

334. Polarization in a Galvanic Cell; Storage Cell.— The gradual weakening of the current possibly observed in Exercise 40, when the circuit is closed for a considerable time, and its recovery when the copper strip is thoroughly rubbed, are phenomena well worthy of attention. The weakening is not due to chemical exhaustion of the solution, but rather to the condition which the action of the cell produces by depositing a layer of hydrogen on the surface of the copper strip. A similar but more striking effect is shown when a current is sent through a cell consisting of two plates of lead in a mixture of sulphuric acid and water. Such a cell, though incapable of producing an electric current by itself at first, gradually, while a current is driven through it, gains the power of independent galvanic action.

Any current which the lead cell, used alone, may yield is due to the peculiar state into which its plates are thrown by the action of the current used in *charging* it. This state is called battery *polarization*. It does not merely offer resistance to the electric current by which it is pro-

duced, for it tends to send a current backward through the cell, and will do so, as experiment shows, when opportunity is given. It was such polarization that reduced the strength of the current in Exercise 40, after a little time, if a reduction was observed there.

Polarization is a serious disadvantage in many galvanic cells, but it is not without its uses; for upon it depends the action of all the so-called *storage batteries*, which since the year 1881 have come to fill an important place in the manifold applications of electricity. It is to be noted that the thing stored in such cells is not, according to the usual terms of science, electricity, but chemical energy in a form peculiarly available for the production of an electric current. The name *accumulators* is also used.

Storage cells are discussed further in § 336.

335. Discussion of Exercise 41.—Any galvanic cell in which the active parts are copper, sulphate of copper, zinc, and sulphuric acid or sulphate of zinc, is called a Daniell cell.

The chemical action in the Daniell cell is somewhat more complicated than that in the single-fluid cell of Exercise 40, but if we consider only the *results* of the reactions they are not difficult to understand.

Within the porous cup we have, as in Exercise 40, zinc replacing the hydrogen of sulphuric acid. The hydrogen thus freed does not appear in the form of bubbles, but in the outer part of the cell hydrogen is replacing the copper of the sulphate of copper, and the copper thus freed is deposited upon the copper plate. It is not to be supposed that the identical atoms of hydrogen replaced by the zinc necessarily find their way to the outer cell, but the outcome of the reactions is represented by the following equation:

$$Zn + H_2SO_4 + CuSO_4 = ZnSO_4 + H_2SO_4 + Cu.$$

It will be seen from this account that the polarization which is discussed in § 334 cannot occur with the two-fluid cell of Exercise 41, as no gas is deposited on either plate, and each plate, though changing in weight, remains unchanged in character.

Various injurious actions are likely to take place in a Daniell cell when it is left for any long time, overnight for instance, in open circuit. Even when the plates are taken out, the porous cup may be spoiled by the deposition of copper upon it, owing to the presence of metallic particles in its walls. The cell should therefore be taken apart when not in use.

336. Other Galvanic Cells.—Before the development of the storage cell there were several other types of galvanic cell, besides the Daniell, in common use. Most of these are now seldom seen and therefore need not be here described. Those which are still much used, for ringing household bells or igniting the charges of gas engines, are, as a rule, of the general type invented by Leclanché and called by his name.

The Leclanché Cell.—This uses zinc in a solution of ammonium chloride, or "sal ammoniac," NH_4Cl, for the *anode* and for the cathode a bar of carbon packed in powdered carbon mixed with the peroxide (black oxide) of manganese. The primary chemical action of this cell is indicated by the following equation:

$$Zn + 2NH_4Cl = \underset{\text{Zinc chloride}}{ZnCl_2} + \underset{\text{Ammonia gas}}{2NH_3} + H_2.$$

The ammonia gas passes off, or may do so, into the air. The hydrogen, if the cell is worked hard without a break more than a few seconds at a time, polarizes the carbon cathode and rapidly reduces the strength of the current. When the circuit is broken this hydrogen is absorbed by chemical combination with the oxygen of the manganese

compound, forming water. Cells of this type are inexpensive and convenient, as their setting up when fresh or renewal when exhausted requires merely, if the solid parts are in good condition, the addition of an easily-made strong solution of a cheap and familiar substance, sal-ammoniac. See Exercise 46.

So-called *dry-cells* have nearly the same active parts as ordinary Leclanché cells, but contain solids, plaster of Paris, starch, etc., which, mixed with the solution, prevent it from being easily spilled.

Storage Cells.—These usually have for the positive electrode (see § 332) plates, or "grids," of lead loaded with oxides of lead, and for the negative electrode similar plates with less oxide of lead. These plates are placed in sulphuric acid of a certain strength, contained usually in a glass or hard-rubber jar.

In the "charging" of such a cell a current is sent in from some dynamo (see Chapter XXXII) by way of the positive electrode and out by the negative electrode. Oxygen is thus sent to the positive plates, adding to the oxides which are already there, and meanwhile hydrogen, going to the negative plates, unites with and takes away some or all of the oxygen there. The original difference between the positive and the negative plates is thus increased. When the charging is nearly complete, bubbles of hydrogen, no longer absorbed by the oxygen at the negative plates, rise in large quantities from those plates to the surface of the liquid.

The method of arrangement of the plates in a storage cell makes them equivalent to two very large plates, one positive and one negative, very close together (see Fig. 239, which shows the top view of a cell). This makes the so-called *resistance* (see Chapter XXXI) of the cell very small, and it is for this reason, mainly, that a storage cell, newly

charged, can give a much more powerful current than a Leclanché cell, for example.

After a time of use the storage cell becomes reduced in "charge," and the current from it speedily grows less. The cell is said to be "run down," and must be charged again.

The positive plates of a storage cell can usually be distinguished from the negative by their brownish color, due to the heavy oxides with which they are more or less loaded,

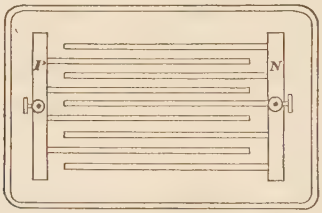


Fig. 239.—Illustrating the storage cell: P, the positive pole; N, the negative pole.

even when the cell is run down. Another means of distinguishing between the positive and negative plates is the fact that, usually, the positive group contains one less plate than the negative group, as in Fig. 239.

Storage cells are called also *secondary* cells, because they have to be charged from other cells or from a dynamo before use. Cells like those described in the first part of this article, which from the start yield a current from their own action, are called in distinction *primary* cells.

CHAPTER XXX.

CURRENT-STRENGTH; ELECTROMOTIVE FORCE.

337. Current Strength.—*The ordinary unit of current strength is called the* AMPERE, from Ampère (1775-1836), the great French investigator of electrodynamic laws.

We have already seen that an electric current is accompanied by chemical change in passing through a liquid containing a chemical compound. We have seen, also, that such a current affects a magnetic needle. Each of these properties can be made, and is made, the means of measuring what is called the *strength* of the current.

338. Chemical Measurement of Current: Voltameter, or Coulometer.—The "international ampere" is defined[1] as a current of such strength as, when passed through a solution of nitrate of silver in water, in accordance with certain specifications, deposits silver on the cathode at the rate of 0.001118 gm. per second.

The same current, passing through a solution of sulphate of copper in water, would deposit copper on a cathode at the rate of 0.000329 gm. per second, nearly.

A current depositing $n \times 0.001118$ gm. of silver per second from a nitrate of silver solution, or $n \times 0.000329$ gm. of copper per second from a sulphate of copper solution, would be called a current of n amperes. In short, a current is regarded as proportional to the amount of chemical change it causes per second in a given cell.

Many other chemical actions beside those just mentioned might be used in defining the ampere.

[1] *Report of the International Conference on Electrical Units and Standards*, 1908.

A cell used for the purpose of measuring an electric current by the amount of its chemical action is called a *voltameter*[1] or a *coulometer*. It may or it may not furnish the motive power of the current which it measures.

If a dozen coulometers were introduced as links of the same electric circuit, with conductors of any sort between them, they would all indicate the same strength of current, provided there were no leakage of current from the circuit. It is upon such facts as this that the description of an electric current as a continuous stream, having everywhere the same strength, is based. (See § 331.)

A common form of coulometer consists of two plates of copper in a solution of sulphate of copper. Such a cell does not *maintain* the current. It merely transmits the current which is driven by some other means.

The anode of this coulometer loses weight and the cathode gains weight, and one might expect the loss of one to balance the gain of the other; but the chemical reaction at the anode is not a mere dissolving of copper. Other changes take place there and so complicate matters that the loss of weight is not a reliable measure of the current. The gain of weight of the cathode is the quantity depended upon.

Another well known coulometer consists of two strips of platinum immersed in water containing a very little sulphuric acid. The acid in some way assists the passage of the current, but the visible chemical change is the decomposition of the water, H_2O, which is resolved into hydrogen, collected in a test-tube over the cathode, and oxygen collected in a similiar tube over the anode. See Fig. 240.

[1] This name is not a good one, as it has no *natural* meaning and is likely to be confused with *voltmeter* (§ 345). Professor T. W. Richards has suggested the name *coulometer* (coulomb-meter), the *coulomb* being the unit quantity of electricity, the amount transmitted by one ampere in one second.

Decompose acidulated water by means of the current from a battery of three or more Daniell cells or two storage cells arranged as in Fig. 240, collecting the gases in tubes above the electrodes.

Observe the relative bulk of the two gases, and show the combustible character of the hydrogen by igniting it, holding the test tube, *filled* with the gas, mouth downward. Before the flame is applied it is well to wrap the tube in a cloth to guard against danger from a possible explosion.

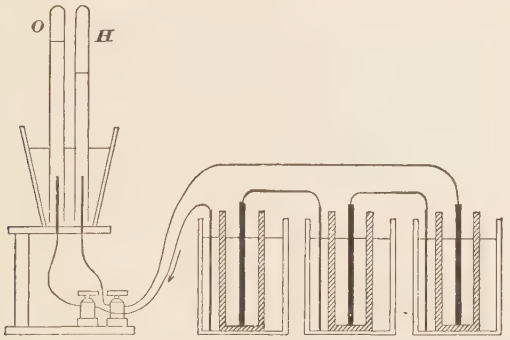


Fig. 240.—Electrical decomposition of water; hydrogen coulometer.

The length of time required for the use of a coulometer makes any such instrument unsuitable for determining the strength of a variable current at any particular instant; but in many cases, in the commercial use of electricity, the quantity to be arrived at is not the strength of the current at any one moment, but the total amount of electricity that passes in a given time—a day or a week, for example. For this purpose coulometers are particularly well suited and have been much used.

A coulometer, properly used, is a very sensitive and satisfactory means of measuring a current of unchanging strength lasting for a considerable time, and coulometers are very often used in "standardizing" other instruments, called galvanometers, which, after being standardized, are

more convenient than coulometers for general purposes of current measurement.

The proper use, however, of a coulometer is a somewhat delicate art, and one should consult some book on electrical measurements before attempting accurate work with this instrument.

Problems.

(1) How much copper can be deposited from sulphate of copper in 1 hour by a current of 5 amperes?

(2) How strong must a current be to deposit 10 gm. of copper from copper sulphate in 3 hours?

(3) In electroplating with copper a deposit of excellent quality is obtained by using a current of 1 ampere for 50 sq. cm. of surface upon which the deposit occurs. If the density of copper is 8.9 gm. per cu. cm., how long must such a current run to make a deposit 0.01 cm. thick?

339. Magnetic Measurement of Current: Galvanometer.— It is customary in the study of pure physics to define the strength of an electric current by reference to the force which a certain length of the conductor transmitting it exerts upon a magnetic pole of a certain strength at a certain distance. On this basis currents are measured by means of the galvanometer, an instrument similar to the galvanoscope of Exercises 40 and 41, but more carefully made.

The principle of such an instrument is that the current flowing in its coils tends to make the magnetic needle point in one way while the magnetic power of the earth tends to make it point in a different way. The stronger the current, other things being equal, the greater will be the deflection of the needle from the line of magnetic north and south (see § 309).

An extended discussion of the measurement of electric currents by their magnetic action is beyond the scope of this book, and it seems best for our purpose to define the

unit strength of current by reference to chemical action, as we have already done in the preceding Article. We may therefore regard the coulometer as the fundamental instrument, by means of which a galvanometer can be tested and "standardized," that is, so studied that from its reading the strength of a current passing through it can be found. *By definition* (§ 338) the strength of an electric current is proportional to the amount of chemical change it produces per second in a coulometer. *By experiment* it is found that the *magnetic force of a current is, other things being equal, proportional to the strength of the current as measured by the coulometer;* this does not imply that the *deflection of the needle* of a galvanometer is proportional to the strength of the current. But see § 343.

We have, in the data obtained from Exercise 41, the means of calculating roughly the strength of current which, in the magnetic field of the laboratory, will produce a particular deflection of the needle of our galvanoscope when the 5-turn section is in use. By means of similar data, obtained with currents of different strength, it would be possible to find the significance, in amperes, of any deflection within a considerable range; but this instrument is hardly worthy of so much study.

An accurate galvanometer must have a well constructed scale of some sort, and its magnetic needle is usually suspended by a very slender fibre liable to troublesome torsion and disastrous breaking, or poised upon a very delicate point which is almost sure to be ruined in the hands of a novice.

The object of Exercise 42 is to give the student some further experience with the magnetic action of electric currents and, in particular, to make him familiar with the general character and course of the lines of magnetic force due to a current running through a galvanometer coil.

340. Lines of Magnetic Force around a Straight Current.—

The lines of magnetic force due to a current in a long straight wire are circles, each in a plane at right angles with the wire, which passes through the centre of each circle.

The direction of the magnetic force along each line depends upon the direction of the current in the wire, and is in all cases such that, *to a person looking along the wire, in the direction of flow of the current, the magnetic needle would lead around each circle in a clockwise direction.*

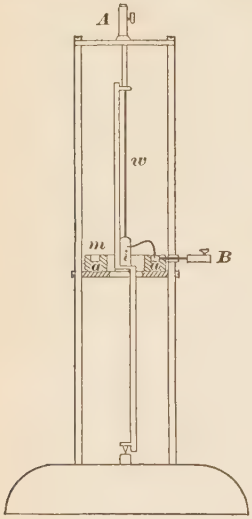


Fig. 241.—Rotation of a magnetic pole around an electric current.

The diagram of Exercise 42 should prove to be consistent with this statement, although the closed curves found in that Exercise are not circles.

The direction in which the north-seeking end of a very short needle *points* is that in which a detached north-seeking *pole* would *move*, if it were free. The following well known experiment makes use of a device for causing one pole of a magnet to rotate around the current while the other pole does not:

Send the current first one way and then the other through the rotation apparatus shown in Fig. 241, and compare the direction of motion of the affected pole, in each case, with the rule given earlier in this Article.

341. The Earth's "Directive Force"; Astatic Galvanometers.—It is evident that in the ordinary use of a galvanometer, with its windings placed north and south, the current in the coil tends to make the magnet at its centre point east or west, while the horizontal force of the earth's magnetism, called the earth's "*directive force*," tends to

make it point north. The position the needle takes is the result of a compromise between these two partly opposing influences. If the earth's directive force could be weakened, other things remaining unchanged, the deflection of the needle would be increased.

In fact, the earth's directive force, under which title is commonly included not merely the magnetic force of the earth proper, but also that of any magnetic bodies, such as iron gas pipes or steam pipes, that may be near the galvanometer, may be very different at different parts of the same room. This fact must be taken into account in careful galvanometric work.

For many kinds of work (see Exercises 43 and 44) a far more sensitive galvanometer is needed than the instrument thus far employed in this course. So-called "*astatic*," that is, *unstable, sensitive,* galvanometers make use of a device which practically reduces the earth's directive force almost or quite to zero.

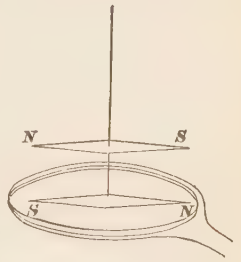


FIG. 242. -Illustrating an "astatic" galvanometer.

This device consists of two magnetic needles of nearly equal power, placed parallel to each other and so connected that they must turn as one, but pointing, with like poles, in opposite directions. These magnets are usually placed one above the other, one being inside the coil of the galvanometer and the other outside, as in Fig. 242.

Such a pair of magnets, if they are perfectly equivalent to each other, will have no more tendency, as a whole, to point north and south than to point east and west. It will point in any horizontal direction under the influence of a very slight net force.

In fact, one of the magnets is usually a little stronger than

the other, so that the pair points naturally in some particular direction, but much less decidedly than one needle alone would.

When a current goes through the galvanometer coil, this current has its full effect upon the magnet inclosed by it. The effect upon the outside magnet is less, but by recalling the observations of Exercise 42, and remembering that the two magnets point with like poles in opposite directions, one can see that the action upon the outside magnet works *with*, not in opposition to, the action upon the inside magnet.

The windings of the astatic galvanometer coil are usually very many and very near the inner magnet, for the purpose of increasing the sensitiveness. If the poles of an ordinary galvanic cell were applied directly to the ends of such a galvanometer coil, without any introduced *resistance* (§ 347), the current through the coil would probably be powerful enough to change the magnetism of the inner needle, thus destroying the equilibrium of the two needles and greatly reducing the sensitiveness of the instrument. Care must be taken to prevent accidents of this kind.

342. D'Arsonval Galvanometer.—We have seen that a current flowing through a coil may deflect a magnetic needle in its neighborhood. It is also true, as we shall see later (§ 359) that the coil itself through which a current is flowing may be deflected, if it lies in a magnetic field and is free to turn. The d'Arsonval galvanometer makes use of this fact. It consists of a coil of fine wire suspended within a strong magnetic field, between the poles of a permanent magnet, in such a way that a feeble current through the coil may produce a sensible deflection.[1]

[1] Attached in some way to the coil and turning with it is a small mirror, from which a narrow beam of light, coming from a lamp or any suitable source, may be reflected to a horizontal scale placed several feet distant from the instrument. A slight turning of the mirror swings the beam of light a comparatively great distance along the scale. See Fig. 18a (p. 578.)

One advantage of this instrument is that ordinary magnetic influences coming from without do not disturb the suspended coil.

343. Ammeters.—For ordinary daily use in commercial or industrial work such a galvanometer as that discussed in § 339, the readings of which are not proportional to the currents through it, is too inconvenient. Accordingly, instruments have been invented in which the readings are proportional to the currents. These are called *ammeters*, a contraction of *amperemeters*. Sometimes they are "direct reading," giving at once the current in amperes; sometimes the readings must be multiplied by some factor, *e.g.*, 0.5 or 0.1, in order to give the number of amperes.

ELECTROMOTIVE FORCE.

344. Electromotive Force of a Galvanic Cell or "Battery."—The power which a galvanic cell has of charging one of its terminals positively and the other negatively (§ 330), so that a current of electricity will flow from one to the other when they are connected by a wire, is called its *electromotive force.*

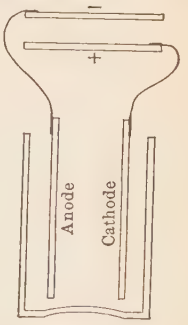


Fig. 243.—Illustrating the electromotoive force of a cell.

The electromotive force, which we shall now call the e. m. f., of a cell or "battery" of cells in *open circuit* can be measured by the difference of potential (§ 324) which it can produce between two metal plates connected with its positive and negative poles respectively. See Fig. 243.

The e. m. f. of a cell or battery in *closed circuit* can be measured by the strength of the current which it can maintain in a circuit of given *resistance* (§ 347).

Owing to polarization (§ 334), and perhaps to some other

causes, the e. m. f. of a battery in closed circuit is usually somewhat less than its e. m. f. in open circuit.

The e. m. f. of a cell depends upon its chemical composition and, to some extent, upon its physical condition. The first of the following experiments will tend to show whether it depends upon the size of the cell:

(1) Take one very small Daniell cell and one large one (see Fig. 244). Join together the copper terminals of the two cells. Then connect the zinc terminal of one cell with one terminal of the lecture-table galvonometer, and the zinc terminal of the other cell with another terminal of the galvanometer, thus forming a circuit consisting of the two cells,

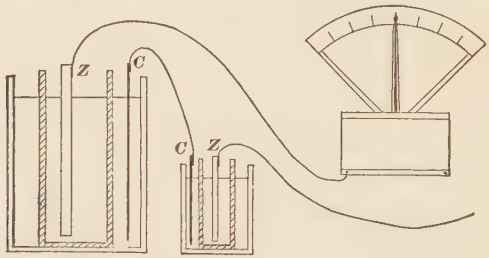


FIG. 244.—Electromotive force and size of cell.

opposed to each other, and the galvanometer-coil windings. Is there now evidence of any current through the galvanometer? What does the experiment indicate as to the comparative electromotive force of the two cells?

The following experiments will give some idea of the electromotive force of various combinations of galvanic cells:

(2) Take three similar Daniell cells. Connect the copper of one with the zinc of a second, and the copper of the second with the copper of a third, as in Fig. 245. Connect the zinc of the first cell and the zinc of the third with the terminals of the galvanometer. With this arrangement the first and second cells, *connected in series*, as the phrase is, and so working in the same direction, are opposed by the third cell. Do they prevail over it?

(3) Join the zinc of one cell to the zinc of a second, as in Fig. 246, and the copper of the first cell to the copper of the second. Connect the two coppers thus joined with the copper of a third cell. Connect the zinc of this third cell with one of the terminals of the galvanometer, and connect the joined zincs of the first two cells with the other terminal of the galvanometer. The first and second cells are now working together, but side by side, rather than, as before, end to end. They are now said to be joined in *parallel*, or *in multiple arc*, or *abreast*. They are opposed by the third cell. Do they prevail over it?

Try four or five cells joined abreast against a single cell, and note the result.

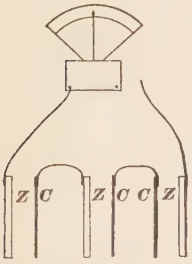


FIG. 245.—Two like cells in "series" against a like cell.

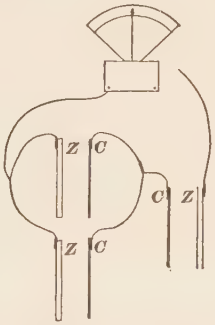


FIG. 246.—Two like cells in "multiple" against a like cell.

The preceding experiments point to several conclusions which more careful experiments would justify:

1. *The e. m. f. of a cell does not depend upon its size.*

2. *The e. m. f. of n similar cells in series is n times as great as that of one cell alone.*

3. *The e. m. f. of n similar cells in multiple is equal to that of one of the cells alone.*

345. Unit of E. M. F.: the Volt.—*The ordinary unit of electromotive force is called the* VOLT, in honor of Volta (§330). The electromotive force of an ordinary Daniell cell is about 1.1 volts.

Instruments now very commonly used to measure e. m. f. are called *voltmeters*.

The way in which voltmeters and ammeters (§ 343) are used together in electric measurements will be illustrated in Exercise 50, an *Efficiency Test of an Electric Motor*.

346. Other Sources of Electromotive Force.—The galvanic cell is not the only seat, or source, of electromotive force.

Thermoelectromotive Force.—If any two wires or bars of different material are soldered together at their ends, so as to make a continuous circuit, an electric current usually flows around this circuit so long as a difference of temperature is maintained between the two junctions. Such a

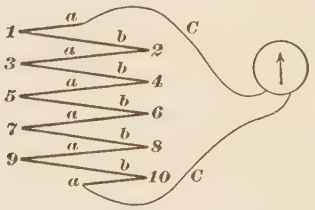


Fig. 247.—Illustrating a thermoelectric "pile."

current is called *thermo*electric, because it is kept up at the expense of heat energy. It is usually very small, but it can be shown by cutting one of the wires and connecting the two ends, kept at the same temperature, with the coil of a sensitive galvanometer.

A pair of wires used in this way is called a thermoelectric *couple*. The power of a couple depends very much on the nature and condition of the two metals composing it. Antimony and bismuth couples are relatively powerful and are much used in so-called *thermopiles*, consisting of several couples joined in series, or *piled*, as in Fig. 247, which repre-

sents five *a-b* couples so connected. When all the odd-numbered junctions are heated and the even-numbered ones cooled, or *vice versa*, the electromotive force is five times as great as if the junction of one couple only were so treated.

Little or no heavy electrical work is done by means of thermoelectric appliances, but they are much used for the detection of very small differences of temperature, such as occur in the study of radiant energy (Chapter XXVI), and for certain instruments called *pyrometers*, which are designed for the measurement of very high temperatures in metallurgical or other scientific operations.

Induced electromotive force, the power of dynamos, will be discussed later. See § 363.

CHAPTER XXXI.

OHM'S LAW; RESISTANCE.

347. Ohm's Law.—Ohm, a German (1787-1854), showed that, other things remaining equal, the current through a given conductor is proportional to the electromotive force which is applied to the conductor. In other words, if E stands for the electromotive force driving a current I through a given conductor, *the ratio $E \div I$ is a constant quantity for that conductor, so long as its physical condition remains unchanged.*

This ratio was taken by Ohm, and has been taken generally since his time, as the numerical measure of the conductor's *resistance.*

The numerical relations of the three quantities E, I, and R, are expressed by the equation

$$R = \frac{E}{I},$$

which, with its other forms,

$$I = \frac{E}{R} \text{ and } E = R \times I,$$

is called *Ohm's Law.*

348. Unit of Resistance: the Ohm.—That resistance through which unit e. m. f., the volt, will maintain unit current, the ampere, is called unit resistance and is named the *ohm.* It is very nearly equal to the resistance, at 0° C, of a column of pure mercury 1 sq. mm. in area of cross-section and 106.3 cm. long.

Problems.

(1) If a battery having an e. m. f. of 5 volts is placed in a circuit the total resistance of which is 100 ohms, how great is the strength of the current?

(2) How great an e. m. f. is required to maintain a current of 10 amperes through a resistance of 5 ohms? (These are pretty nearly the requirements of an ordinary arc-light.)

(3) How great is the resistance of a 16-candle "incandescent" electric lamp, if an e. m. f. of 100 volts maintains through it a current of 0.6 ampere?

349. Resistance of a Conductor; Specific Resistance.— The resistance of a conductor depends upon its material, its dimensions, and, in most cases, upon its temperature.

Other things being equal, the resistance of a conductor is proportional to its length and inversely proportional to its area of cross-section. That is, if several conductors of various sizes and shapes, but made of the same material in the same state of purity, hardness, etc., are tested, it is found that the quantity $(R \times S) \div L$, where R is the resistance of the conductor, L its length, and S its area of cross-section, is the same for all. This quantity, for which we shall use the letter r, has different values for different materials. Its value for any one material is called the *specific resistance* of that material.

When we write $r = (R \times S) \div L$, we see that $r = R$ when S and L are each equal to 1. That is, the specific resistance of a material is equal to the resistance of a *centimeter cube*[1] of the material. A measurement of r for copper is undertaken in Exercise 43.

Measurement of the rate of change of the resistance of copper wire with rise of temperature is undertaken in Exercise 44.

[1] It is not safe to say *cubic centimeter* here; for a cubic centimeter may be in the form of a long, thin wire or a short, wide bar, etc., and the resistance is different in the different shapes.

Problems.

(1) If the resistance of 45 m. of copper wire 1 sq. mm. in cross-section is 1 ohm, what is the resistance of 90 m. of like wire 4 sq. mm. in cross-section? How great is the specific resistance of the material?

(2) If the resistance of 2.5 m. of German-silver wire 3 sq. mm. in cross-section is 0.4 ohm, what is the resistance of 10 m. of like wire 0.5 sq. mm. in cross-section? How great is the specific resistance

(3) If the resistance of 10 m. of wire 1 mm. in diameter is 3 ohms what is the resistance of 25 m. of like wire 2 mm. in diameter? How great is the specific resistance of the material?

(4) How long must a piece of wire of the same quality as that of the preceding problem be, in order that its resistance may be 15 ohms, the diameter being 0.15 cm.?

350. Resistance of Conductors in Combination; Conductance.—When conductors are joined *in series*, that is, end to end, or *in tandem*, so that the whole current goes through each one of them, the resistance of the whole combination is the sum of the separate resistances.

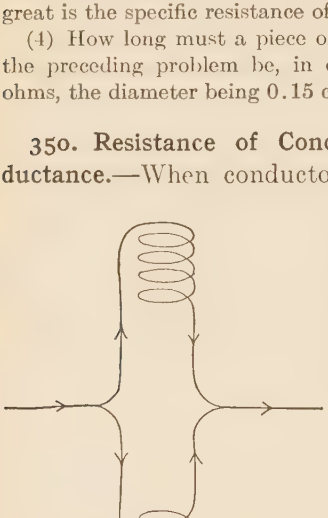


FIG. 248.—Resistances in "multiple," or "parallel."

When two perfectly similar conductors, each of resistance R, are joined together *in parallel*, or *in multiple*, that is, in such a way that the current can divide, as in Fig. 248, half going through each one, their resistance is $R \div 2$, the same as if they were really one conductor having twice the cross-section of either alone.

If n similar conductors are joined in parallel, the resistance of each being R, the resistance of the whole combination is $R \div n$.

Problems.

(1) If the resistance of a certain wire is 12 ohms, what would be the resistance of three such wires joined in *series?* joined in *parallel?*

(2) If the resistance of an arc lamp is 5 ohms, what would be the resistance of six such lamps joined in *series?* joined in multiple?

In discussing the resistance of a multiple combination in which the individual conductors have *unequal* resistances we have a more difficult matter, which may be dealt with by means of illustrative problems.

(3) If two wires, one having a resistance of 20 ohms, the other a resistance of 5 ohms, are joined in parallel, what is the resistance of the combination?

Solution: The 5-ohm wire is equivalent to four 20-ohm wires joined in parallel. The whole combination is therefore equivalent to five 20-ohm wires joined in parallel. Its resistance is, therefore, $20 \div (1+4) = 4$ ohms.

Observe, for use below, that this is the same as $1 \div (\frac{1}{20} + \frac{1}{5})$.

(4) What is the combined resistance of two wires, in parallel, of 17 ohms and 4 ohms respectively?

Solution: The 4-ohm wire is equivalent to seventeen 68-ohm pieces in multiple; the 17-ohm wire is equivalent to four 68-ohm pieces in multiple; the two in multiple are equivalent to twenty-one 68-ohm pieces in multiple; the two in multiple have, therefore, a resistance of $\frac{68}{21}$ ohms $= 3.24$ ohms.

Observe that this is the same as $1 \div (\frac{1}{17} + \frac{1}{4})$ ohms.

In each of these cases, R_1 and R_2 being the single resistances and R the multiple resistance, we have

$$R = 1 \div \left(\frac{1}{R_1} + \frac{1}{R_2}\right), \text{ or } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

With n conductors in multiple,

$$R = 1 \div \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_n}\right), \text{ or } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \frac{1}{R_n}.$$

The "reciprocal of the resistance," $1 \div R$, is called the *conductance*, or conducting power. If we write k for conductance, our rule for multiple becomes

$$k = k_1 + k_2 + k_3 + \ldots k_n.$$

That is, the *conductance* of any number of conductors in multiple is the sum of their separate conductances.

The *resistance* of any number of conductors in multiple is found by dividing 1 by their multiple conductance.

(5) What is the resistance of two conductors in parallel, the separate resistances of which are 4 ohms and 12 ohms? 6 ohms and 11 ohms?

(6) What is the resistance in multiple of four wires, the separate resistances of which are 4, 6, 8, and 10 ohms?

351. Resistance Coils.—It is frequently necessary in electrical work to introduce resistance into a circuit, and so-called *resistance coils* are part of the ordinary equipment of a physical laboratory. They consist of spools upon which are wound pieces of silk-covered or cotton-covered wire, long or short, thick or thin, according to the particular service each coil has to perform. To avoid magnetic action from these coils the wire is wound upon them in such a way that half the turns carry a current in one direction and half in the other direction. This can be easily done if the wire is doubled before it is wound on the spool.

Much attention has been given to the choice or invention of materials for resistance coils. The qualities most needed are constancy of resistance (through changes of temperature, lapse of time, etc.), absence of troublesome thermo-electric[1] qualities, high specific resistance (§ 349), and convenience of working.

Certain alloys of two or more metals are found to have very great specific resistance and to change in resistance far less with change of temperature than most pure metals. Accordingly, alloys are generally used in the construction of resistance coils. German silver, the specific resistance of which is about thirteen times as great as that of copper

[1] A beam of sunlight falling upon a junction of dissimilar metals may utterly derange delicate electrical experiments.

and may increase as little as 1 per cent. with a rise of 20° C., has been very commonly employed. But another alloy, *manganin*, composed of copper, manganese and nickel, is now preferred to German silver for the best work. The resistance of manganin changes far less than that of German silver with change of temperature.

A set of resistance coils is usually arranged in a wooden case with some convenient means for putting any or all of them into or out of the circuit. A set of resistances so encased is called a "resistance box."

352. Theory of Wheatstone's[1] Bridge.—Accurate measurements of resistance are made by means of a device called *Wheatstone's Bridge*. It requires the use of a sensitive

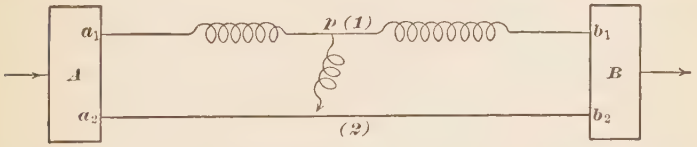


FIG. 249.—Illustrating the Wheatstone "bridge."

galvanometer (§§ 341 and 342) and one or more conductors of known resistance. Such conductors are usually in the form of resistance coils of some convenient number of ohms.

The theory underlying the use of the bridge is indicated in the following discussion:

If A and B (Fig. 249) are two thick pieces of metal connected by a thin wire, $a_1\ b_1$, of comparatively high resistance, and a thick wire, $a_2\ b_2$, of lower resistance, an electric current entering at A will divide on its way to B and the greater part of it will go through $a_2\ b_2$, the smaller part through $a_1\ b_1$. The resistance of A being very small,

[1] Wheatstone, though he was not and did not claim to be, the inventor of this method, was the one to make it generally known.

the two points a_1 and a_2 are of the same potential, very nearly, there being practically no tendency to flow from either of these points to the other. The same is true of the points b_1 and b_2 on the block B. Accordingly the fall of potential along a_1 b_1, that is, the difference between the potential of a_1 and the potential of b_1, is equal to the fall of potential along a_2 b_2, that is, the difference between the potential of a_2 and the potential of b_2. In other words, if P_{a1} stands for the potential of a_1, P_{b1} for the potential of b_1, and P_{a2} stands for the potential of a_2, P_{b2} for the potential of b_2, we have $P_{a1}-P_{b1}=P_{a2}-P_{b2}=D$, let us say.

Now (see § 344), difference of potential is of the same nature as electromotive force, in the sense that the e. m. f. of a battery is proportional to, or is measured by, the difference of potential which the battery can maintain between its poles in open circuit. Accordingly, difference of potential is measurable in *volts* and Ohm's law (§ 347) holds not merely for a whole circuit of e. m. f. E, but also for any part $a\ b$ of a circuit in which part we have no *source*[1] of e. m. f., but merely a difference of potential, D, maintained by some e. m. f. outside this part of the circuit. That is, we can write for the current, I, in our part $a\ b$ having resistance R, $I=D\div R$.

In the case shown by Fig. 249 we have, since D is the same for both wires, $I_1R_1=I_2R_2$. If now we connect one end of a wire[2] to the point p on wire 1 and touch the other end to some point on q wire 2, it is fairly evident that we may get a current in either direction through our cross-wire, or *bridge* wire, according to the position of q. Evidently, too, we may be able to find such a point of contact q, that there will be no current through our cross, or bridge, wire.

[1] No battery action, thermoelectric action, etc. See § 346.

[2] This wire was originally called the *bridge*, a name now given to the main apparatus indicated in Fig. 249.

In this case the points p and q must be at the same potential. Let us now indicate

by D'_1 the fall of potential from a_1 to p, by D''_1 the fall from p to b_1,
" D'_2 " " " " " " a_2 " q, " D''_2 " " " " q " b_2,
" J " resistance from a_1 to p, by K the resistance " p " b_1,
" L " " " " a_2 " q, " M " " " " q " b_2.

In order that p and q may have the same potential we must have $D'_1 = D'_2$ and $D''_1 = D''_2$. But this is equivalent to $I_1 J = I_2 L$ and $I_1 K = I_2 M$.

Dividing the first of these last two equations by the second, we get $J \div K = L \div M$, or $J : K :: L : M$, as the necessary condition for having p and q at the same potential.

In the actual use of the Wheatstone bridge method of measuring a resistance we have a sensitive galvanometer in the part that we have called the *bridge*, the part that leads across from path 1 to path 2, and when, connecting some point p on (1) with some point q on (2), we get no current through this galvanometer, we know that we have found equipotential points.

If in this case we know all the resistances in the equation or proportion last given except one, J let us say, we can at once find the value of J by the equation $J = K(L \div M)$. Indeed, if we know the value of K and the value of the *ratio* $L \div M$, we can find J without knowing the value of L or M alone.

But L and M, being merely the resistances of the two parts of the wire 2, will, if this is a straight uniform wire, have the same ratio as the lengths of the parts $a_2\ q$ and $b_2\ q$ of this wire. Hence, if we call these lengths l and m respectively, we have

$$J = K \times (l \div m).$$

In the Wheatstone bridge, as we shall use it, the part corresponding to (2) of Fig. 249 is a uniform wire of German-silver, manganin, or some other highly resistant alloy, one

meter long, stretched over a meter rod, so that the measurement of l and m is readily made. Evidently, if either J or K is known, we can with help of the ratio $l \div m$ find the other. We shall, however, in Exercise 43 indicate by K a known resistance, one or more coils of a resistance box, while J will be the resistance to be found. The resistance of the wires used to connect the J coils and the K coils with the end blocks of the apparatus will be so small, compared with J and K, that they need not be considered in the calculations.

353 Temperature-Coefficient of Resistance.—The electrical resistance of most conductors changes with change of temperature. In metals, and most alloys of metals thus far tested, the resistance increases with rise of temperature. The exact relation between the two changes through a great range of temperature is probably very complicated, but for many purposes it is near enough to the truth to assume that the resistance increases *in proportion* to the increase of temperature. This relation can be put into mathematical form thus:

$$R = R_0 + \gamma R_0 t = R_0(1 + \gamma t),$$

where R_0 = the resistance of the conductor at 0° C.,
" R = " " " " " " t° C.,
" γ = a constant quantity, called the *temperature coefficient* of resistance. Solving for γ, we get

$$\gamma = (R - R_0) \div R_0 t.$$

Problems.

(1) If the resistance of a certain G.-s. wire is 15 ohms at 0° C., what is its resistance at 70° C., the temperature-coefficient being 0.00044?

(2) If the resistance of a certain silver wire is 20 ohms at 0° C., and 25.7 ohms at 75° C., what is the temperature-coefficient?

Evidently if we measure R_0 at 0° C., and R at a known temperature, t°, above zero, we can find γ at once. But

sometimes it is not convenient to measure the temperature at 0° C. We may then proceed as follows:

Measure the resistance, R_1, at some known temperature t_1;
" " " , R_2, " " " " t_2.

From the first measurement we get $R_1 = R_0(1+\gamma t_1)$;
" " second " " " $R_2 = R_0(1+\gamma t_2)$.

These two equations contain the two unknown quantities R_0 and γ. Eliminating R_0, we find γ. See Exercise 44.

354. Resistance of Cells in Combination; Resistance of the Whole Circuit; Further Use of Ohm's Law.—From what we have already learned concerning resistance, it is possible to make a shrewd guess as to the effect upon battery resistance of change in size or distance of the plates in a cell, or change from series to multiple arrangement of cells in a battery. Nevertheless, Exercise 45, in which these matters are touched upon, may not be unprofitable.

The fact is, that *the resistance of n similar cells in series is n times as great as that of one of the cells alone*, and *the resistance of n similar cells in multiple is equal to* $1 \div n$ *that of one cell alone.* See § 350.

If the resistance of a battery is called R_b, and that of the part of the circuit external to the battery is called R_e, Ohm's law (§ 347) takes the form

$$I = \frac{E}{R_e + R_b}$$

Observe that if R_e is very large in comparison with R_b—one hundred times as large, for instance—doubling or trebling R_b will make but little difference in the value of I. But if R_e is not large compared with R_b, doubling or trebling the latter will make a great difference in the value of I.

Recall the fact (§ 344) that combining cells in *series* makes the e. m. f., E, of the battery proportional to the

number of cells, while combining them in multiple leaves the e. m. f. of the battery equal to that of one cell.

Accordingly, when R_b is very small compared with R_e, the current I will be greatest when the cells are joined in series; for the advantage of maximum E more than offsets the disadvantage of maximum R_b in this case. See Exercise 45.

If R_b is large compared with R_e, it may be of advantage to combine the cells in multiple, sacrificing E to some extent, for the sake of reducing R_b. See Exercise 45.

When one has a given number of similar cells to be arranged at will for the purpose of sending the maximum current through a certain known external resistance, the proper rule to follow (of which no proof will here be given) is this: *Join the cells in such a way as to make the resistance of the battery equal, as nearly as may be, to the resistance of the external part of the circuit.*

This rule will sometimes require all the cells to be arranged in series, which will make the battery resistance a maximum, but it must not be supposed that this arrangement is adopted *for the sake* of making the battery resistance large. It is adopted for the sake of making the *electromotive force* large, and *in spite of* the fact that it increases the battery resistance.

To illustrate the working of the rule, let us suppose that we have to arrange 12 cells, each having an electromotive force of 1 volt and a resistance of 1 ohm, in such a way as to send the strongest current through an external resistance of 3 ohms. With the cells all in series we have

$$I = \frac{E}{R_e + R_b} = \frac{12}{3+12} = \frac{4}{5} \text{(ampere)};$$

with the cells 2 abreast and 6 in series,

$$I = \frac{6}{3+3} = 1 \text{ (ampere)};$$

with 3 abreast and 4 in series,

$$I = \frac{4}{3 + \frac{4}{3}} = \frac{12}{13}(\text{ampere});$$

with 4 abreast and 3 in series,

$$I = \frac{3}{3 + \frac{3}{4}} = \frac{4}{5}(\text{ampere});$$

with 6 abreast and 2 in series,

$$I = \frac{2}{3 + \frac{1}{3}} = \frac{3}{5}(\text{ampere});$$

with all the cells abreast,

$$I = \frac{1}{3 + \frac{1}{12}} = \frac{12}{37}(\text{ampere}).$$

We have in this discussion assumed the resistance of a cell to be a fixed quantity. The fact is, however, that the resistance of a cell, like its electromotive force, is somewhat variable, being dependent upon the strength of current and certain other conditions.

When, as sometimes happens, one part of a battery is set up in such a way as to oppose the other part (see experiment in § 344), *the effective e. m. f. of the battery is equal to the difference of the e. m. f.'s of the opposing parts.* The total resistance, however, is the same as if both parts were directed in the same way.

Problems.

(1) If the resistance of a single galvanic cell is 0.8 ohm, how great is the resistance of 4 such cells joined in "series"?

(2) What would be the resistance of 4 of these cells joined abreast, or in multiple arc?

(3) What would be the resistance of the same number of these cells joined in a square, two cells wide and two cells long?

(4) If each of these cells had an electromotive force of 1.8 volts, what would be the electromotive force of each of the combinations mentioned in the three preceding problems?

(5) If each combination in turn were used in a circuit having an

external (outside the battery) resistance of 100 ohms, how great would the current be in each case?

(6) How great would the current be in each case if the external resistance were 1 ohm?

(7) How great would the current be in each case if the external resistance were 0.1 ohm?

(8) If 6 cells, each having an e. m. f. of 2 volts and a resistance of 0.5 ohm, were joined in series in opposition to a series of 12 cells, each having an e. m. f. of 1.5 volts and a resistance of 1 ohm, the external resistance being 2 ohms, which set of cells would prevail, and how great would the current be?

(9) If an arc lamp has a resistance of 5 ohms and requires a current of 10 amperes, what e. m. f. does it require?

(10) If a sixteen-candle power incandescent electric lamp requires a current of 0.5 ampere and an e. m. f. of 100 volts, what is its resistance?

(11) If the e. m. f. available for an electric circuit is 2000 volts, and if the resistance of a single arc lamp is 5 ohms, how many such lamps placed in series would reduce the current to 10 amperes, a suitable strength for such lamps?

(12) If an incandescent lamp has a resistance of 150 ohms and requires a current of 0.5 amperes, how many cells connected in series, each having an e. m. f. of 1 volt and a resistance of 0.5 ohm, would be required to operate it?

(13) If an arc lamp having a resistance of 5 ohms requires a current of 10 amperes, would 1000 cells like those of the preceding problem, arranged 100 in series and 10 abreast, be more or less than sufficient to operate it?

CHAPTER XXXII.

ELECTROMAGNETISM AND INDUCED CURRENTS.

ELECTROMAGNETISM.

355. Discovery: Electromagnets.—Oersted of Copenhagen (1777–1851) discovered the directive action of the electric current upon the magnetic needle in 1820. In the same year Arago, a Frenchman (1786–1853), and Davy (§ 147) discovered that an electric current can produce magnetization in soft iron. These discoveries have been followed up by a multitude of experimenters, and now *electromagnets*, that is, masses of iron or steel made magnetic by the passage of an electric current through a wire coiled about them, are familiar objects to nearly everybody.

Connect the poles of a galvanic cell by means of a piece of uncovered copper wire and dip one part of this wire into a mass of iron filings. It was in this way that Arago made his discovery.

Show the magnetizing action of the current by passing it through a coil of wire surrounding a core of soft iron.

One of the most familiar uses of the electromagnet is found in the ordinary electric door-bell, or call-bell. This application is studied in Exercise 46.

356. Electromagnetic Telegraph.—The essentials of a telegraphic outfit are very simple.

The *battery* may be of one or more galvanic cells, according to the resistance of the circuit.

The *sounder* (Fig. 250) consists of an electromagnet m, a lever l, pivoted at p and carrying a piece of soft iron i, which iron, attracted by the electromagnet, is drawn downward, stretching the spring s, and bumping the end of the lever against the lower stop. It is this bumping,

against either its lower or its upper stop, which gives the familiar click of the instrument in operation and justifies its name.

Two clicks coming close together, one caused by closing, the other by opening, the electric circuit, are called a *dot;* two clicks about twice as far apart are called a *dash.* Perhaps it would be more accurate to say that the space

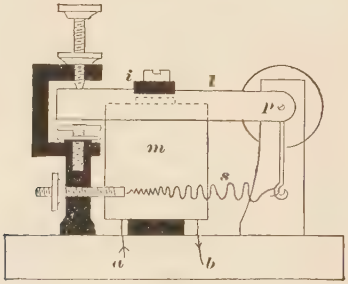


Fig. 250.—Telegraph sounder.

between the click of closing and the click of opening is the dot or the dash. Various groups of dots and dashes indicate the letters of the alphabet. In the following table a dot is indicated thus, -, and a dash thus, —; this table gives the international alphabet or "code:"

A- —	J - — — —	S - - -
B — - - -	K — - —	T —
C — - — -	L - — - -	U - - —
D — - -	M — —	V - - - —
E -	N — -	W - — —
F - - — -	O — — —	X — - - —
G — — -	P - — — -	Y — - — —
H - - - -	Q — — - —	Z — — - -
I - -	R - — -	

The first code of this kind was invented by Morse, an American (1791–1872), who was one of the foremost leaders in the development of telegraphy. The alphabet now used in this country, America, and called the Morse code, differs from that given above in the following cases:

C - - -	O - -	X - — - -
F - — -	P - - - - -	Y - - - -
J — - — -	Q - - — -	Z - - - -
L — —	R - - -	

The *key* (Fig. 251) consists of a lever pivoted at *p*, subject to a spring *c*, operated by the hand pressing down the knob *k* against the stop *m*, which operation closes the cir-

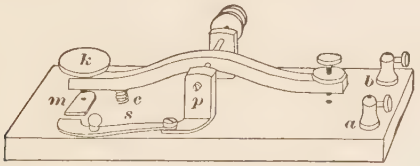


Fig. 251.—Telegraph key.

cuit; for *m* is connected permanently, by a wire not shown in the figure, with one of the binding posts *a* and *b*, while *p* is connected with the other, and by wires leading from these posts the instrument is connected with the rest of the circuit. The action of the spring makes it possible to work the lever up and down rapidly.

When not in use the key is *shunted*, that is, *side-tracked*, by a metal bar *s*, called a switch, connecting *m* and *p*.

The arrangement of a short telegraphic line is indicated in Fig. 252, where the left-end switch is open and the right-end switch is closed. The message now enters at the left, and at every make or break of the key contact there the sounder at the farther end of the line, as well as the one near the sending key, clicks.

On permanent lines the "return wire," the lower, long, horizontal line of the figure, is dispensed with, connection being made with the earth at each end of the line, as the dotted lines of the figure indicate. The earth-connection must be such as to reach moist soil, as dry soil is not a sufficiently good conductor of electricity to transmit the current to or from the wires readily.

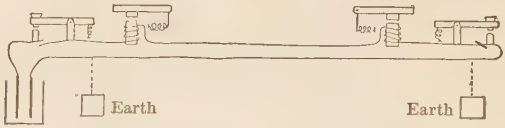


FIG. 252.—Illustrating a telegraph system.

When a line is very long the battery at one end may not be sufficient to work the sounder effectually at the other end. In such a case it is made to operate an electromagnet called a *relay*, slight pulsations of which bring into action another battery at the distant station, and the current from this second battery works the sounder.

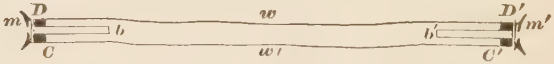


FIG. 253.—Illustrating an early form of telephone circuit.

Exercise 47 gives the student an opportunity to set up and use a very simple telegraphic circuit.

Wireless telegraphy, so-called, is considered later. See § 383.

357. The Telephone.—The necessary apparatus of a telephone line is as simple as that of a telegraph line, but its adjustment is far more delicate and difficult.

In Fig. 253, *m* is a mouthpiece behind which, held rather loosely, is a diaphragm, *D*, of thin soft iron. Beyond the

diaphragm is a steel magnet *b*, on the nearer end of which is a coil of fine wire, *C*. At the other end of the line are the counterparts *m'*, *D'*, etc.

Each of the diaphragms lies all the time in a magnetic field. Any movement of the diaphragm *D*, for instance, makes a slight change in the strength or distribution of the lines of magnetic force in and about the end of the magnet *b*. This change *induces* (§ 363) a current of electricity in the coil *C*, and this current, passing also through *C'*, produces a slight magnetic change in or about the end of *b'*, and so causes a movement of the iron diaphragm *D'*.

The chief new wonder revealed by the telephone was, that movements so slight as those of the second diaphragm

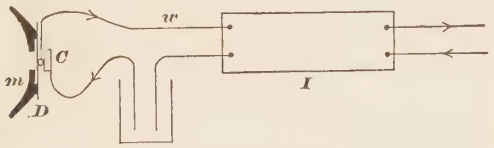


Fig. 254.—Illustrating further development of the transmitting end of a telephone circuit.

could produce audible sounds, and that these sounds could simulate and reproduce human speech. See foot-note on p. 273.

The arrangement of apparatus shown in Fig. 253 can be used for short distances only. It consists of two Bell *receivers*, which are used also as *transmitters*. For long lines the Blake *transmitter* was for a time much used in conjunction with a battery and an induction-coil (§ 377). This device does not require the voice to furnish the energy of the electric current, but uses it merely to control or vary the strength of a current coming from the battery.

The plan of the Blake transmitter is shown in Fig. 254, where *m* is the mouthpiece, *D* the metal diaphragm, be-

tween which and the soft carbon "button" C there is a bit of platinum suspended in metallic connection with the wire w. I is an induction coil. When the apparatus is not in use a small current of electricity runs from the battery through C to the platinum, which presses C lightly, then through w to the primary coil of I. The secondary of I is in connection, by means of two wires or one wire and the earth, with the distant telephone.

Speech directed against D varies many times a second the pressure of D against the platinum and of the platinum against C. This variation of pressure changes the resistance of the circuit, which resistance is largely at the point of contact with C, and so varies the strength of the current sent through the primary of the induction coil. Accordingly, induction-currents, corresponding to the vibrations of D, are sent to the receiving telephone, which reproduces in speech what is spoken into the transmitter.

An electric contact the resistance of which varies greatly with pressure is called a *microphone* contact, because it has been much used in devices for making very slight sounds audible. It was not original in the Blake transmitter, though it found there a most useful application. A more recent form of transmitter, which has for the most part displaced the Blake instrument, uses, in place of the metal diaphragm D, the platinum bead and the carbon button, granules of carbon between two carbon plates, one of which is agitated by the pulses of the voice.

358. Definition of Electric Motors.—An electric motor is a machine for producing mechanical energy or work from the energy or work of electric currents.

The possibility of such a machine is shown by the cases already described in this book of magnets moved by electric currents. In fact, every galvanometer is in some sense an electric motor; but in the ordinary sense of the term an

electric motor is a contrivance by means of which continuous rotation of a coil of wire carrying an electric current, or of a combination of magnets and coils, is maintained by current action.

The fact that this motion, which always accomplishes some mechanical work, requires the *expenditure* of electric energy, and tends to weaken the electric current that maintains it, will be discussed later (§ 365).

Certain forms of apparatus will presently be described which illustrate the development of electric motors from the stage of mere scientific interest to that of commercial importance.

359. An Elementary Motor.—Fig. 255 represents a coil of wire, C, mounted in such a way as to revolve freely between the poles, N and S, of a horseshoe magnet. The ends of the coil are soldered to plates of metal m and m', which are fastened to the upright axis a, but are insulated from it, that is, separated from it by some non-conductor of electricity. The arch at the top of the figure is of brass, and supports two binding-posts, p and p', which are insulated from it. From p and p' elastic strips of metal, s and s', reach to the plates m and m', respectively.

A current of electricity entering at p will pass by s to m, then through C to m', and out by way of s' and p'. The current used with this apparatus may be supplied by one or more Daniell cells or by a storage cell.

At the centre of a coil the line of the magnetic force due to a current in the coil itself is (see Exercise 42) at right angles with the plane of the windings, and so directed that, to an observer looking *forward* along it, the current circulates *clockwise* in the coil. This line of magnetic force will now be called the *axial magnetic line of the coil.*

The lines of magnetic force of the *magnet* are directed from N to S. Experiment shows that, under the conditions

illustrated by Fig. 255, the coil C will turn or tend to turn in such a way as to make the direction of its own axial magnetic line the same as the direction of the lines of the field in which the coil is placed. Of course, then, if the

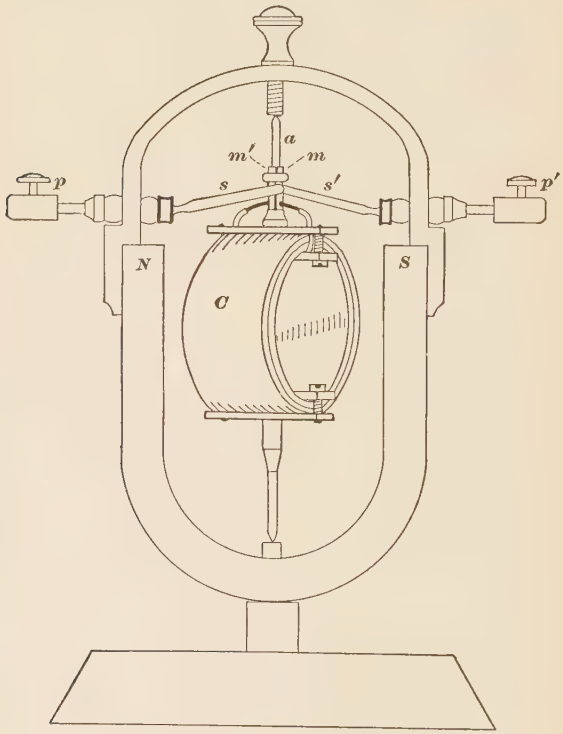


FIG. 255.—Illustrating an elementary electric motor.

coil is placed so as to make this agreement, it is at a *dead-point*, and will start in neither direction.

The coil can be made to rotate past its *dead-point* by its own momentum, and, if the contacts of the springs s and

s' against m and m' are so contrived as to send the current in through m and m' alternately, the changes taking place at the dead-points, the coil can be kept in continuous rotation in one direction.

360. Electromagnetic Fields and Armatures.—The apparatus just seen is an electric motor, with a *magnetic field* produced and maintained by a *permanent* magnet, and a revolving part, or "*armature,*" containing no iron. The direction of rotation of the armature in such a motor depends upon the direction of the electric current.

As electromagnets are more powerful than permanent magnets, it is an obvious improvement to use an electromagnet to furnish the field, and this is done in all large motors. It is less obvious, but equally true, that there is a great advantage in furnishing the armature with an iron *core* upon which the coil is wound, and this is usually done.

These changes give power, and they bring another advantage, namely, that the armature revolves always in the same direction without regard to the direction of the current. This fact can be shown by means of a simple form of apparatus like that of Fig. 256. In this apparatus the current entering at p divides at the point o, part going through the windings of the *field-magnet* to p', where it joins the other part, which has passed through the windings of the armature.

A conductor that leads off from the main circuit and returns to it farther on is called a *shunt,* and a motor like that of Fig. 256, in which the current is thus divided, is called a "shunt-wound" motor. Motors for business purposes are usually shunt-wound.

It is easy to modify the connections of Fig. 256 (see dotted lines) so as to make the whole current go through the windings of the armature and the windings of the field magnet. The motor is then said to be "series-wound."

The armature of the series-motor, like that of the shunt-motor, revolves in the same direction whether the current enters at p or at the other terminal, the fact being that reversing the current reverses the direction of the magnetic lines of the field and reverses the magnetic direction of the

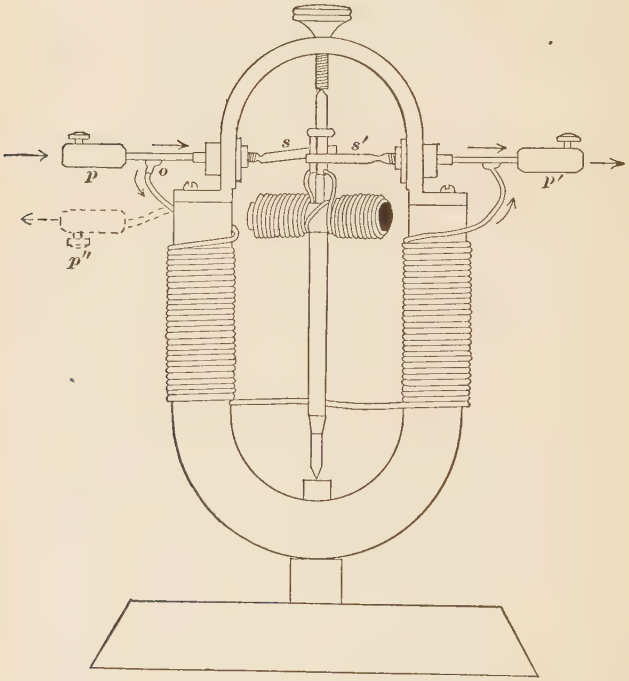


Fig. 256.—Electric motor: electromagnetic armature and field.

armature at the same time. If only one of these conditions were reversed, the direction of rotation would be reversed.

The fact that direction of rotation of the armature is, in motors with an electromagnetic field, independent of the direction of the driving current suggests that motors may

be driven by *alternating currents*, currents undergoing frequent and regular changes of direction. See § 369.

361. Ring Armatures and Drum Armatures.—The two plates m and m', on the axis of revolution in Fig. 255, by means of which, when the coil is in revolution, the current in the coil is reversed at every passage of a "dead-point," make what is called the *commutator* of the motor. Motors are usually made with a considerable number of coils, or "sections," set at different angles around the axis of revolution of the armature, and the commutator consists of a number of bars, one for each "section" of the windings, placed upon the axis.

Fig. 257 shows an armature with four sections, S_1, S_2, S_3, S_4, and four commutator-bars, or plates, b_1, b_2, b_3, b_4. The

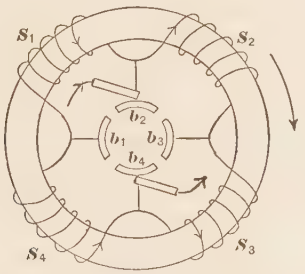


Fig. 257.—A ring armature, with commutator.

sections are wound on a ring of soft iron, and each section is connected directly by wires with two of the commutator-bars. Two conductors, called the "*brushes*," corresponding to the springs s and s' of Fig. 255, lead the current to the commutator, and take it away after it has traversed the sections of the armature. If the current is just now entering at the bar b_2, by way of the upper brush, it divides there, part going through sections S_1 and S_4, the other

through sections S_2 and S_3, and these parts, uniting at the bar b_4, where the second brush touches, go out as one current.

If now the magnetic lines of the field in which the armature revolves are horizontal and extend from left to right, it will be seen, from what is said in § 359, that all the sections tend to rotate in the same direction, that is, clockwise. But the only way the sections have of rotating is to turn the whole armature about its axis, and this they do.

The form of armature shown in Fig. 255 is called a *ring* armature. The sections are, one may say, wound on the *rim* of a wheel, the soft iron ring.

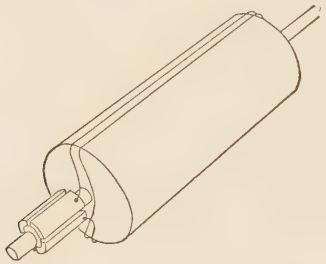


Fig. 258.—A drum armature.

Another form of armature has no ring, but rather a *hub* of soft iron upon which the sections are wound, as in Fig. 258. This form is called a *drum* armature. The principle of the two forms is the same. The ring armature has an important place in the history of electromagnetic machinery, and its details are more easily shown in a drawing than are those of the drum armature, but the latter is now much more generally used.

Exercise 48 gives an opportunity to study the parts as well as the working whole of a small motor.

INDUCED CURRENTS.

362. Number of Lines of Magnetic Force Through a Circuit.—We have heretofore used the term *line of magnetic force* to indicate merely a line along which a compass will lead in a magnetic field, and, as the number of possible positions of the compass in any given cross-section of the field is countless, the number of such lines of force in any magnetic field, or even any small portion of it, is countless.

If, however, one undertakes to make a diagram of a magnetic field, as in Exercises 39 and 42, drawing in a certain number of continuous lines of force, one will find that these lines lie closest together in those parts of the diagram where the strength of the magnetic field is greatest. In fact, it is very convenient, and thus very common, to speak of a magnetic field of unit strength (§ 315) as having one line of magnetic force per unit area, 1 sq. cm., of cross-section—to speak of a magnetic field of strength H as having H lines of magnetic force per unit area of cross-section, etc.

If, therefore, a loop of wire, inclosing an area of A sq. cm., is placed in a magnetic field of strength H, with its plane at *right angles* with the direction of the magnetic force, it is customary to say that the number of lines of magnetic force embraced by this coil is $A \times H$.

If the coil is placed with its plane *parallel* to the lines of force, it embraces none of the lines.

If the coil is so placed that its plane is neither at right angles with nor parallel to the lines of force, the number of lines embraced by it is something between 0 and $A \times H$.

Faraday invented this quantitative method of treating lines of magnetic force, and it is of very great convenience in summarizing the laws of induction of electric currents, which laws he discovered.

363. Induced Currents.—Faraday discovered in 1831–32 that, *When the number of lines of magnetic force embraced*

by a closed conducting circuit is increased or diminished, a current of electricity is set up in the circuit, which current is proportional to the rate at which the change is occurring, and lasts only so long as the change is in progress. A current produced in this way is called an INDUCED CURRENT, *and it is said to be due to an* INDUCED E. M. F. *The direction of the induced current depends upon the direction, increase or decrease, of the change in the number of lines of force embraced.*

If there is already a current flowing in the circuit, due to some other cause than induction, the induced current is combined with the other, causing sometimes an increase and sometimes a decrease of the total current.

(1) Connect the binding-posts p and p' of the apparatus shown in Fig. 255 with the terminals of a sensitive galvanometer. Then whirl the coil quickly about on its axis, first in one direction and then in the other, noting the effect upon the galvanometer.

Observe the direction of the induced current in each case, and compare it with the direction of the current which, if sent through the armature from a battery, would cause rotation of the armature in the same direction.

(2) Try similar experiments with the apparatus of Fig. 256.

364. Dynamos; Relation Between Dynamos and Motors.— We have now seen a number of machines which, driven by the power of an electric current, produce mechanical power. When used in this way they are called *electric motors* (§ 358).

The experiments of the preceding Article illustrate the fact that when such machines are driven by mechanical power they produce a current of electricity. When used in this way they are called *dynamos.*

The terms *field, armature, commutator, brushes, series-wound, shunt-wound,* etc., have the same meaning when applied to dynamos as when applied to motors. This fact, however, is to be noticed, that in the shunt motor the current from the outside line is divided between the field-mag-

net coils and the armature, while in the shunt dynamos the current from the armature is divided between the outside line and the windings of the field magnet.

365. Energy Absorbed in Motors and Dynamos.—When we see that a motor, driven by a current, yields mechanical energy, we may be sure that it absorbs electrical energy, since, on the whole, no machine can give out more energy than it takes in; but it is not at first obvious how the current going through a motor in operation is doing therein any more work than it would do if the motor were at rest.

The first experiments of § 363, however, illustrate the fact that when the armature of a motor is driven by *mechanical means*, a current is *induced* in it, opposite in direction to the current which would drive the armature in its present direction of revolution, the direction of the *field* meanwhile being unchanged. Accordingly, when a motor is driven in the ordinary way, by a current, the revolution *tends* to set up in the armature a current opposite in direction to that actually flowing through it. In other words, it is harder to maintain the given current in the revolving armature than it would be to maintain it in the same armature at rest. This opposition is analogous *in effect* to polarization in a battery (§ 334).

This opposing force is called *counter-electromotive force*, and it is in overcoming this curious opposition that electrical energy is turned into mechanical energy. All the rest of the electrical energy spent in the motor goes to *heat* its windings and its cores, and is worse than wasted.

In the best motors the counter-e. m. f. is nearly as great as the e. m. f. applied to the armature from without, so that the current which passes through the latter is only a small part of what would pass through if the armature were at rest, with the same difference of potential between its brushes. Indeed, most large motors are used in such a way

that, if the armature could be suddenly stopped, without reduction of the e. m. f. applied from the external line, the windings of the armature, if not especially protected, would be ruined almost instantly by the too powerful current which would flow through them. This damage is prevented by a device which automatically breaks the circuit through the armature when the speed falls below a certain limit. When the motor is to be started, a considerable resistance is introduced into the armature branch at first, and this is gradually diminished as the speed increases.

A further safety device, which is used with a great many kinds of electrical apparatus, is a short piece of easily melted wire, called a *fuse*, which, placed in series with the part to be protected, melts before this part suffers harm.

Law of Lenz.—The law, that *every induced current of electricity is in such a direction as to oppose the change, of position or state, which produces it*, is called, from its discoverer, the law of *Lenz*. It follows from the law of conservation of energy (§ 142); for if a certain change, motion of an armature for instance, caused a current tending to *maintain* that motion, we should have *perpetual motion*, that is, a continual supply of energy without corresponding expenditure of energy.

In the case of the dynamo, it is not at first obvious that the mechanical power which drives it is doing anything more than to overcome the resistance of mechanical friction. But, in fact, the mechanical power has to supply the energy of the electrical current also, as experiment shows.

A motor absorbs electrical energy and gives out a partial equivalent in mechanical work; not a full equivalent, for some energy is wasted in heating the apparatus.

A dynamo absorbs mechanical energy and gives out a partial equivalent in electrical work; not a full equivalent, for, here too, some energy is wasted in heating the apparatus.

Electricity has worked many wonders within the last century, but it has not created one foot-pound of energy. It is a carrier, not a producer.

Exercise 49 gives an opportunity to study the parts, as well as the working whole, of a small dynamo.

366. Further Account of Armature Action.—It may be inferred from the opening paragraph of § 363 that the current in any particular winding of a dynamo armature is different in direction at different parts of a single rotation of the armature in the magnetic field. This is true. In

Fig. 259.

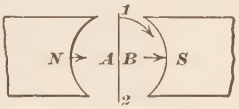


Fig. 260.

Fig. 261.

Fig. 262.

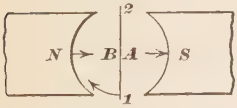


Fig. 263.

Fig. 264.

Figs. 259 to 264.—Illustrating various phases of the cycle of a one-turn armature.

Fig. 259 the line (1)–(2) represents the end view of a single turn of wire on a drum armature, revolving clockwise in a magnetic field the lines of which extend from left to right. Evidently the number of lines of force (in the Faraday sense, § 362) embraced by this turn of wire is just now increasing and, according to the law of Lenz, the induced current in the wire will be in such a direction as to oppose this addition of lines embraced, in such a direction as to

favor the setting up of magnetic lines from right to left through the winding in question. That is, the induced current will run from (2) toward (1).

In Fig. 260 the number of lines embraced by the turn of wire is a maximum, is for the instant neither increasing nor decreasing, and therefore there is no induced current in the wire, if this wire is considered alone. In Fig. 261 the number of lines embraced by the winding is diminishing, and the induced current is now from (1) toward (2). In Fig. 262 the current is still from (1) toward (2); in Fig. 263 it is once more zero; in Fig. 264 it is again from (2) toward (1), as at first.

It must not be supposed that the current in this one-turn armature would remain of uniform strength in one direction until it suddenly changed to the other direction. The *rate of change* of the number of lines of force embraced by

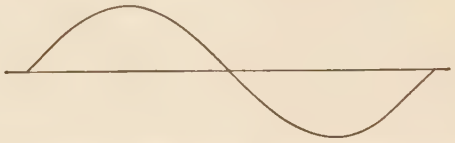


Fig. 265.—Illustrating the phases of e. m. f. in the armature of Figs. 259 to 264.

the winding varies gradually from a maximum, when the plane of the winding is parallel to the lines of the magnetic field, to zero, when the plane is perpendicular to these lines, and the current in the winding varies accordingly.

If one were to move a pencil point up and down on a sheet of paper, in such a way that its distance above or below a given horizontal line would be at every instant proportional to the strength of the current in the one-turn armature, while the paper itself moved with uniform horizontal velocity, one would trace upon it a curve like Fig. 265, the part above the line indicating a current from (1)

toward (2), let us say, in Figs. 260 to 263, and the part below the line indicating a current from (2) to (1), as in Figs. 263, 264, 259, 260.

Such a varying flow of electricity is called an *alternating* current, the abbreviation for which is *A. C.*

367. Alternating-current and Direct-current Dynamos.— If we wish to have an alternating current in that part of the circuit which is outside the dynamo, we have merely to make the brushes connect with the armature as they do in

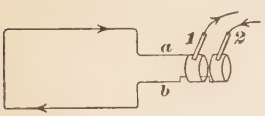


FIG. 266.—Commutator giving an alternating current outside the dynamo.

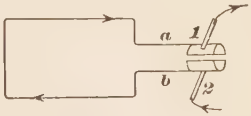


FIG. 267.—Commutator giving a direct current outside the dyanmo.

Fig. 266, the brush (1) connecting always with the *a* end, and the brush (2) always with the *b* end of the winding. This is done in many cases, alternating currents being very extensively used, as we shall see later, particularly in the long-distance transmission of power by electricity.

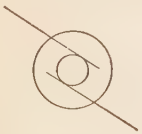


FIG. 268.—Indicating an alternating-current dynamo.

FIG. 269.—Indicating a direct-current dynamo.

If, on the other hand, we wish to have the current always in one direction outside the dynamo, a so-called *direct* current (*D. C.*), we can use a brush connection like that indicated by Fig. 267, where (1) and (2) connect alternately with the *a* end and the *b* end of the winding.

An alternating-current dynamo can be indicated in a diagram by a sketch like Fig. 268, a direct-current dynamo by one like Fig. 269.

Multipolar Dynamos.—The magnetic field of a dynamo may have more than two poles, as in Fig. 270, where four are shown. Here the lines of magnetic force are curves having the directions indicated by the arrow points. If a loop of wire, the end view of which is shown, is swung around the field through the successive positions 1, 2, 3, 4, the direction of the lines of force through it is reversed four times in each revolution.

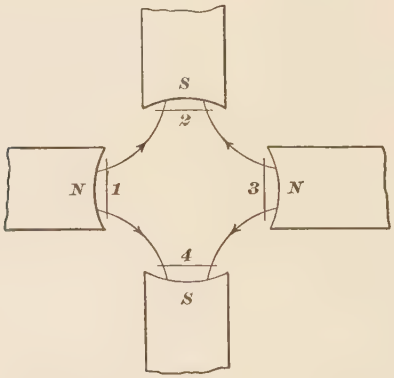


FIG. 270.—A "multipolar" arrangement.

368. Cycles.—In position 1 (Fig. 270), and again in position 3, the loop in question is facing a north pole, and the lines of magnetic force go from the pole through it. Practically, then, the loop is in the same state at 3 as at 1. It has gone through a complete electric and magnetic *cycle* in going from 1 to 3. Evidently it goes through two of these cycles in every complete revolution of the armature of which it is a part.

The whole curve of Fig. 265 represents one complete cycle of the alternating current, corresponding to each cycle of the armature state. The number of cycles per second of an alternating current is called the *frequency* of the current. For ordinary "incandescent"-lamp service the frequency is about 60.

369. Direct-current and Alternating-current Motors.—A direct-current motor is one that is driven by a direct current, a current that comes *to* the motor in a non-alternating form, though in any particular winding of the armature in motion the current is usually changing direction at least twice in each revolution. Figs. 259 to 264 would, with a mere change of direction of the magnetic field, serve to illustrate the armature action of a motor driven by a direct current. In the early history of motors nearly all were of the direct-current type.

Attention is called in § 360 to the fact that a motor may be driven, and driven always in the same direction, by an alternating current, provided the changes of direction of the armature current and the magnetic field occur at the same instant. This can hardly be well managed unless the changes of direction of the driving current come when the armature is at what may be called a *dead-point* (see § 359), a condition in which (see Fig. 260) the planes of the armature windings are at right angles with the lines of the field, so that the armature has for the instant nothing except its own momentum to keep it in motion. But to have this adjustment we must have the number of revolutions of the armature regulated by the number of alternations per second, the "frequency," of the driving current; that is, the motor armature must keep step with the armature of the dynamo which produces the current, an arrangement which is called *synchronism*. It is not very difficult to maintain after it is once set up, but, as the dynamo must be going at

a high rate before it can generate sufficient current to drive the motor, some special contrivance is needed to start the motor and get it up to speed.

370. Measurement of Alternating Currents.—It is evident that an alternating current, if the alternations are frequent, cannot be measured by means of an ordinary galvanometer (§ 339) or an ordinary coulometer (§ 338), for in each of these instruments reversal of direction of the current reverses the effect produced in the instrument, and a rapid reversal of effects is practically equivalent to no effect.

But we can put in place of the suspended needle of an ordinary galvanometer a suspended coil of wire through which flows the same current that passes through the fixed coils, and the suspended coil will turn, as the needle turns, when the current is sent through the instrument, the fixed coils acting magnetically on the suspended coil. In this case, however, reversal of the current simultaneously in both the suspended and the fixed coils will leave the direction of the magnetic action unchanged, so that the instrument can be used for measuring alternating currents.

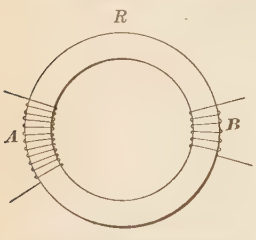


FIG. 271.—Illustrating a "transformer."

371. Transformers.—If R in Fig. 271 is a ring of soft iron having wound upon it one coil, A, consisting of many turns of wire insulated from each other, and another coil, B, having a smaller number of turns, also insulated, an alternating current acting in either A or B will generate by induction an alternating current of the same frequency in the other coil. If the primary current is in A, the secondary, or induced, current in B will be of smaller e. m. f., lower "voltage," than the primary. If the primary current is in B, the secondary current will be of higher voltage than

the primary. Any device like this for changing the voltage of an alternating current without changing its frequency is called a *transformer*. If the secondary current has a higher voltage than the primary, the apparatus is a *step-up* transformer. If the secondary has a lower voltage than the primary, the apparatus is a *step-down* transformer.

These terms have come into use with the application of alternating currents to business purposes, the furnishing of light and power on a large scale. The main wires, or "mains," which carry currents, direct or alternating, from a distant power-house or central station to the various places where work is to be done or light is to be furnished by these currents, are usually in a state of potential, or voltage, which would make them very dangerous in buildings. When the alternating current is used, a step-down transformer, attached to the outside of a building, furnishes a convenient and economical way of reducing the high external voltage to the comparatively low value, usually not more than 110 volts, considered allowable for use in ordinary buildings. When direct currents are used, the step-down operation is effected by means of a "*motor-generator*," a machine in which the high potential current drives a motor, this motor drives a dynamo, and this dynamo produces a low potential current.

The fact that a transformer usually costs less and wastes less power than a motor-generator makes the use of alternating currents advisable for certain purposes, though in some respects they are less convenient than direct currents.

The loss of energy in a good, large-scale, transformer, that is, the fraction of the entering energy which is turned into heat in the transformer instead of leaving it as electrical energy in the secondary, or induced, current, need not be more than 2 or 3 per cent. This loss is mainly due

to the fact, that in the constantly repeated magnetizing and demagnetizing of the iron *core* of the transformer, a resistance, a sort of internal friction of the iron resisting change of magnetic state and delaying this change, is encountered, and the loss in question is called the *hysteresis* (*lagging*) loss.

A transformer is frequently indicated in a diagram by a sketch like Fig. 272.

372. "Rectifiers."—Certain pieces of apparatus will transmit a current of electricity freely in one direction but not in the opposite direction. If one of these is put into the path of what would naturally be an alternating current, only the positive or only the negative pulses get through it, so that within this apparatus the current is in one direction only. Such a contrivance is called a *rectifier* of the current, because it gets a direct, though intermittent, current from an alternating one. By properly combining two rectifiers both parts of the alternating current can be made to issue from the apparatus as direct current, suitable for charging a storage battery, for example. The mercury-arc lamp (§ 376) in certain forms is used as a rectifier.

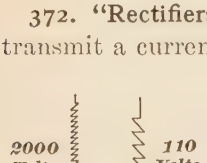


FIG. 272.—Indicating a transformer.

373. Work of an Electric Current; Comparison of Water Power and Electric Power.—A steady electric current flowing from a point of higher potential to a point of lower potential, is in some respects like a stream of water flowing down hill with unchanging velocity. In such a stream gravity does work upon the descending water, and this work is spent in overcoming friction, thereby producing heat, or, perhaps, in driving machinery. So upon the electrical stream, work is done between the two points by the power that drives the current, and this work may take the form

of heat, in the wire, or mechanical energy in an electric motor (§ 365).

One very important practical difference between water power and electric power must be borne in mind. The former power we find ready-made by the natural processes of evaporation and rainfall. The latter power we do not find ready to hand in any usable form and quantity. We have to manufacture it at the expense of other power when we would use it. A battery current uses up chemical energy; a thermoelectric current uses up heat; a dynamo uses up mechanical energy.

374. The Watt; Electrical Horse-Power.—An electric current of one ampere, flowing between two points the difference of potential of which is one volt, absorbs electrical energy between those two points, and yields heat or some other form of energy, at a rate called one *watt* (from Watt, see § 203). If the number of amperes is I and the number of volts is D, the number of watts is ID. *Seven hundred and forty-six watts are equivalent to one horse-power* (§ 138).

This method of reckoning electrical work is very common among those who have much to do with dynamos and motors. Instruments called *watt-meters* are so constructed that an index shows the product at any instant of the present current-strength (in amperes) and the difference of potential, or the e. m. f., (in volts) required to maintain this current in the part of the circuit which is under consideration.

One watt, if used entirely in producing heat, generates 0.24 calorie (§ 176) per second.

Since, according to Ohm's law (§ 347), we have, in the simple case of a current of I amperes maintained through a resistance of R ohms by a difference of potential of D volts, $D = IR$, we get in such a case, as the number of calories developed per second in overcoming resistance,

$$h = 0.24\, ID = 0.24 I^2 R.$$

Questions and Problems.

(1) A battery, the e. m. f. of which is 40 volts, sends a current of 100 amperes through a certain circuit. What is the number of watts? What is the rate of work in horse-power?

(2) A battery working at the rate of 3 horse-power has an e. m. f. of 80 volts. What is the strength of the current?

(3) What must be the e. m. f. of a battery which maintains a current of 10 amperes and works at the rate of 21 watts?

(4) How many calories are generated per *minute* in a wire of 10 ohms resistance through which a current of 5 amperes is maintained?

(5) Make a diagram indicating in the simplest way a water wheel driving a dynamo, a transformer producing from the 2000-volt alternating current of this dynamo an alternating current of 20,000 volts for long distance transmission, a transformer at the other end of the line for getting from this current an alternating current of 250 volts, and a motor driven by this final current. (See Figs. 269, 272.)

375. The Efficiency of a Motor or Dynamo.—If W_i is the amount of energy which is put into a machine and W_o the amount of energy, of the desired form, which comes out from the machine, the ratio $W_o \div W_i$ is called the *efficiency* of the machine.

Sometimes, as in the transmission of power from one place to another or in the transmission of heat from a boiler to a distant part of a building, the energy put in is of the same kind as the energy taken out; but often the case is different. For example, we put mechanical work into a dynamo and get from it as our desired product electrical energy; we put electrical energy into a motor and get from it as our desired product mechanical work. In order to reckon the efficiency of an apparatus, we must *express* W_i and W_o in the same units, however unlike the two forms of energy may be. If we do this, the efficiency will be the same whatever unit of energy is used. In Exercise 50 the efficiency of a small electric motor is measured.

376. Electric Lamps.—We may well distinguish four classes. They are nearly always run by currents from

dynamos, though battery currents are sometimes used for illumination on a small scale.

The Incandescent Lamp.—This consists of a thread, or *fibre*, of carbon or of some highly-resistant metallic substance such as *tungsten* or *tantalum*, heated to incandescence by the passage of the current and kept in a vacuum to prevent its burning out, which would occur very quickly in air. The nominal power of the incandescent lamps most used in this country is 16 candles, and the proper difference of potential at the terminals of one of these lamps is usually 110 volts. The number of watts required to keep a good carbon-filament 16 c. p. lamp at standard brightness is about 55.

For an equally powerful tungsten or tantalum incandescent lamp the number of watts is less, as a somewhat larger proportion of the electrical work spent is turned into light in tungsten or tantalum filaments than in carbon filaments.

The resistance of carbon, unlike that of metals, decreases with rise of temperature, so that a carbon filament, other things being equal, carries a stronger current when hot than while it is growing hot.

Problems.

(1) From the data given above find how strong a current is required in a 16 c. p. 110-volt carbon lamp.

(2) How great, then, is the resistance of such a lamp?

The Nernst Lamp.—In this lamp the filament, or *glower*, which is shorter and thicker than the filament of a carbon lamp, consists of or contains certain earthy materials, oxides of certain rather rare metals, such as thorium, which are practically non-conductors of electricity at low temperature but become conductors when heated to incandescence. The preliminary heating, which is done by sending a current of electricity through fine platinum wire near the

glower, requires a good many seconds, so that this lamp is not so good for instantaneous use as a common incandescent. It is, however, very brilliant, white, and strong.

The Arc Lamp.—By far the most powerful lamp in use is that in which the electric current leaps across a gaseous or vacuous space, between two solid rods which are usually of carbon, making in this space a tremendous glow, the *arc*. Familiar, though not always correct, figures for such a lamp are: potential difference, 50 volts; current, 10 amperes; maximum candle-power (in some direction), 2000.

To prevent rapid consumption of the incandescent ends of the carbons it is now common practice to inclose the arc within a glass globe that is nearly air-tight. The oxygen originally within this globe is soon used up by consumption with carbon, and after this the carbons waste away but slowly. Such inclosed-arc lamps require a somewhat higher voltage than the open-arc lamps.

Sometimes metallic substances are mixed with the carbons and by their combustion in air give a *flaming* arc, which, if the right materials are used, may be more luminous than the ordinary arc.

The Mercury-Vapor Lamp.—The vapor pressure of mercury at the ordinary temperature of a room is so small that merely as a pressure it can for most purposes be ignored. If, however, the air is pumped out from a vessel containing mercury and an electric discharge is then sent through the vessel (see §379), between liquid mercury as one electrode and liquid mercury or some other metal or carbon as the other electrode, a vivid greenish light, due to the electric agitation of mercury vapor, fills the space between the two electrodes. The color of the light makes such a lamp unsuitable for ordinary purposes of illumination, but valuable for lighting pictures which are to be copied photographically.

Questions and Problems.

(1) (*a*) How great is the resistance of an arc light using 50 volts and 10 amperes?

(*b*) If this is a 1000 c. p. light, how many watts are required per c. p.?

(2) (*a*) How many volts would be required to maintain 12 such lights in series?

(*b*) How many amperes?

(*c*) How many watts?

(*d*) What electrical horse-power?

(3) What would be the answers in Problem (2) if the lights were in parallel?

(4) (*a*) If a 16 c. p. incandescent lamp on a 220 volt circuit requires 0.25 amperes, how great would be the resistance of 12 such lamps arranged in parallel rows, with 4 lamps in series in each row? (Make a diagram.)

(*b*) How many volts difference of potential between the ends of the rows would be required in this case?

(*c*) How great a current would be required in the single main leading up to or away from such a combination of lamps?

(*d*) How many watts would be used in the combination and what horse-power?

377. The Induction Coil: Description.—A so-called *induction coil*, or *Ruhmkorff coil*, is a contrivance for producing transient but rapidly repeated induced currents of electricity. It is, in fact, a step-up transformer of a less simple plan than that shown in § 371.

The general plan of the apparatus is shown in Fig. 273. The part *i* is a core of straight soft-iron wires, around which is a coil, P_1P_1, called the *primary coil*, of comparatively coarse wire; outside the primary, and insulated from it, is a *secondary coil*, P_2P_2, consisting of many turns of very fine carefully covered wire, of which coil *t* and *t* are the terminals.

The main current enters the primary by *a* and after traversing the windings comes out at the farther end, where it passes by the stop *p* through the spring, or vibrator, *V*,

at the top of which is a piece of soft iron, back to the wire b by which it leaves the apparatus.

This current magnetizes the core i, which attracts away from p the soft iron at the end of V and so breaks the circuit. This interruption of the current causes the core to lose its magnetism, and this change induces a strong, though momentary, electromotive force in the secondary, sending a spark across a considerable air space between t and t. The core having lost its magnetism, V swings back to its

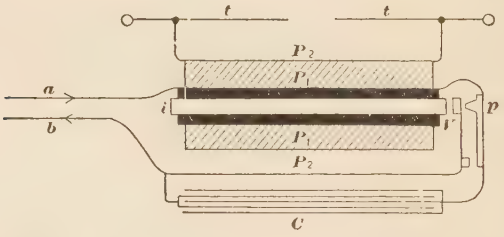


FIG. 273.—Lengthwise section through an induction coil.

normal position, thus closing the circuit, and the whole operation is repeated with a rapidity which depends greatly upon the character of the vibrator.

At the point of rupture, between V and p, a spark occurs, and this spark, by making a heated conducting *arc* (see § 376) across the air, may cause the primary current to die out so slowly that no strong induction effects are produced. This difficulty is removed, to a certain extent, by connecting V and p with the opposite sides of a *condenser* (§ 325), C, which is usually placed in a wooden case beneath the coil. When the break occurs, there is a flow of electricity for an instant into the condenser, which lessens the flow across the air from p to V, the final result of the rather complex operation being a more sudden demagnetization of the core and a more powerful effect in the secondary.

378. Uses of the Induction Coil.—Evidently the terminals of the secondary may be made of such length that the spark between them will occur a long distance away from the main apparatus. This fact makes it possible to use the induction coil for lighting gas jets in places not easily accessible, and for exploding charges of powder, etc., in mines.

The coil is used to some extent in the practice of medicine. Any one who connects the terminals of a small induction coil, in operation, with his bare hands will experience a very peculiar and powerful stimulus of the nerves in his hands and arms. Such a stimulus is considered beneficial in certain forms of nervous debility. The experiment of connecting the terminals by means of the hands should not be tried with a powerful coil.

A very conspicuous use of the induction coil is to send momentary currents of electricity through so-called *vacuum-tubes*.

379. Vacuum-Tubes.—The electrical resistance of gases at ordinary temperatures and at atmospheric pressure is very great, but electricity passes with comparative readiness through gases in a certain state of rarefaction. If the

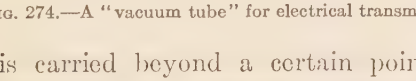


FIG. 274.—A "vacuum tube" for electrical transmission.

rarefaction is carried beyond a certain point, which is different for different gases, the resistance begins to increase, and finally it becomes much greater than that of the gas under atmospheric pressure. This fact leads us to the belief that the "ether" (§ 300) of an absolute vacuum is a non-conductor of electricity, in the usual sense of the word conductor; although it may be that electricity can be *forced* through it by means sufficiently violent.

A *vacuum tube* (see Fig. 274) is usually a glass tube containing some highly rarefied gas, penetrated by two or more platinum wires, called electrodes, which are sealed into the wall of the tube. When the secondary terminals of an induction coil in operation are connected with the electrodes of such a tube, transient currents of electricity pass through the gaseous space, producing curious and often very beautiful effects.

Vacuum tubes in which the glass itself has very complicated forms, and which are used largely to produce beautiful color effects, are called *Geissler* tubes, after a celebrated maker of such apparatus. Tubes using still higher vacua, provided with electrodes terminating in disks or portions of spheres, and often with much other internal furniture of metal, mica, etc., are commonly called Crookes tubes, after an English investigator whose experiments made such tubes widely known.

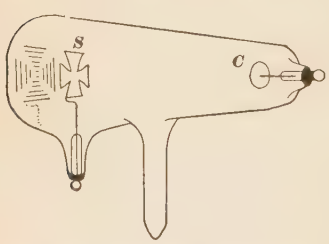


FIG. 275.—Illustrating cathode rays.

380. Cathode Rays.—It was noticed long ago that the *cathode* of a vacuum tube, the electrode by which the positive current leaves the tube or the negative current enters it, shows phenomena of peculiar interest. In highly rarefied gases, like those of Crookes tubes, some kind of influence appears to proceed in straight lines from the cathode; for certain effects are observed at spots within the unobstructed reach of such lines and not at all or very little elsewhere. Thus Fig. 275 shows a shadow cast by a screen, *S*, placed in the course of rays from the cathode, *C*, within a Crookes tube.

Crookes believed, and the opinion has now become general, that there is an actual projection of some material

from the cathode to the points affected. It is believed that the particles of matter thus projected are smaller than ordinary atoms. See foot-note, § 333.

About the year 1892 Hertz (§ 382) and Lenard found that cathode rays could penetrate, or at least produce an effect through, opaque matter, such as thin films of metal.

381. Roentgen Rays.—In the latter part of 1895 Professor Roentgen of Würzburg discovered that from a vacuum tube in which the cathode rays are active some influence goes out in straight lines which can penetrate through many opaque bodies—wood and flesh, for example—and produce photographic effects beyond.

It appears that the Roentgen rays are not the same as cathode rays, but are produced at, and given out from, any solid body, such as glass or metal, upon which the cathode rays strike.

Roentgen rays falling directly upon the eye do not give the sensation of light. But they can produce photographic effects upon ordinary photographic plates and fluorescent effects (§ 306) upon layers of various substances, and in these indirect ways they serve the eye.

The fact that gives to the Roentgen rays a peculiar and even a weird interest is this, that they penetrate flesh much more readily than bone, and as they go always in straight, or nearly straight, lines, they produce by photography or fluorescence well-defined shadows of the skeletons of living bodies.

There has been much discussion as to whether the Roentgen rays are of the same nature as light-waves. The fact that they are reflected and refracted very little made people cast about for some theory to explain them as something different from ether-vibrations, but it now seems probable that they are really very short ether-pulses, akin to light-waves, but much shorter and less regularly spaced.

The uncertainty as to their nature caused Roentgen to call them *X-rays*, and by this name they have become widely known.

Roentgen rays have been produced from a Crookes tube agitated by the discharge from a powerful Holtz electrical machine (see Appendix VI), a piece of apparatus which may be available where dynamo currents and induction coils of sufficient power are not to be had.

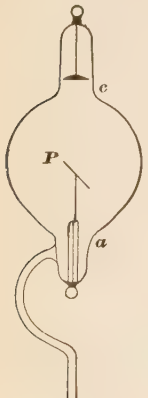


Fig. 276.—Tube for generating X-rays.

A common form of Crookes tube for the production of Roentgen rays is shown in Fig. 276, where *c* is the cathode and *P* is a plate of platinum, which, receiving the cathode rays, gives out the Roentgen rays. *P* is also the anode.

ELECTROMAGNETIC WAVES.

382. The Experiments of Hertz.—The wave theory of light, and so by inference the existence of a "luminiferous ether," was proved early in the nineteenth century by Young and Fresnel. See § 234.

The experiments of Faraday led him to believe electrical and magnetic forces, "lines of force," to be transmitted by some medium, probably the luminiferous ether. He expressed this opinion in 1851.

Maxwell, a close student of Faraday's works, developed Faraday's idea mathematically, and arrived at certain definite conclusions which could be tested by experiment. Some of them were soon tested, and the results did much to confirm the theory; but a real demonstration, capable of making an impression on the public, was not accomplished until 1887, when Hertz, a young German, showed that electromagnetic waves may be sent out from a circuit

connected with the secondary of an induction coil and travel for considerable distances through the surrounding space.

The instrument used by Hertz to detect these waves at a distance from their source he called an *electrical resonator*. It was a loop of wire of such dimensions and shape that electrical oscillations within it had the same time of vibration as the waves the instrument was intended to detect. The analogy between the use of this instrument and that of the acoustic resonators of Helmholtz (§ 222) is obvious.

The waves discovered by Hertz, which are now often called by his name, were some meters long, whereas the waves which affect our eyes as light are only a few hundred-thousandths of a centimeter long. His waves were of electrical origin, and were traced through space by their electrical effects. They were beyond question electromagnetic waves, yet they were reflected like light waves, they were refracted like light waves, and they traveled with about the same velocity as light waves. There could be no doubt that the two kinds of waves are essentially the same.

Thus was completed one of the greatest intellectual triumphs of the nineteenth century, the proof of the electromagnetic nature of light.

383. Wireless Telegraphy.—The step from the experiments of Hertz to the sending of messages thousands of miles by "wireless telegraphy" is a very simple one in theory, though in practice it proved somewhat difficult. It was accomplished by a young Italian, Marconi, who made the original electrical oscillations exceedingly powerful and used a much more sensitive receiver than Hertz employed. It has been found that vertical oscillations are better for the purposes of wireless telegraphy than horizontal oscillations are.

There are now many arrangements of wireless telegraph apparatus that work successfully. Fig. 277 shows a trans-

mitter, Fig. 278 a receiver. In Fig. 277, when the key K is depressed for a moment, thus sending an alternating current through the primary of the transformer Tr, the secondary of the transformer is excited by the alternations in the primary and a succession of sparks leaps across the air gap S. In circuit with this gap, above it, is a condenser,

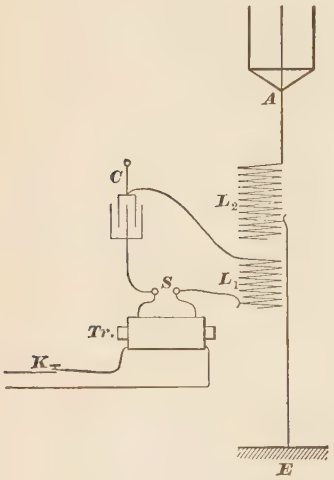


FIG. 277.[1]—Sending end of a wireless telegraph system.

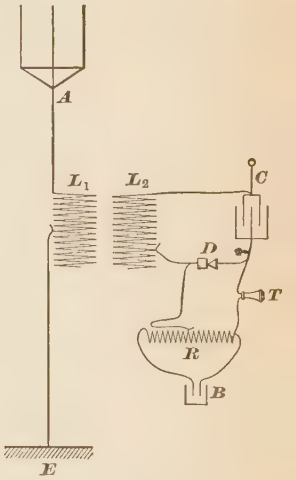


FIG. 278.[1]—Receiving end of a wireless telegraph system.

C, of variable capacity, and a coil of wire, L_1, so arranged that a greater or smaller number of its turns can be used at will.

Every spark sent across S by the secondary of Tr sets up electric oscillations, to and fro surgings of current in the circuit S—C—L_1, and the rapidity, the "vibration frequency," of these oscillations depends on C and L_1. In fact, the transformer Tr acts on this S—C—L_1 circuit some-

[1] These figures are taken, with some changes, from the *Principles of Wireless Telegraphy*, by G. W. Pierce (McGraw-Hill Book Company).

what as the clapper acts on a bell, the frequency of the resulting vibrations having no necessary relation to the rapidity of the exciting blows. The circuit S—C—L_1 is not closed, in the ordinary sense, as the plates of C are separated by a non-conductor which is not traversed by a spark; but the condenser becomes charged, the upper side + and the other −, or the upper side − and the other +, the two states alternating, and discharges across the gap S several times during the continuance of a single spark, so that powerful alternating currents pass through the connected turns of L_1. These currents induce currents in the coil L_2, which is in circuit with the *antenna* A and the earth. If this vertical circuit is *tuned* to resonance with the S—C—L_1 circuit, by using the right number of turns in L_2, the up and down surgings of electricity may be very powerful and send out in all directions over the earth's surface vertical ether-disturbances, which by means of suitable apparatus can be detected thousands of miles from their source. It must be remembered that these waves are sent out only while the key, K, in the primary circuit of the transformer, is closed. Evidently the dot and dash alphabet of the ordinary telegraphic code (§356) can be used in the transmission of messages.

The receiving apparatus, as shown in Fig. 278, has a vertical circuit very like that of the transmitter. The coil L_1 has a variable number of turns in use, for the purpose of attuning the antenna circuit to the corresponding circuit of the sending apparatus. The oscillations in L_1 induce oscillations of like frequency in the attuned adjacent circuit containing L_2, connected with which is the circuit containing a receiving telephone T. This instrument must not be placed directly in circuit with L_2 and C, for the large number of turns of wire in its coil would be objectionable, and the frequency of the vibrations in the L_2—C

circuit is so great that the diaphragm of the telephone would not respond audibly to them.

There is, however, a periodic interruption or change of strength in the procession of vibrations from the sending apparatus, corresponding to the alternations of current in the primary of the transformer *Tr* in Fig. 277, which is of the right frequency to affect the telephone. Thus, let us suppose that the key, *K*, of Fig. 277 is held down, making "contact," for the space of four alternations, *aaaa* (see Fig. 279), of the primary current, or four sparks, *ssss*, of the transformer secondary at *S* (Fig. 277), each spark setting up in the sending and receiving vibrating circuits a *train* of several high frequency pulses, as Fig. 279 indicates. If

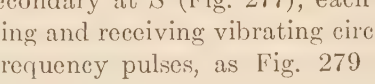


FIG. 279.—Dot and dash in wireless telegraphy.

the telephone is properly connected, according to the indications of Fig. 278, its diaphragm will, as a result of this contact at *K*, move forward and back four times, with audible effect, a *dot* of the telegraph code. If the contact of *K* is made longer, lasting through eight alternations, let us say, we have in the telephone a sound of corresponding duration, a telegraph *dash*. Thus, Fig. 279 gives the letter *A* of the telegraphic code.

In the application of the telephone, a "detector," D (Fig. 278), is used, which is common to the L_2—C circuit and the telephone circuit. This is usually some form of "rectifier" (§ 372), that is, a contrivance which transmits electric currents more freely in one direction than in the opposite direction. An excellent and much used detector consists of a piece of silicon, or of any one of several crystalline substances, imbedded in soft metal on one side and

touched on the other side by a metal point. Every "train" of high frequency vibrations, sends more current in one direction than in the other through D[1] and therefore accumulates a charge in the condenser C, which charge is released, at the end of the train, through the telephone circuit. The result is a tone in the telephone corresponding in pitch to the *train frequency* of the vibrations, that is, to the alternation frequency of the primary current in the sending station of Fig. 277. The battery B, though not essential to the working of the receiver, is helpful and is commonly used. Its current passes through a resistance R, with which the telephone circuit is connected in such a way as to get an e. m. f. of about 0.1 volt from the battery.

384. Wireless Telephony.—A listener at the telephone of a wireless telegraph receiving station hears only a succession of tones of a fixed pitch, lasting for shorter or longer times according to the dots and dashes of the telegraphic alphabet. If the transmitting station is made to send out a continuous stream of vibrations of a frequency too high to affect a telephone receiver audibly, no sound will be heard in the instrument so long as the strength of these vibrations remains constant; but, if a telephone transmitter is introduced into the sending apparatus in such a way that words spoken into it vary the strength of these high frequency vibrations, the same receiving apparatus which is indicated in Fig. 278 may give out these spoken words through the telephone of the final circuit.

There are several methods of producing the continuous stream of high frequency vibrations, one of which makes

[1] In the early days of wireless telegraphy the detector commonly used was a "coherer," consisting of a small body of metal filings lying loosely in a glass tube between two metal plugs. Every "train" tended to reduce the naturally high resistance of the coherer and so allow the battery B to send through its own circuit a current sufficient to work an electric bell or "buzzer." The telephone cannot well be used with a coherer.

use of the so-called singing-arc, an electric arc in hydrogen gas, coupled in parallel with a condenser in series with a coil of wire. By certain adjustments[1] the pitch of the note given out by the arc can be raised above the limit of human hearing, about 35,000 vibrations per second, and it thus becomes suitable for use in wireless telephony by the method already indicated. Verbal messages sent out in this way have been received many miles from their source, and it seems altogether probable that great advances in this field of electrical art will soon be made.

Questions and Problems.

(1) A magnetic pole of 20 units strength is placed at a distance of 10 cm. from a similar pole of 50 units strength. Find the force between them.

(2) A magnetic pole of 4 units strength is placed in a field of 0.5 unit strength. How great is the force exerted upon the magnetic pole?

(3) What length of wire 0.3 mm. in diameter will have the same resistance as 15 m. of the same kind of wire 2 mm. in diameter?

(4) A wire 3 ft. long has a diameter of 0.064 cm. What must be the diameter of a wire of the same nature 27 ft. long, if it offers the same resistance?

(5) If the resistance of a manganin wire 1 m. long and 1 mm. in diameter is 0.6 ohm, what length of manganin wire 0.25 mm. in diameter will it take to make a resistance coil of 4.8 ohms?

(6) If a foot of iron wire 0.05 in. in diameter has a resistance of 0.03 ohm, what is the resistance of 10 miles of iron telegraph-wire having a diameter of 0.2 in.

(7) A wire 10 m. long and 0.5 mm. in diameter has a resistance of 8 ohms. Calculate the specific resistance.

(8) Two points in a circuit are joined in multiple arc by two wires whose resistances are 50 and 80 ohms respectively. Find the resistance of the circuit between these two points.

(9) Three wires whose resistances are 20, 50, and 60 ohms respectively are joined in multiple arc. Find their joint resistance.

[1] See Pierce's *Wireless Telegraphy*, already mentioned.

(10) The resistance of a wire between two points is 40 ohms, but on adding another wire in multiple the joint resistance falls to 35 ohms. What is the resistance of the second wire?

(11) The resistance between two points is found to be 100 ohms. This must be reduced to one-half this amount. How many wires, each having a resistance of 1000 ohms, would be needed in multiple with the original resistance?

(12) A battery which is producing a current of 1 ampere delivers this current through two wires, side by side, having a resistance of 30 and 50 ohms respectively. What is the strength of the current in each wire?

(13) How strong a current in a Daniell cell will be required to make the copper plate gain 0.5 gm. per hour?

(14) A battery of 30 cells is connected in series with an external resistance of 20 ohms. If the e. m. f. of each cell is 1.8 volts, and its internal resistance 1.5 ohms, what is the current-strength?

(15) If the cells in the previous example had been arranged 3 abreast by 10 in series, what would have been the current strength in the external part of the circuit?

(16) How many cells in series, each having an e. m. f. of 2 volts and an internal resistance of 1.5 ohms, will produce 1 ampere current through an external resistance of 7.5 ohms?

(17) A battery of 6 cells, each having an e. m. f. of 1.8 volts and an internal resistance of 0.5 ohms, is arranged in series and delivers its current through 3 wires, of 10, 8 and 6 ohms, respectively, in multiple. How much is the current in each of these wires?

(18) A battery of 20 cells, each having an e. m. f. of 1.5 volts and an internal resistance of 1 ohm sends the strongest possible current through an external resistance of 30 ohms. Find the strength of the current.

(19) What arrangement will cause a battery of 48 storage cells, each having an e. m. f. of 2 volts and an internal resistance of 0.02 ohm, to send the greatest possible current through a resistance of 0.5 ohm and how great will this current be? (Try all in series; then 24 in series, 2 abreast; then 16 in series, 3 abreast, etc.)

(20) (*a*) Which of the first two arrangements suggested in Problem (19) would work the individual cells the harder?

(*b*) At what rate, in watts, would energy be spent against the external resistance in each of these two cases? Against the internal resistance?

(*c*) How many horse power would be the total work in the first case? In the second case?

(*d*) How many calories would be generated per second in the external part of the circuit in each case?

(21) If 50 storage cells, each having a resistance of 0.02 ohm and an e. m. f. of 2 volts, are being charged in series by the application of 105 volts difference of potential, how strong is the charging current?

(22) Two lamps arranged in multiple arc, each having a hot resistance of 30 ohms, are connected with a dynamo whose e. m. f. is 50 volts and internal resistance 3 ohms. What is the strength of the current going through each lamp?

(23) If a difference of potential of 110 volts, maintained between the terminal binding-posts of a shunt-wound D. C. motor having a field-coil resistance of 110 ohms and an armature-resistance of 5 ohms, keeps up a current of only 10 amperes through the motor,

(*a*) How great is the current through the field coils?

(*b*) How great is the current through the armature?

(*c*) How many volts is the "counter e. m. f." of the armature?

(*d*) At what rate, in watts, is electric energy supplied to the motor? What horse-power?

(*e*) At what rate, in watts, is energy spent in overcoming the *resistances*, proper, of the motor.

(f) At what rate, in watts, is energy spent in overcoming the counter e. m. f. of the armature?

(*g*) Taking what is found in (f) as a measure of the effective work-rate of the motor, what do you find as the efficiency of the machine?

DIRECTIONS FOR THE LABORATORY EXERCISES MENTIONED IN THE TEXT-BOOK.

EXERCISE 1.

DENSITY, WEIGHT OF UNIT VOLUME, OF A BLOCK OF WOOD.

Apparatus: A block of wood (No. 5). A spring-balance (No. 3), or a platform balance (No. 4). A measuring rod (No. 2). Thread for suspending the block.

Find the weight of the block in grams.[1]

Measure in centimeters,[2] to the nearest 0.05 cm., the length of each of the four edges which are parallel to the grain of the wood, take the average of these measurements and call it the *length* of the block.

Measure the length of each of the four long edges which are crosswise to the grain of the wood, and call the average of these four measurements the *width* of the block.

Measure the length of each of the four short edges and call the average of these four measurements the *thickness* of the block.

[1] The smallest divisions on the scale of the spring balance are probably 10 gms. each, and when the pointer lies between two lines of the scale, as it usually does when there is a load on the hook, the experimenter should estimate as well as he can what particular gram of the 10 gm. interval it indicates.

[2] The shortest divisions on the rod are *millimeters*, that is *thousandths* of a meter, or tenths of a centimeter. The block should be placed on a sheet of white paper in a good light, the measuring rod laid upon it near and parallel to the edge which is to be first measured, with some *centimeter* mark, preferably the 10 cm. mark, exactly over one end of the block as in Fig. 1e. The other end of the block will then probably lie not exactly under any mark on the rod, but under the space between two marks. If it lies nearer either mark than it does to the point midway between the marks, record its position as if it were exactly under this mark. If it lies nearer to the midway point than it does to either mark, record its position as if it were exactly under this midway point.

The reason for not trying to put one end of the rod just over one end of the block is that the end division of the rod is usually imperfect.

The weight in pounds can be found from the weight in grams by dividing by 453.6.

From the length, width, and thickness in centimeters the length, width, and thickness in *feet* can be found by the rule that 1 ft. = 30.5 cm., but it is shorter to find the volume in feet from the volume in cubic centimeters by the rule that 1 cu. ft. = 28,300 cu. cm.

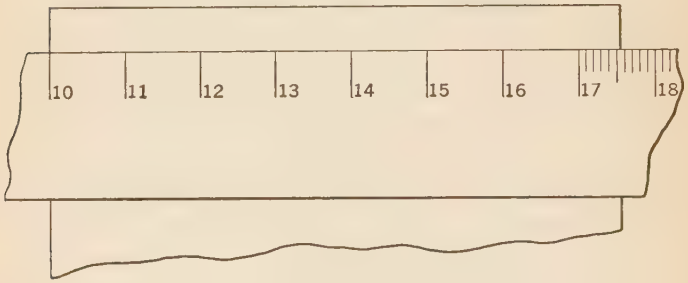


Fig. 1e.

Calculate, 1st, how many grams, or what part of a gram, 1 cu. cm. of the block weighs; 2d, how many pounds, or what part of a pound, 1 cu. ft. of such wood weighs.

EXERCISE 2.

SPECIFIC GRAVITY OF A SOLID BODY THAT WILL SINK IN WATER.

Apparatus: A spring balance (No. 3). A gallon glass jar (No. 6), nearly filled with water. A lump of sulphur (No. 7). Thread.

Weigh the sulphur out of water; then in water.

We know that a body immersed in water loses in apparent weight an amount equal to the weight of the water displaced by it. It is easy, therefore, to get from the two weighings just made the ratio which we have undertaken to find in this Exercise.

If time permits, find in this Exercise, by the same method that is used for the sulphur, the specific gravity of other solids that will sink in water—such as glass, coal, etc.

EXERCISE 3.

SPECIFIC GRAVITY OF A BLOCK OF WOOD BY USE OF A SINKER.

Apparatus: A rectangular block of wood (No. 5). The spring-balance (No. 3). The gallon jar (No. 6) nearly filled with water. A lead sinker (No. 8). Thread.

We have to find two quantities, by experiment: 1st, the *weight of the body;* 2d, the *lifting effect of water upon it when it is immersed.*

Weigh the wood in air and record its weight.

Now put the block into water. You see that it floats. To make it stay under water you must *hold it down.* Try this, putting your fingers on the block. In this case, you see, the lifting effect of the water, when the block is wholly beneath its surface, is greater than the weight of the block. We must find out how much it is.

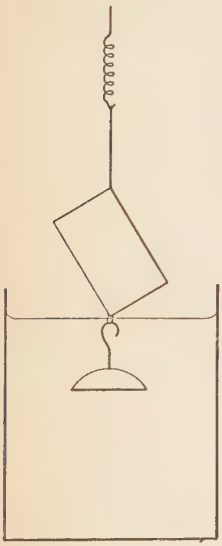


Fig. 2e.

We shall use the lead sinker to hold the block under water, and we need to know the weight of the sinker alone under water. Weigh it in this position and record the weight.

Now suspend the block from the balance[1] and the lead sinker from the thread under the block, and consider how much the two, block and sinker, would weigh in the position shown by Fig. 2e, the block out of water and the sinker in water. You can tell this from the weighings already made. Write it down.

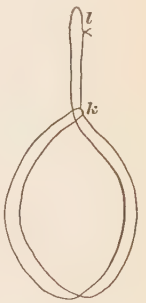


Fig. 3e.

Wt. of block in air + Wt. of sinker in water = ... + ...

Now lower the block and sinker till both are covered by the water and weigh the two together in this position and record:

Wt. of block and sinker together in water = ...

[1] The following method of attaching the block to the balance is recommended: Take a thread 2 feet long and tie the ends together. Then make of it a slip-noose by passing one end, *l* (Fig. 3e), through the other end, *k*. The block may then be placed in the noose and the loop *l* slipped upon the hook of the balance.

Just before the *block* entered the water, the sinker being already in, the weight was . . . . Just as soon as the block also was covered the weight was only . . . . The difference is the *lifting effect of the water upon the block*. We have now all that we need for calculating the specific gravity of the block by means of the formula,

$$Sp.\ grav. = \frac{Wt.\ of\ block}{Lifting\ effect\ of\ water\ upon\ block\ immersed}.$$

EXERCISE 4.

SPECIFIC GRAVITY BY FLOTATION METHOD.

Apparatus: A gallon jar (No. 6) nearly filled with water. A slender wooden cylinder (No. 9). A support for holding this cylinder upright in water (No. 10). A measuring-stick (No. 2).

If a cylinder floated upright with its top just level with the top of the water, we should at once know its specific gravity to be 1. If it floated just half in and half out of water, we should know its specific gravity to be 0.5. The cylinder that we have to use will not float all in water or exactly half in water, but if we float it, and find the length of the part then in the water, we shall, by comparing this with the length of the whole cylinder, find some way of getting the specific gravity of the cylinder.

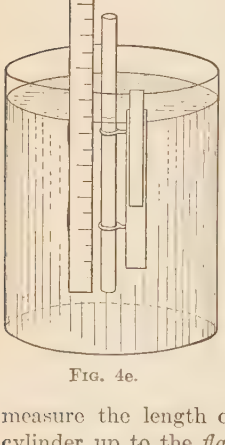


Fig. 4e.

Measure the length of the whole cylinder.

Float the cylinder in the jar (Fig. 4e), keeping it upright by means of the holder, which is attached to the side of the jar. Joggle the cylinder to make sure that it is free to take its proper position. After each joggling it should come to rest at the same depth as before. The rings of the holder must not *grip* the cylinder at all. When sure that the cylinder floats as it should, measure the length of the submerged part, from the bottom of the cylinder up to the *flat* surface of the water.

To find the specific gravity from the two measurements now made, begin by recalling the fact that the water displaced by the floating

cylinder weighs just as much as the cylinder itself. Put this fact into the form of an equation, letting

w = the weight of 1 cm. length of the wood,
w' = " " " an equal bulk of the water,
L = the length of the whole cylinder,
l = " " " " submerged part,

and with the help of this equation find the specific gravity of the wood.

EXERCISE 5.

SPECIFIC GRAVITY OF A LIQUID: TWO METHODS.

Apparatus: A gallon jar nearly filled with water, and a smaller jar (No. 11) nearly filled with a solution of sulphate of copper.[1] A small glass bottle (No. 12). A spring balance (No. 3). Thread.

First Method.

Weigh the bottle empty. Dip the bottle into the jar of sulphate of copper and let it fill with the liquid. Holding the bottle over the jar, put the stopper in place, thus crowding out the excess of liquid, then wipe the outside of the bottle and weigh it carefully with its contents.

Pour the sulphate of copper back into its jar, rinse out the bottle, then fill it with water, just as it was before filled with the other liquid, and again weigh the bottle and its contents.

From the three weighings now made the specific gravity of sulphate of copper can easily be found.

Second Method.

We know that a body going from air into water loses in apparent weight an amount equal to the weight of its own bulk of water. So a body going from air into a solution of sulphate of copper will lose in apparent weight an amount equal to the weight of its own bulk of the solution. This gives a method of finding the specific gravity of the solution. As a *body* to be weighed first in air, then in water, then in the solution, we will use the bottle with enough water in it to make it sink in either liquid. We may, indeed, use the bottle *full* of water, just as it was left at the end of the first part of this Exercise.

[1] This solution may be made by putting 2 lbs. of sulphate of copper crystals into about 3 qts of *warm* water in a glass vessel and stirring occasionally till the crystals are dissolved. A saturated solution of common salt is preferred by some teachers.

EXERCISE 6.

SPECIFIC GRAVITY OF A LIQUID BY BALANCING COLUMNS.

Apparatus: Articles 13, 14, and 15. Water for one of the tumblers and a solution of sulphate of copper for the other.

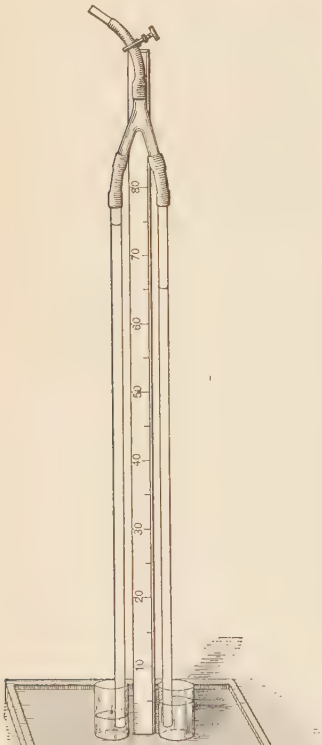


Fig. 5e.

Place the free ends of the glass tubes one in the water and the other in the sulphate of copper, and fix them in a vertical position against the upright (see Fig. 5e). Observe and record the height to which each liquid rises in its tube by capillary action, the pinch cock being open.

Applying the lips[1] to the short tube at the top, draw out some of the air, thus raising the two liquids in their respective tubes. When the taller column reaches nearly to the top of its glass tube, pinch the rubber tube near the lips, and then close the pinch cock. Watch the columns to make sure that there is no falling of their surfaces, as there will be if the connections are not air-tight. (It is well to moisten the connections occasionally with a little glycerine to keep them air-tight).

When assured that the columns are not sinking, measure carefully the height of each from the *present* level of the liquid surface in the corresponding tumbler.

Subtract from the height of each column as thus measured the

[1] This use of the lips can be avoided by using a small air-pump. To prevent too sudden action of the pump a large bottle should be inserted between it and the upright tube.

height to which it was raised by capillary action at the beginning.

From the data now obtained find the specific gravity of the solution of sulphate of copper.

EXERCISE 7.

COMPRESSIBILITY OF AIR: BOYLE'S LAW.

Apparatus: No. 16, and No. 15 to support it. Sufficient mercury to fill at least the long arm of the tube. A barometer (No. 17).

Pour into the bent glass tube sufficient mercury to fill the bend and rise to the straight part in each arm. At first the mercury will stand a little higher in the long arm, but by tipping the tube and letting out a little of the imprisoned air the level can be made the same in both arms, as in Fig. 6e, or left only one or two mm. higher in the long arm than in the other.

Fig. 6e.

Now measure the length of the imprisoned air-column, from the curved top of the mercury surface to the curved top of the bore of the tube, taking care not to handle the short arm much, lest the warmth of the hand expand the air, and write the length thus found under the letter V[1] in a table like that here indicated.

V	P	$P \times V$
(.. .—....)=..	.. + (....—....)=..	....
.....	.....	....
.....	.....	....

The pressure upon this air is now, *if the mercury-level in the same in both arms,* equal to that upon the unimprisoned air; it is as much as would be exerted by a column of mercury as tall as that of the barometer; take, then, a reading of the barometer, and record this reading under the letter P[2] in the table. But *if the level is a little higher in the long arm than in the short arm,* write under P the height of the barometer column + the difference of mercury-level in the two arms of the tube.

[1] The *length* of the air column is equal to *its volume*, if we take for our unit of volume in this experiment the space contained in unit length, 1 cm., of the tube. The bore of the tube is supposed to be of the same size throughout the short arm.

[2] It is common practice to measure pressure in terms of centimeters of depth of mercury.

Pour in more mercury till the difference of level in the two arms is about 20 cm., then measure again the length of the confined air-column. Record this length under V, and record under P the barometric height *plus* the present difference of mercury-level in the arms of the tube.

Proceed in this way, by fairly uniform stages, measuring and recording at each stage, till the volume of the confined air column is about one-half what it was at first.

Multiply each number under V by the corresponding number under P, and write the products in the column headed $P \times V$. An examination of this column will probably show a pretty close confirmation, but not an exact confirmation, of *Boyle's* law.

EXERCISE 8.

DENSITY OF AIR.

Apparatus: Nos. 17, 18, 19 (supported by 15), 4, 20 and 21.

Set up the apparatus according to Fig. 7e, after moistening with glycerine the junctions at which leakage might occur, and pour mercury into the U-tube till both arms are about half full.

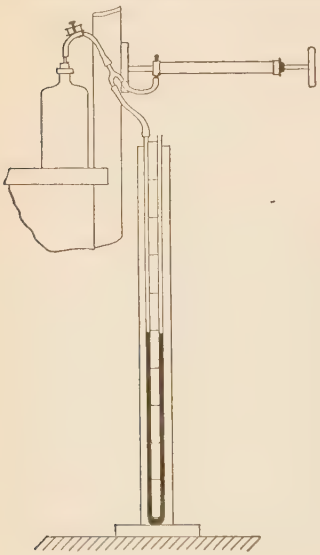


FIG. 7e.

Work the pump and draw air from the bottle until the difference of mercury-level in the arms of the U-tube gauge is about 70 cm., then pinch the rubber tube that leads from the pump very hard and watch the mercury in the gauge for a short time to see whether it is changing level. If it is changing, there is some leak which should be looked for and stopped. If there is no leak, read and record the difference of level in the arms of the pressure gauge, and then immediately close the pinch cock near the bottle, making it thoroughly tight.

Disconnect the bottle, with the attached tube and pinch cock, from

the rest of the apparatus and place it on the platform balance, after making sure that this balance is in good condition. Weigh the bottle and attachments *very carefully*, to the nearest tenth of a gram, and then, without removing the bottle from the balance, open the pinch cock and let the air enter. Then weigh again, as carefully as before.

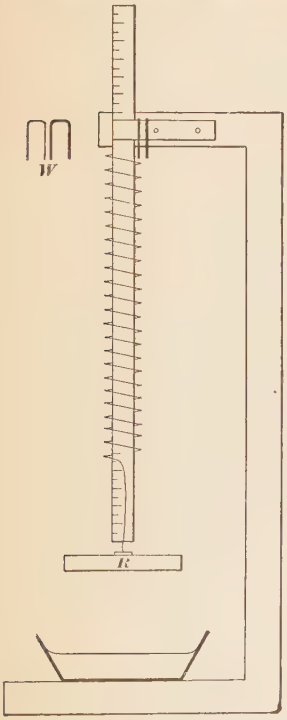


Fig. 8e.

The difference of the two weights thus found is the weight of the air originally pumped out from the bottle. The reading of the pressure gauge, already recorded, compared with the height of the barometric column, which should now be read, will show what fraction this air was of the whole amount of air contained by the bottle when open to the atmosphere, and so the weight of this whole amount of air can be found.

It is now only necessary to find the capacity of the bottle, in cu. cm., in order to be able to calculate the density of the air. The capacity of the bottle may be found by finding how many grams of water it will hold.

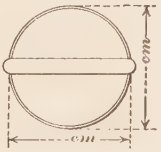


Fig. 9e.

EXERCISE 9.

MEASUREMENT OF SURFACE TENSION.

Apparatus: Nos. 22 and 23.

Measure the diameter of the ring *R* (Fig. 8e) in two directions at right angles to each other and record as in Fig. 9e.

Adjust the wooden scale of Fig. 8e within the spiral spring so as not to touch any of the suspended parts.

Using as an index any convenient point on one of the hooks at the bottom of the spiral, find how much extension is caused by loading the ring with the light copper staple, 1 gm., the heavy staple, 2 gm., the two staples together. Record in tabular form. For example,

TEST OF SPRING BALANCE.

	Position of index. cm.	Stretch per gm. cm.	
With no load			Mean stretch per gm. .....
" 1 gm.	.....	.....	
" 2 "	.....	.....	
" 3 "	.....	.....	

SURFACE TENSION OF ALCOHOL.

Put a small amount of alcohol in the saucer and raise it beneath R till the whole of the lower edge of the ring is immersed; then gradually lower the vessel, watching meanwhile the extension of the spring and the behavior of the liquid adhering to the ring. Continue until the film of liquid breaks and releases R, noting the position of the index just before the break occurs. Repeat the test several times. Record somewhat as follows:

Position of Index cm.	Stretch cm.	Total pull of film
Without load	.....	(..) ÷ (..) = .. gm.
With alcohol film		

Surface tension of alcohol = (..) ÷ (..) = .. gm. per cm.

SURFACE TENSION OF WATER.

Put clear water in place of the alcohol and proceed as in the preceding case, making the record as before.

BEHAVIOR OF SOAP SOLUTION.

Put soap solution in place of the clear water and find the surface tension of this solution, making use, for this purpose, of the *maximum* pull exerted by the soap film, which may not be the pull just before it breaks.

Estimate (you cannot measure accurately) and record the height of the film when this maximum pull is exerted.

Note and record the position of the index when the height of the liquid film is about 1 cm., and make, with care, a drawing showing the shape of the film, whether straight up and down or not, in this case.

Do the same when the height of the film is about 2 cm.

Is the total *downward* pull of the film on the ring greater or less when the height is 2 cm. than when it is, for example, 0.2 cm.? Does the shape of the film in the two cases help you to explain the difference of downward pull?

EXERCISE 10.

THE STRAIGHT LEVER.

Apparatus: The lever (No. 24). Three ounce[1]-scale-pans (No. 25, A, B, C). A set of ounce weights (No. 26). A spring balance (No. 3).

First Part: Two Loads.—Suspend the spring balance from any convenient fixed support over the table and hang the lever upon it as in Fig. 10e, but without the pans. Read the balance with its present load and record this reading as *b*, the weight of the bar.

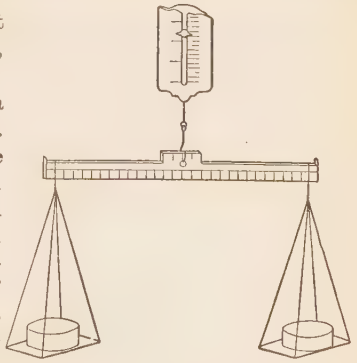


Fig. 10e.

Hang one scale pan, carrying a *load* of 7 oz., at a distance of 14 cm. to the left from the middle of the bar. Hang the other pan, with an equal load, on the right-hand end of the bar, at such a distance from the middle that the lever will *balance*, that is stay horizontal, if once placed so, even when the apparatus is jarred somewhat by tapping the spring balance. Then make a record, in a single horizontal line, under headings like those given in the table below, where

W_l = the left-hand load *plus* the weight of the pan,
W_r = " right- " " " " " " " " "
D_l = " left-hand distance from the central mark,
D_r = " right- " " " " " "
F = the reading, R, of balance with whole load, *minus b*;

[1] The weights in this exercise are taken in ounces, instead of in grams, because smaller numerical values can thus be used.

W_l	D_l	W_r	D_r	R − b = F	$W_l + W_r$	$W_l \times D_l$	$W_r \times D_r$
1+7=8	14.0	1+7=8	...	(..)−(..)=..	16	112	.....
1+5=6	14.0	1+8=9	...	(..)−(..)=..	.....	.....	.....
1+3=4	14.0	1+9=10	...	(..)−(..)=..	.....	.....	.....

Continue the tests and make the record according to the indications given in the table.

By studying the record table find,—

1. How F compares with $W_l + W_r$,
2. How to compute either D when the other D and both W's are known,
3. " " " " W " " " W " " D's " "

Second Part: Three Loads.—Use three pans, one on the left and two on the right arm of the lever. Make tests and record the results according to the indications of the following table, where

$F = R - b$, as before; $S = W_l + W_r + W_{r'}$;
$M_l = W_l \times D_l$; $M_r = (W_r \times D_r) + (W_{r'} \times D_{r'})$.

W_l	D_l	W_r	D_r	W_r'	D_r'	F	S	M_l	M_r
1+5=6	14.0	1+4=5	8.0	1+2=3	...	...	14	84	....
"	"	1+3=4	7.0	1+3=4	...	...	"	"	....
"	"	1+2=3	6.0	1+4=5	...	...	"	"	....

By studying this record table find,—

1. How F compares with $W_l + W_r + W_{r'}$,
2. How to compute any D when the other D's and the W's are known,
3. " " " " W " " " W's " " D's " "

EXERCISE 11.

CENTRE OF GRAVITY AND WEIGHT OF A LEVER.

Apparatus: A lever (No. 24), and a small block (No. 27), the two being fastened toegther, as in Fig. 11e, so as to make one body, *the whole of which will be called the lever* in this Exercise. A slender wooden cylinder (No. 9). A 1-oz. scale-pan (No. 25). A 1-oz. wt. and a 2-oz. wt. from No. 26.

To find the centre of gravity of the lever, balance it as nearly as you can, bar and block fastened together, in a horizontal position on the cylinder laid on the table (see Fig. 11e), the cylinder being kept at right angles with the lever. Find in this way at what particular mark of the bar the centre of gravity is, and record this mark—for example, 9.1 cm.—on a simple sketch of the apparatus.

Suspend the 1-oz. scale-pan, carrying no weight, from any convenient point near the free end of the bar, letting this end project beyond the edge of the table-top and balance the whole as now arranged, as nearly as you can, on the cylinder laid on the table as before (see Fig. 12e).

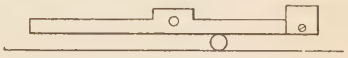


Fig. 11e.

Now record the mark from which the scale-pan hangs, 33.4 cm. let us suppose, and the mark which is just over the middle of the cylinder when the whole balances, 17.6 cm. let us say, on another very simple sketch of the apparatus.

We have in this case left-hand weight 1 oz.; left-hand distance (from fulcrum) = 33.4 − 17.6 = 15.8 cm.; right-hand weight = the weight of the lever = W oz. (unknown); right-hand distance = 17.6 − 9.1 = 8.5 cm., that is, the distance from the support in Fig. 12e to the centre of gravity of the lever (bar and block).

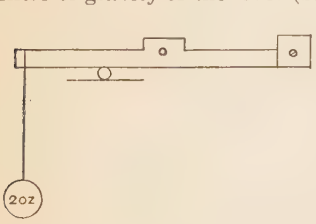


Fig. 12e.

We do not yet know W, but we should now be able to find its amount by calculation. We have $1 \times 15.8 = W \times 8.5$. That is $W =$ 1.86 oz., nearly, from the data here assumed.

Repeat the experiment, using now the pan with a load of 1 oz., 2 oz. in all, making the record, with a sketch, as before, and work out the value of W from the new set of data.

Do the same with a load of 2 oz. in the pan, 3 oz. in all. Make the record and calculation as before.

Compare the three values of W thus found, take their mean and reduce this value to grams.

Compare this mean value with the weight of the bar and block as found by direct use of a balance reading to 1 gm.

EXERCISE 12.

THREE FORCES IN ONE PLANE, ALL APPLIED AT ONE POINT: PARALLELOGRAM OF FORCES.

Apparatus: Three 500 gm. spring balances (No. 3). The rectangular block (No. 5) and a straight-edge ruler (No. 28). A sheet of paper about 30 cm. square. Thread and string. Three bricks, or better, iron weights about as heavy. A measuring stick (No. 2).

Arrange the apparatus on a table, according to the indications of Fig. 13e. The threads connecting the balance hooks should be long

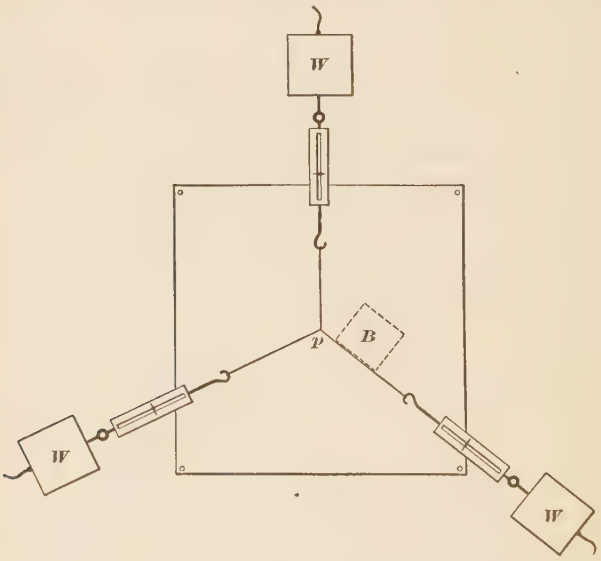


Fig. 13e.

enough to allow the block B to come between each hook and the central meeting point, p, without touching any knot. This point, p, should be near the centre of the sheet of paper, which should be held in some way to prevent slipping. Each balance hould indicate a pull of not less than 150 gm., or more than 300 gm., and the slot in each balance case should be in line with the thread leading from its hook, to prevent friction within the balance. Heavy weights, W, on strings

attached to the rings, hold the balances in place under pull, leaving the experimenter's hands free.

Record now on the paper the direction of pull and the amount of pull of each balance. To record the direction, place one side of the block *B* alongside one branch of the thread, as in Fig. 13e, taking care not to push the thread out of place, and then run the point of a well sharpened pencil along the edge of the block under the thread. Draw the other two lines in the same way, all carefully.

Alongside each mark thus made write the pull indicated by the corresponding spring balance, putting the record into such a form as to make sure that the "zero-point[1] error" of the balance in its horizontal position is allowed for. For example,

160 gm. (actual *reading*) +20 gm. (zero-point error).

If more than one experimenter works with this one set of apparatus, each in turn should make a set of lines and record alongside them the proper forces. The directions of the pulls should be varied somewhat by each new experimenter, in order that his case may not be exactly like that of any one else.

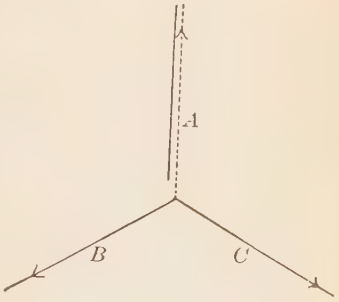


Fig. 14e.

Take now the wooden ruler (not the measuring-stick, which is usually not quite straight) and extend the three direction lines toward each other till they meet at one point. This they will do if they have been drawn originally just *under* the threads. If they do not all meet at one point, a new line should be drawn parallel to one of them in such a position as to pass through the crossing of the other two lines, and this new line, the dotted line in Fig. 14c, is then to be used in place of the original line. The three lines as now drawn will represent accurately the *directions* of the three forces.

Now measure off from the common point along the line *A* a distance of 1 cm. for each 30 gm. of the force which was exerted along

[1] To find this correction, lay the balance flat on its back, with the hook free, and tap the case lightly till the spring contracts or extends as much as it will; then see how far the index is *behind* the zero mark. This distance, interpreted in grams, gives the "zero-point error."

that line, and put a small arrow-head (see Fig. 14e) at the end of this measured distance. Erase that part of line A which lies beyond the arrow-head.

Do the same with lines B and C that has been done with A. The three arrows thus obtained, all reaching from the same point, represent the magnitude and the direction of the three forces exerted by the spring balances.

With A and B of Fig. 14e as two of the sides draw a parallelogram, taking pains to make it accurate.[1] Next make a parallelogram with B and C as sides, then one with A and C as sides. See Fig. 69

From the central point draw a dotted line, equal in length to A but exactly opposite in direction, and mark this as A'; another, equal and opposite to B, and mark it B'; another, equal and opposite to C, and mark it C'.

How closely does A' correspond to the diagonal of the parallelogram BC? B' to the diagonal of AC? C' to the diagonal of AB?

EXERCISE 13.

FOUR FORCES AT RIGHT ANGLES IN ONE PLANE.

Apparatus: Four spring balances (No. 29). No. 30. Four beds (No. 31) for the balances. Hard-twisted cord capable of holding at least 30 lbs., but not over bulky. Rest the board on four steel bicycle balls, well apart.

Using the balances carefully, and having the iron pegs set well down into the holes, so that the strings must be near the surface of the board, find by experiment a case of equilibrium, with one of the forces acting north,[2] one south, one east, and one west, no two strings lying in the same line.

Have the distance between parallel lines of force not less than three spaces and use no force less than 10 lbs.

Record this case of equilibrium thus:

Hole	Force	Direction	
. . . .	. . . .	. . . .	} Equilibrium.
. . . .	. . . .	. . . .	
. . . .	. . . .	. . . .	
. . . .	. . . .	. . . .	

[1] To draw a line parallel to A, for instance, place one edge of the block on A and then guide it to a new position at any desired distance from A by sliding it along the edge of a ruler at right angles with A.

[2] Call the 1–7 line on the board west to east; call the 1–43 line on the board north to south.

Find and record at least two other cases of equilibrium, varying the arrangements as much as practicable, but observing the conditions above mentioned.

Compare the magnitudes of the forces in each of these cases of equilibrium. Is there any fixed relation between the magnitude of the force north and that of the simultaneous force south? between that of the force east and that of the force west? between that of the force north and that of the force east? etc.

Calculate, in at least one of the cases of equilibrium, the moments of the various forces with respect to some one of the pegs used, and compare the sum of the positive moments with the sum of the negative moments. In this calculation use the mean of the N and S forces instead of these forces singly and the mean of the E and W forces instead of these forces singly. Do the same with respect to each of the other pegs used in this case of equilibrium, and with respect to one or two other points on the board. State carefully any general conclusion inferred from the Exercise.

EXERCISE 14.

ELASTICITY: BENDING.

Apparatus: Articles 32a, 32b, 33, 34, 35, 36, 37.

First Case.—Mark lightly two points on the narrow rod 100 cm. apart and mark the point midway between them. Support the rod on two of the wooden prisms placed 100 cm. apart (see Fig. 15e), each prism being just under one of the end marks on the rod, the central mark on the rod being just over one of the holes in the table top.

Suspend the scale pan from the middle of the rod, so that it may hang beneath the table top, taking care to prevent the suspending string from touching the table.

Near one end of the thin slip of wood (No. 34) make a light cross-mark, and call this mark 1. Make a similar mark 5 cm. from the first, and call this second mark 2. Make a third similar mark 25 cm. from mark 2, and call this last mark 3. This slip of wood is to be used as an index.

Place the third prism parallel to the rod and opposite the middle, and rest the index on the top ridge of the prism as a fulcrum, the mark 2 being exactly over this ridge and mark 1 being exactly under and *touching* the nearer edge of the rod. Place the upright scale

alongside the index, so that the face of the scale, upon which the divisions are marked, shall be just as distant from the rod as the mark 3 is.

It is now evident that, if the centre of the rod is pushed down any distance, the height of the point at which the index passes the face of the scale will be raised five times this distance.

Everything being carefully adjusted,[1] read the point on the scale where the upper edge of the index passes it, and record this reading in the third column of a table, made like that below, under the heading "Readings without load."

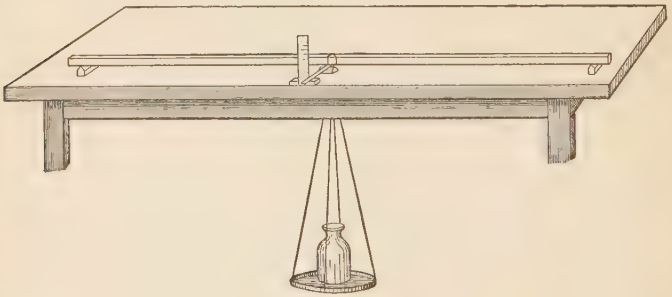


FIG. 15e.

Then place 100 gm. in the pan, watching the apparatus to see that nothing is deranged in this operation; take the reading, and record it in the column headed "Readings with load," placing alongside in the column headed "Load" the number 100. Then carefully remove the weight and make another reading, to be recorded in the column "without load."[2]

Then put 200 gm. in the pan, read and record as before, and make

[1] The rod is sometimes warped so that it will not rest fairly on the prisms at the ends. In this case one end of one of these prisms should be elevated as much as may be necessary to give a good bearing at both ends of the rod. Thin flat slips of card-board serve very well for making this adjustment.

Sometimes a rod has such a bend that its centre is considerably higher than the fulcrum, and the index does not touch the front *edge* of the rod. In such a case the fulcrum should be raised by means of the card-board slips.

[2] Slight differences may be expected in the "Readings without load," but these differences should not be as great as 0.1 cm.

another reading "without load." Then take 300 gm. and 400 gm. in turn.

In this way the observations are made and recorded in the first three columns of the table. The other columns explain themselves. The last column, which shows how much the rod bends per gram in each case, should be particularly noted.

Load	Readings with load	Readings without load	Mean	Rise of pointer on scale	Deflection of rod	Deflection per gm.
gm.	cm.	cm.	cm.	cm.	cm.	cm.
		3.00				
100	3.40		3.00	$0.40 \div 5 = 0.080$		0.00080
		3.00				
200	3.75		2.98	0.77	0.154	0.00077
		2.96				
...	....		....	....	.....	.....
		....				
...	....		....	.....	.....	.....
		....				
					Mean	.....

Second Case.—Use the narrow rod with supports 50 cm. apart. Use first 1000 gm., and then double this load, following the same method of procedure and record as in the First Case.

Third Case.—Use the broad rod on its broad side with supports 100 cm. apart. Use 400 gm., and then double this load.

Fourth Case.—Use the broad rod on edge with supports 100 cm. apart. Use 1000 gm., and then double this load.

Find for each case the mean deflection per gram.

EXERCISE 15.

FRICTION BETWEEN SOLID BODIES.

Apparatus: A spring balance (No 3). A rectangular block (No. 5). Set of weights (No. 26). A flat board (No. 38) about 15 cm. wide and 50 cm. long covered on one side with paper for the block to slide on. Some means of raising one end of the board and adjusting it so that the block will just slide down it; another block similar to the one which slides, or any similar object, will do for this purpose. A 30-cm. measuring stick. Thread.

First Part, Surface Horizontal.—*Consider first the velocity of the motion; that is, find whether the force required to keep up a slow steady motion is greater or less than that required to keep up a more rapid steady motion.* To do this:

Lay the block on one of its broad sides and attach it to the spring-balance by a thread passing around but not under the block. Load the block with weights until the force required to maintain a slow steady motion is about 3 oz. Draw the block parallel to its *grain* along the paper-covered board, laid horizontal, several times with a very slow steady motion, and then several times with an equally steady motion two or three times as fast. (As the paper is likely to grow somewhat smoother under the repeated rubbing, the experimenter should not make all his slow trials first, but should change from slow to fast and fast to slow a number of times.)

Record your conclusions as to whether the slow or the more rapid motion requires the greater force.

Next try to find out whether, the total weight being the same as before, it is easier or harder to draw the block on a narrow side than on a broad side. To do this:

Use the same block and the same load of weights, pulling it now, as before, parallel to its grain. (The side upon which the block slides should in all cases be clean, and the broad and narrow sides which are compared should be, as nearly as practicable, equally smooth. The thread must not be between the rubbing surfaces in any case.)

Record your conclusion as to whether the broad side or the narrow side offers the greater resistance to the motion.

Finally, ask what connection there is between the total mass drawn and the force required to draw it.

For this purpose vary the weights placed upon the block, using not less than 6 oz. for the least, and as much as 16 oz. for the greatest, load.

It is well to begin with the lightest load, proceed in regular order to the heaviest, then go back in exactly the reverse order to the lightest, recording both trials made with each load and taking the mean of the two for final study.

Add to the load in each case the weight of the block itself, and make the record in the following form, W being the load, b the weight of the block, and F the friction, that is, the force required to maintain steady motion:

$W + b$	F	Mean F	$F \div (W + b)$
6 + ..	.... →	....	.....
10 + ..	.... →	....	.....
16 + ..	.... →	....	.....
10 + ..	....		
6 + ..	....	Mean	.....

The ratio $F \div (W + b)$ is called the *coefficient of friction.*

Second Part, Surface Inclined.—Place one end of the board on the table and the other on the support. Vary the steepness of the board by varying the position of the support, until such an inclination is found that the block, once started slowly, will barely continue in motion down that part of the board, or that part of the paper, which was used in the First Part.

Then lay off on the table beneath the board a distance of 30 cm., measured from the edge where the board rests upon the table, and from the end of this line measure H, the vertical distance up to the under side of the board. Make the record of these distances on a diagram in the note book. The *coefficient of friction* will be $H \div 30$.

Compare the value thus found with the mean value found in the First Part.

EXERCISE 16.

COMPARISON OF MASSES BY THE ACCELERATION TEST.

Apparatus: Nos. 39, 40, 41. Lead scraps for one of the carriages. String.

Connect the free ends of the rubber tubes by means of a string about 20 cm. long.

Place the two planks side by side and incline each one until one of the cars,[1] when started very slowly down it, will continue in motion from the upper end half way down without gaining or losing velocity.

[1]It is quite likely that the two cars will require different inclinations, but with a good car on a good plank the inclination need not be greater than 1 in 50. The object of this adjustment is to make the inclination of the planks compensate for the friction of the cars, so that we may proceed with the test as if using frictionless cars on a level surface. The bearings of the cars should be oiled; but if too much oil is used, it is likely to come into contact with the rubber tubes and spoil them.

The Acceleration Test.—Load one car with four of the iron weights, the other with an unknown amount of lead. Place each car on its proper incline, put the string around the two pulleys and draw each car to the top of its incline until its rear wheels touch the cleat, all as in Fig. 16e. Make a pencil mark straight across both planks a little above where the front wheels of the cars will be when the tubes cease to pull.

Let one person release both cars at the same instant, and let another watch sharply to see which car is ahead when it reaches the pencil mark.

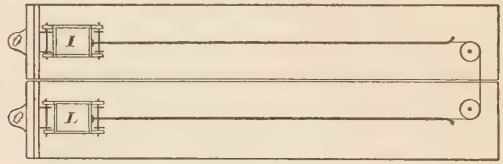


Fig. 16e.

Then increase or decrease the load of lead, as the case seems to require, and repeat the test and readjustment of load several times, very carefully at the last, until no difference can be detected in the time of arrival of the two carriages at the pencil mark.

The Weight Test.—Weigh the carriages with their loads, as finally adjusted in the acceleration test, and thus see how accurate the work of that test has been.

Repeat both tests, with only *three* iron weights in one car; then with only two.

EXERCISE 17.[1]

MEASUREMENT OF GRAVITY ACCELERATION, g.

Apparatus: No. 42 and something to serve the purpose of the stand s in Fig. 18e. Carbon paper, and rubber bands for fastening it to the pendulum rod.

Hang the pendulum P Fig. 17e on its supporting pin p. Suspend the ball b by means of a silk thread passing over the eccentrically

[1] This exercise is a modification of one devised by the late Doctor Harold Whiting, which was described in the first edition of the Harvard *Descriptive List of Elementary Exercises in Physics.* The original method used a ball and thread pendulum and required the vertically falling ball to collide with the pendulum ball, a rather difficult condition. I am indebted to Mr. C. H. Andrews of the South High School, Worcester, Mass., for the suggestion of a rod-pendulum and carbon paper. E. H. H.

mounted disk C and turn C until b hanging at rest is barely clear of P. Test the verticality of P, by lowering b to the position b' and noting whether in this position also it hangs barely clear of the rod. If it does not, turn the eccentrically mounted disk d to the right or the left, thus changing slightly the position of the centre of gravity of the pendulum, until the ball in position b' does hang barely clear of the rod.

Cover the lowest 20 cm. of the face of the rod next b with white paper well fastened on, and over this secure, by means of rubber

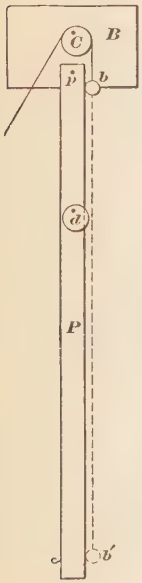


Fig. 17e.

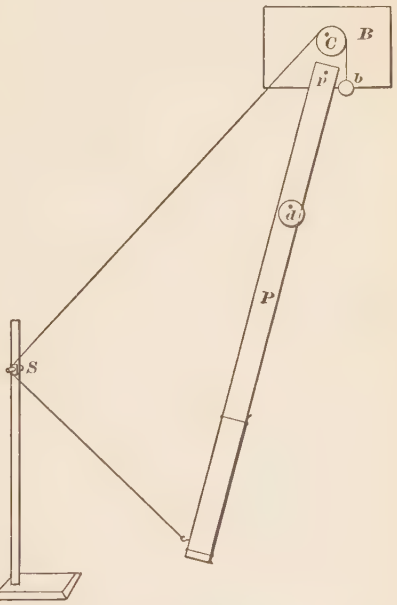


Fig. 18e.

bands at the top and bottom, a strip of carbon paper with the black side inward. Then arrange the apparatus according to Fig. 18e, make note of the height of the centre of b in relation to the lower edge of the supporting bar B, then burn off the thread t at a point just to the left of the eccentric C, thus releasing b and P at the same time. The pendulum, while passing swiftly through its middle position, will strike the falling ball through the carbon paper, and a

black dot made on the white paper beneath will show the point of contact.

Measure from the middle of this dot to the position of the middle of the ball just before its release and call this distance s. Repeat the fall experiment several times and find the mean value of s.

Pull the pendulum back to the position shown in Fig. 18e, then release it and count the number of single swings it makes in 30 seconds. This should be done several times, and the mean number of swings found should be used in calculating the time of a single *half-swing;* for only a half swing of the pendulum occurs between its release and its collision with the falling ball. Call this time t.

From s and t find g.

EXERCISE 18.

ACTION AND REACTION IN CASES OF COLLISION.

(For two or three working together.)

Preliminary Discussion.

We do not in this Exercise find the actual velocities of the ivory balls used or the actual momenta of these balls, as this operation would be too difficult. We make use of *relative* velocities and *relative* momenta, that is, quantities which bear the same ratio to each other that the actual velocities and the actual momenta bear, and this is all we need for the test we have in mind, which is to find whether the total momentum of the two balls is the same just before their collision as just after their collision.

For example, suppose that in a certain case we are told not the real momenta, M_1, M_2, M_1', M_2', of two bodies before and after their collision, but the relative momenta, M_1k, M_2k, $M_1'k$, and $M_2'k$, k being the same quantity throughout. We can tell whether

$$\begin{array}{ccc} \text{Momentum before} & & \text{Momentum after} \\ M_1+M_2 & = & M_1'+M_2', \end{array}$$

even when we do not know the value of k. For if

$$M_1k + M_2k = M_1'k + M_2'k,$$

it must be that

$$M_1 + M_2 = M_1' + M_2'.$$

In getting the ratios of the velocities we shall make use of the fact, which can be shown by experiment, that, if two pendulums of equal length are started at the same instant, one at the end of an arc of

length l, the other at the end of an arc of length nl, where n may be any convenient number, 2, 3, 4, etc., neither arc being more than a small fraction of the length of the pendulum, the velocity of the second pendulum will be, at any instant of its swing, nearly n times as great as the velocity of the other pendulum at the same instant. Accordingly, if two pendulums of equal length are suspended in such a way that the balls, A and B, barely touch each other, when each is hanging at rest in its natural position, then, if these balls meet in this position after one has made a swing of length D_a, while the other has made an opposite swing of length D_b, the velocities with which they meet may properly be called kD_a and kD_b, respectively, provided both D_a and D_b are small compared with the length of each pendulum. This will hold even if one of the D's is zero, that is, even if one of the balls is at rest when hit by the other. It is true that we do not know how great k is, but that is of no consequence, provided it has the same size for all the D's.

But we need also a measure of the velocity of each ball when it separates from the other at the end of collision, which lasts so short a time that each ball may be considered as at its zero position all through the collision. Now a pendulum swinging freely, without resistance or collision from end to end of its arc, travels the same distance from its zero position in both directions. Accordingly, the distance a pendulum swings from its zero position may be taken as a measure of the velocity with which it leaves that position. If, then, d_a is the distance A swings after collision and d_b the distance B swings after collision, we can take kd_a and kd_b as the respective velocities with which the two part company at the end of the collision, the k here having the same size as before.

The four velocities we have to consider, then, are kD_a, kD_b, kd_a and kd_b; but, as we are concerned with their relative size only, we may as well let D_a, D_b, d_a and d_b represent these velocities.

The Exercise.

Apparatus: All the articles of No. 43, the beveled suspension-board being attached to the highest[1] convenient support. About 20 gm. of putty.

Place the wires in slots 1 and 2 (Fig. 19e), so that the suspended

[1] The distances numerically given in this Exercise are suited to a suspension 2m. long; but it would be well to use a suspension twice as long and double the horizontal distances accordingly. No distinction is here made between the arcs proper of the swings and the horizontal reach of these arcs. The numerical difference is slight.

balls, will barely touch each other, with their centres about 4 cm. above the graduated rod.

Place the detents about 50 cm. apart, and so adjust the base-board that both of the suspension wires shall lie in the line of sight through the middle of the slots of the uprights (see Fig. 20e), the point of contact of the balls being exactly over the middle point, the 50-cm. mark, of the base board.[1]

Call the large ball A, the small ball B; the mass of the large ball, m_a; the mass of the small ball, m_b. Put A on the side toward the smaller numbers on the base-board. See Fig. 21e.

Call the distance moved by A before collision D_a; the distance moved by B before collision, D_b; the distance moved by A after collision, d_a; the distance moved by B after collision, d_b. Call movements from small to large numbers on the scale $+$; movements from large to small on the scale, $-$.

Find the movements of each ball by watching that side of each which is turned toward the other ball.

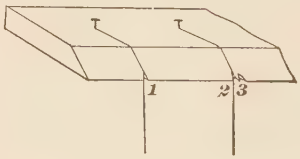


Fig. 19e.

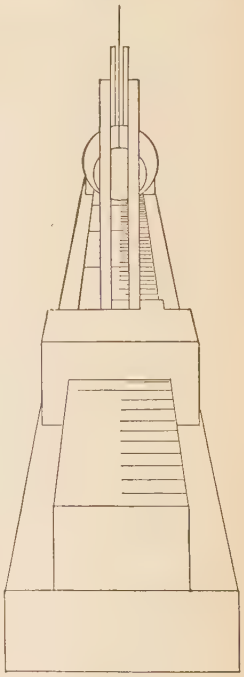


Fig. 20e.

First Case.—Make $D_a = O$; that is, leave A in its position of rest. Make $D_b = -30$ cm.; that is, place the detent in such a position that, when B is released, it will have to move 30 cm., measured horizontally from large numbers toward smaller numbers on the scale, before it can strike A, and record by means of a sketch like Fig. 21e.

[1] The adjustment of the base board is very important, and there should be some means of fastening it in place after the adjustment is made.

After making sure that A is at rest, as nearly as may be, release B suddenly but quietly, and let it swing against A. Notice carefully over what part of the scale[1] the B-ward side of A makes its first stop after the collision and over what point the A-ward side of B makes its

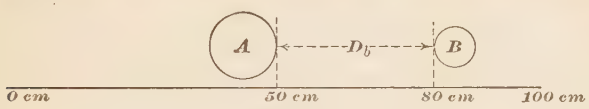


FIG. 21e.

first stop after the collision, and mark each of these readings on the scale line in a sketch like Fig. 22e.

Repeat this experiment several times, and finally, having decided upon the values of d_a and d_b, see whether this equation holds true:[2]

Representing momentum before collision.		Representing momentum after collision.
$(m_a \times D_a) + (m_b \times D_b)$	=	$(m_a \times d_a) + (m_b \times d_b)$.

In this equation test it should be borne in mind that a pendulum does suffer some slight loss of motion from the resistance of the air. As the momentum just before collision is here estimated from the

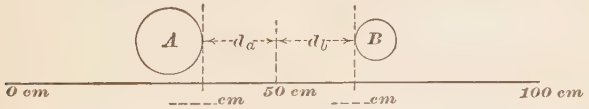


FIG. 22e.

length of swing preceding contact, and the momentum just after collision from the swing following contact, it is evident that the former estimate will be a little larger, and the latter a little smaller, than it should be. A rough estimate of the error caused in this way can be obtained by studying the rate of decrease of the pendulum-swings.

[1] It is well in making this observation to have a line of sight between two vertical lines, one of which may be a thread stretched across a hole and the other a pencil-mark (Fig. 23e). The uprights carrying these lines are fixed to a base which is movable along the base-board and is perforated to give a view of the scale.

[2] No account is here taken of the weight of the suspending wires. If it seems worth while to consider them, one-half the weight of its own suspending wire may be added to the weight of each ball. As the ball A will necessarily move toward the small numbers on the scale after the collision, d_a will be a *minus* quantity in this equation; d_b will be *plus*, if B rebounds.

Second Case.—Make $D_a = 15$ cm. and $D_b = 0$. Determine d_a (the onward movement of A after the collision) and d_b by several trials, making a graphical record as in Figs. 21e and 22e, and then compare the total momentum before collision with the total momentum after collision, as in the preceding Case.

Fig. 23e.

Third Case.—Make $D_a = 15$ cm. and $D_b = -15$ cm. Release both the balls *at the same instant.* (This must be done by one person. It is well to attach a string to the upright of each detent, as in Fig. 24e, pass the middle of the string through a hole or screw-eye near the centre of the base-board, and, when all is ready, pull suddenly upon this part of the string, taking care not to derange the base board.)

Determine d_a and d_b by several trials, making a record as before, and then compare the total momentum before collision with the total momentum after collision, as in the preceding Cases.

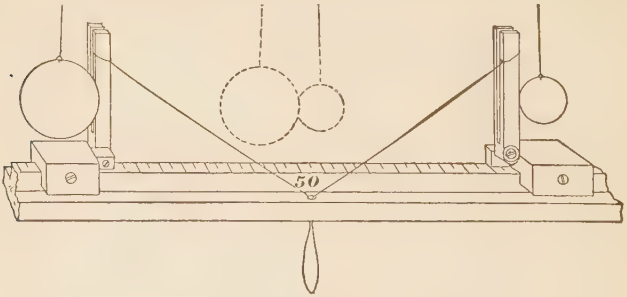


Fig. 24e.

Fourth Case.—Let the suspending wires hang in slots 1 and 3 (Fig. 19e). Put around the smaller ball a belt of putty about 1 cm. wide and 0.3 cm. thick, so that this belt will just touch the large ball when both are hanging at rest (Fig. 25e). (The object of the putty is to destroy the elasticity of the collision, in order that the conditions of the momentum-test in this Case may be very different from those of the other Cases. It will do little harm if the balls stick together at collision, though this may be prevented by covering the putty with thin paper at the point of contact.)

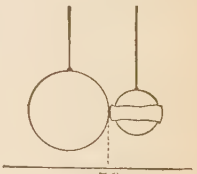


Fig. 25e.

Make $D_a = 30$ cm. and $D_b = 0$ cm. Determine d_a and d_b by several trials, making the record as usual, and then apply the momentum test as before, taking for m_b the mass of the ball B and its belt of putty.

EXERCISE 19.

TESTING A MERCURY THERMOMETER.

Apparatus: Nos. 17, 44, 45, 46, and 47. Snow or ice. A short rubber tube and a pinch cock.

Examine the thermometer, holding it all the time bulb downward, to see whether any mercury has separated from the column and lodged at the top of the stem If any is so lodged, it can probably be freed by grasping the thermometer near the middle of the stem (bulb outward) and swinging it rapidly back and forth, the arm being held at full length downward.

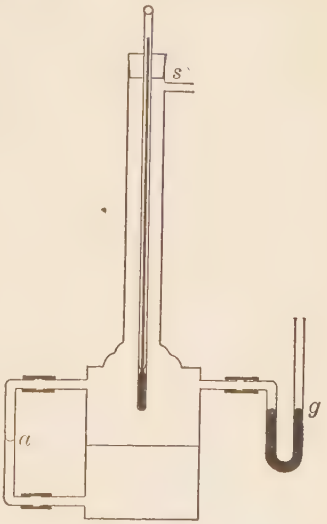


Fig. 26e.

To test the freezing-point: Break the ice into fine pieces, the finer the better, and fill the dipper with it. Add only enough water to fill the space between the lumps of ice. Thrust the bulb of the thermometer down into the middle of the cup until the point marked 0° is only 1 or 2 mm. above the surface of the water. Give the mercury-column time to descend as far as it will, and then record the reading. (If its shortest divisions are a millimeter or more in length, try to read to tenths of these divisions. Place the eye in such a position that a straight line drawn from its centre would strike the thermometer at right angles at the top of the mercury column.)

To test the boiling-point: Fill the boiler (Fig. 26e) with water to the depth of 3 or 4 cm., screw the top on firmly, attach the small mercury pressure-gauge, g, and thrust the bulb and stem of the thermometer

down through the perforated stopper until the point marked 100° is not more than 2 or 3 mm. above the top.

Boil[1] the water and keep it boiling, the steam escaping from the side tube, *s*, near the top, until the mercury column in the thermometer rises as far as it will, and then record the reading. (The flame of the burner beneath the boiler should not be allowed to flare out beyond the bottom, lest the current of hot gases rising from it should overheat the rubber connection of the water gauge, *a*, on the side of the boiler. It is well to wrap this connection with asbestos paper.)

Record the reading of the barometer.

To test the effect of increased pressure upon the boiling temperature: Nearly close the steam outlet by means of a short rubber tube and a pinch cock, the flame still burning, and watch the thermometer and the pressure gauge, *g*. Adjust the pinch cock at the outlet so that the pressure in the boiler, as shown by the gauge, shall be 4 of 5 cm. of mercury greater than the atmospheric pressure. When everything is satisfactory, record both the rise of temperature and the rise of pressure which have occurred.

Calculate, from boiling-point data now obtained, (*a*) how great a rise of temperature here[2] corresponds to a rise of 1 cm. in pressure, (*b*) what the reading of the thermometer would be[3] in steam at 76 cm. of pressure, the standard condition.

The difference between 100° and the reading found under, (*b*) is the "boiling-point error" of the thermometer.

To test the effect of leaving the stem of the thermometer exposed to the air while the bulb is heated: Give the steam free escape once more, and then, after noting the reading again, draw the thermometer up through the stopper until only the bulb and a few degrees of the stem are exposed to the steam. Watch and record the effect upon the reading as the stem cools.

Finally, if time permits, test the freezing-point again, taking care, as before, to have the snow or ice reach the bottom of the dipper. It is not sufficient to have snow or ice *floating* in the water.

[1] To save time, it is well to begin heating the water before making the freezing-point test. The water gauge should be looked at frequently during every experiment in which the boiler is used, and the water should not be allowed to fall very low in it.

[2] At very different pressures and temperatures the ratio between rise of temperature and rise of pressure would be very different from the ratio here found.

[3] This assumes, what is usually true, that the barometric pressure at the time of the experiment is not exactly 76.

The operation of determining the freezing-point and boiling-point is profitable for the student on account of the light it throws on the theory of the thermometer, but the inaccuracies detected in the operation will not, unless they are greater than 1°, be serious in the heat experiments of this course. It will be important, however, not to exchange thermometers during such experiments as require careful observation of small changes of temperature.

EXERCISE 20.

LINEAR EXPANSION OF A SOLID.

Apparatus: Nos. 44 (without the pressure gauge), 46, 47, 48, 49, 50. A barometer. A meter rod. A rubber tube 30 cm. long, to carry steam from the boiler to the heating tube; another, 15 cm. long, to carry the steam and water from the same tube (Fig. 27e). A small tumbler to catch the water thus drained off. (It is better not to place this tumbler on the wooden rack. Steam or water coming from it might wet some part of the rack and cause a swelling, in or near the back post, that would seriously affect the position of the index. A similar, though less, danger threatens if much water or steam escapes from the other end of the heating tube.)

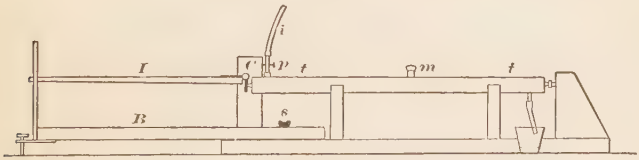


Fig. 27e.

Fill the boiler to a depth of 4 or 5 cm. with water and place the lighted Bunsen burner beneath it, all outlets of the boiler being closed except the lower side tube.

Raise the long arm of the index, *I* (Fig. 27e), and place the heating-tube, *tt*, the brass rod within it, in position on the rack, the unpointed end of the rod resting against the screw back-stop, and then gently lower the index until the short arm bears against the sharp point of the rod.

Attach the 30-cm. rubber tube to the inlet tube, *i*, making no connection as yet with the boiler, turn the heating tube in its bed till the inlet tube rests against a firm support, *p*; attach the outlet

rubber tube, leading to the tumbler; put the thermometer bulb through the middle side opening, *m*, into the steam cylinder alongside the rod.

Measure and record the length of the brass rod. This measurement need not be made with great care, as an error of 2 or 3 mm. here would make very little difference in the final result.

Measure, with about the same care, and record the length of the long arm of the index, from the centre of the pivot to the vertical scale.

Measure with much care and accuracy the vertical short arm of the index, from the centre of the pivot to the point where the tip of the brass rod bears against it. As the centre of the pivot is not marked, this measurement requires some contrivance. It is well to proceed as follows: Make a fine pencil mark on the edge of the vertical arm just on a level with the tip of the brass rod. Measure very carefully the distance from the base board of the apparatus up to this mark. See Fig. 28e. Then lift the horizontal arm of the index and turn it

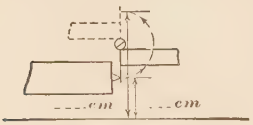


Fig. 28e.

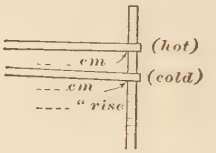


Fig. 29e.

back till it lies upon the heating-tube. The short arm of the index will now point upward, and the pencil mark which it carries will be as far above the centre of the pivot as it previously was below this centre. Measure very carefully the present distance from the base board to the mark on the index. One half the difference of the two measurements now made is the distance required, the effective length of the short arm of the lever index.

Read the thermometer, then remove it, and put a stopper in its place. There is no further use for the thermometer, as the higher temperature, that of the steam, can be found better from the barometric pressure. (See Exercise 19.)

Make sure that the brass rod still rests against the back-stop screw, then read, as accurately as you can, the position of the index, upper or lower edge according to convenience, upon the vertical scale. See Fig. 29e. (The levelling screw at the end of the bar *B*, Fig. 27e, which carries this scale, is for use in a subsequent Exercise. In the present

Exercise it should not be allowed to rest on the table. The scale should be brought close to, but not into contact with, the index before use, by turning *B* around the pin which reaches into the base board. *B* can then be fastened in the right position by means of the screw *s*.)

By this time a good flow of steam should be coming from the boiler. Slip the end of the inlet rubber tube over the vent of the boiler and let the steam take its course through the tube on the rack. See that there is a free escape of steam from the other end of this tube. Watch the index until it ceases to rise along the vertical scale, then read its position as carefully as before.

Read the barometer.

From the difference of the first and last readings upon the vertical scale we can, making use of the ratio of the index arms, calculate the increase of length which the rise of temperature has produced in the rod.

From the original length of the rod, the increase of its length, and the rise of temperature, is to be calculated the *coefficient of expansion of brass for* 1 *Centigrade degree*—that is, the fraction of itself by which the length of a bar of brass is increased when its temperature is raised 1 Centigrade degree. The calculation will assume that the expansion is equally great for each and every degree of the rise of temperature. This is not quite true. The quantity which the calculation gives is the average, or mean, value of the coefficient between the limits of temeperature used.

(It is well to do this Exercise more than once, repeating all the measurements. In order to do this properly within the time of a two hour laboratory period it will usually be necessary to cool the rod somewhat rapidly. This can be done by removing the heating tube from the rack and running cold water through it for a short time. This treatment is likely to leave the rod a little cooler than it was at the beginning, so that its expansion may be a very little greater at the second heating than it was at the first heating.)

EXERCISE 21.

BEHAVIOR OF A GAS HEATED, (*a*) AT CONSTANT VOLUME, (*b*) AT CONSTANT PRESSURE.

Apparatus: Nos. 44 (without the pressure gauge), 47, 49, 50*a*, 51, 52, 52*a*, and 53. A meter rod, a barometer, rubber stoppers and connecting tubes, a quantity of snow or pounded ice.

In handling the tube that contains the dry air and the mercury, keep the sealed branch nearly horizontal, do not lower the other branch enough to let any mercury flow out, and avoid shocks lest the mercury column be broken so as to include bubbles of air.

First Case.—*To find the increase of pressure with volume constant:* Fill the boiler with water to a depth of several centimeters, put on the top, close all the steam outlets except the lowest, and place the flame, not more than 8 cm. tall, beneath.

Fig. 30c.

Measure the distance from the brass ring r (Fig. 30c), touching the rubbber tube, to the sealed end of the bore of the glass tube. (If the sealed end of the bore *tapers*, do not measure to the extreme tip, but, as well as you can, to the spot where the bore would end if it stopped without tapering, but had the same total volume as now.) Call this distance D.

Push the sealed end through a short stopper into the cooling tray, t (Fig. 31c), and cover the whole length of the imprisoned air-column with snow, or with water kept at 0° C. by means of pounded ice. Place a spirit level on the platform which supports the outer glass tube and, by means of the screw s, level the platform. Adjust the outer glass tube, by means of the arm c, in such a way that the two ends of the mercury column shall be at the same height above the

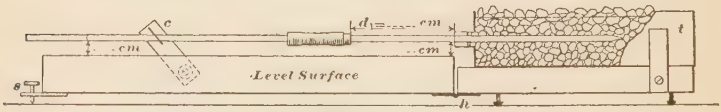


Fig. 31c.

leveled platform, pulling the sealed tube out through the stopper till the inner end of the mercury column is just visible. The imprisoned air will now be at atmospheric pressure, the two ends of the mercury column being at the same level (even though intervening parts of this column may be at a lower level). Leave the tube in this condition for a few minutes, readjusting slightly if need be, till contraction of the air column has ceased; then measure the

distance from the brass ring r (Fig. 30e) to the inner end of the mercury column. Call this distance d_1.

The difference, $D - d_1$, between the two measurements now recorded is to be taken as the length, l_o, of the imprisoned air-column at 0° C. under atmospheric pressure.

The barometer reading should now be noted. Call this p_o.

Take the tube from the cooling tray to the heating apparatus (Fig. 32e). Push the sealed end through the perforated rubber stopper in the heating tube till the length l_o has disappeared; that is, till the distance from the stopper to the brass ring at the elbow of the glass tube is, as before, equal to d_1. Rest the outer glass tube, turned high, on S.

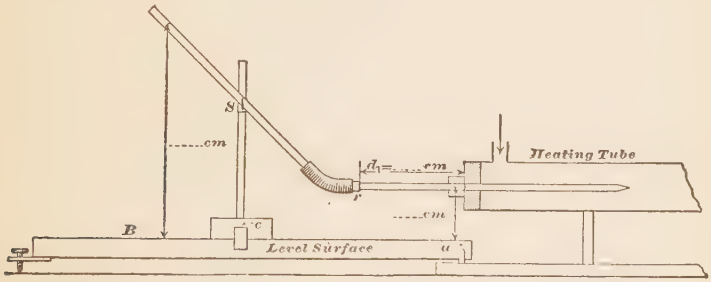


Fig. 32e.

Lead steam from the boiler through the heating tube, as in Exercise 20. While the apparatus is heating up, level the bar B.

After the steam has been flowing some minutes through the apparatus, raise or lower the outer glass tube, by raising or lowering S or by sliding c to the left or to the right, until the inner end of the mercury column is just at the outer end of the rubber stopper, thus making the length of the imprisoned air column l_o, just[1] what it was at 0° C. under atmospheric pressure.

After making sure that expansion of the imprisoned air has ceased, measure the difference in height of the two ends of the mercury column above the leveled bar B. Call this h. This difference represents the *increase* of pressure, $p_t - p_o$, required to keep the volume

[1] The expansion of the *glass*, very small compared with that of air, is neglected in this Exercise.

of the air the same at the temperature of steam as at the temperature of melting ice.

Calculate α_1, the *coefficient of expansion of pressure with constant volume*, thus:

$$\alpha_1 = (p_t - p_o) \div p_o(t - 0),$$

where p_o = pressure (atmospheric) upon the air at 0° C.,
p_t = total pressure upon the air at steam temperature,
t = temperature of steam (found from the barometer reading).

Second Case.—*To find the increase of volume with pressure constant.* Proceed exactly as in the First Case until the air tube is placed in the heating tube, then place and *keep* the two ends of the mercury column at the same level, the inner end being *kept* just at the outer surface of the rubber stopper, and, when expansion ceases, measure the distance from this end to the brass ring at the elbow. Call this distance d_2. See Fig. 33e.

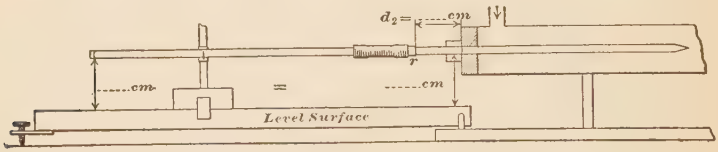


Fig. 33e.

It is evident that, when this measurement is made, the air is under the same pressure as when it was in the cooling tray. The increase of length of the air column, representing its increase of volume, is found from the measurements now recorded.

Calculate α_2, the *coefficient of expansion of volume with constant pressure*, thus:

$$\alpha_2 = (l_t - l_o) \div l_o(t - 0),$$

where l_o = length of the air column at 0° C.,
l_t = " " " " " " the steam temperature,
t = the temperature of steam.

EXERCISE 22.

SPECIFIC HEAT OF SHOT.

Apparatus: No. 4, 44 (without the top or the mercury gauge), 45, 46, 47, 54, 55. A piece of pasteboard to cover the top of the dipper.

Half fill the boiler with water and place a small flame beneath it. Adjust the balance and weigh the calorimeter.

Weigh out in the calorimeter about 500 gm. of shot, pour[1] them into the copper heating-dipper, place this dipper in the boiler, the bottom in the water, and cover it with the pasteboard. See Fig. 34e.

Weigh out in the calorimeter about 100 gm. of water, already cooled to a temperature 7 or 8 degrees below the temperature of the room.

Push the bulb of the thermometer through a hole in the pasteboard cover down into the shot, and note the rise of temperature, stirring[2] the shot frequently with the thermometer. When the temperature has finally ceased to rise, record it as t and withdraw the thermometer.

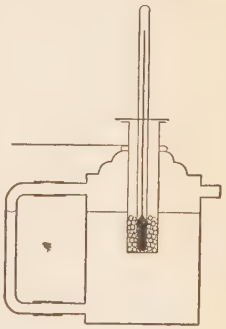


Fig. 34e.

After the thermometer has cooled 40 or 50 degrees in the air, place the bulb in the water within the calorimeter, stir this water very thoroughly and take its temperature with care, reading to 0.1 of a degree. Meanwhile the shot should be occasionally stirred with a pencil or other convenient instrument.

Remove the thermometer, take the dipper, still covered, from the boiler, pour the shot quickly into the calorimeter, put in the thermometer and stir the water and shot thoroughly until the temperature is the same in the shot as in the water above. Then read this temperature, t_m, and record it.

The reading t was probably not the true temperature, t_s, of the heated shot. (See Exercise 19). To find t_s proceed as follows: Take a reading, t', with the bulb only of the thermometer in the boiling water. Read the barometer and from its reading calculate t_b, the true temperature of the boiling water. Record t_b and t'. It may then be assumed that $t_s - t = t_b - t'$, or $t_s = t_b - (t' - t)$.

It may be assumed that the whole calorimeter rises in temperature with the water it contains, when the shot are poured in. The

[1] It is well to *practice* pouring the shot from the dipper into the calorimeter in order to acquire the art of doing it quickly and surely when there is need.

[2] If the common paper-scale thermometer is grasped as near the bulb as possible, it may be used rather vigorously as a stirrer, with little danger of breaking. It is, however, much more likely to be broken in stirring the shot dry than in stirring a mixture of shot and water.

"thermal capacity," or "water equivalent," of the calorimeter, if it is of brass, may be found with sufficient accuracy by multiplying its weight by 0.1, that being, approximately, the specific heat of brass.

Record and Calculation of Result.—Since the heat lost by the shot in cooling to the final temperature is gained by the water and the calorimeter in rising to the final temperature, it will be possible to state the losses and gains in the form of an equation:

Let m_w = mass of water;
m_s = mass of shot;
t_w = temperature of water before mixing with shot;
t_s = temperature of shot in heater;
t_m = temperature of mixture;
m_c = mass of calorimeter;
h_c = specific heat of material of calorimeter;
x = specific heat of shot.

Heat lost by shot $= m_s \times x \times (t_s - t_m)$.

Heat gained by water $= m_w \times 1 \times (t_m - t_w)$, the specific heat of water being 1.

Heat gained by the calorimeter $= m_c \times h_c \times (t_m - t_w)$, as this is supposed to rise in temperature just as much as the water does.

We now write the equation,

loss of heat by shot = gain of heat by water and calorimeter,

or $m_s \times x \times (t_s - t_m) = m_w \times (t_m - t_w) + m_c \times h_c \times (t_m - t_w)$.

From this we find $x = \dfrac{[m_w + (m_c \times h_c)] \times (t_m - t_w)}{m_s \times (t_s - t_m)}$.

EXERCISE 23.

LATENT HEAT OF MELTING ICE.

Apparatus: Nos. 4, 44 (without the top), 46, 47, 54, 56. About 150 gm. of ice in large clear lumps. (It is important to have the ice as dry as may be when it is put into the water. If it is pounded fine long before being used, the melting, which is going on continually at the surface of each small lump in a room at ordinary temperature, fills the spaces between with water, and serious error may result.) Placed in a canvas bag and struck smartly against any firm resisting surface, the large lumps can be quickly powdered when the ice is

needed. They should be kept in some convenient vessel—a saucer, for instance—until this time comes.

Half fill the boiler with water and place the flame beneath. Weigh the calorimeter. When the water in the boiler is near 50° C., pour about 200 gm. of it into the calorimeter and weigh it with an accuracy of 0.5 gm.

Immediately transfer the lumps of ice to the bag, taking care not to *pour* them in lest water should go with them, and pound them fine. Stir the water in the calorimeter thoroughly, take its temperature, and then put into it about two-thirds of the ice, avoiding the wetter portions. The weight of ice added need not be determined with accuracy at first, as it can be found by carefully weighing the calorimeter and its contents after the hurry is over.

Stir the water thoroughly, though not violently, with the thermometer, and record the temperature as soon as all the ice is melted. If so much ice has been put in as to cool the water below 5° C., it is well to dip out the ice remaining unmelted at that temperature, taking as little water with it as possible.

Weigh the calorimeter and its contents, in order to find more exactly the weight of ice added.

From the data now obtained the *latent heat of fusion* of ice is to be found, that is, the number of units of heat required to change 1 gm. of ice, taken at 0° C., into water at the same temperature. The thermal capacity of the calorimeter is to be taken into account in this Exercise as in the one preceding.

Record and Calculation of Result.—It must be noticed that the ice is first melted, and then the water which results from the melting is raised to the final temperature. The heat gained in these two operations must equal that lost by the hot water and the calorimeter which contains it.

Let m_c = mass of the calorimeter;
m_w = mass of water;
m_i = mass of ice;
t_w = temperature of hot water;
t_m = temperature of mixture;
x = latent heat of ice.

Call the specific heat of the brass calorimeter 0.1.

With these directions the student should be able to form the necessary equation, corresponding to, though not exactly like, that of Exercise 22, and to find from it the value of x, the quantity which is to be determined.

EXERCISE 24.

FINDING THE DEW-POINT.

Apparatus: Nos. 46 and 54. Ice, or snow, and salt.

Pour a little water at the ordinary temperature into the cup, and into this put the bulb of the thermometer. Gradually cool this water by small additions of ice-water or ice, and finally of salt if necessary, watching the outside of the lower part of the cup after each addition, and continually stirring its contents. As soon as a mist begins to form on the bright surface near the bottom of the vessel, note the temperature and cease adding ice.

It is likely that the temperature thus observed will be a little lower than necessary, a little below the true dew-point. Therefore, reverse the process, making small additions of water, taken at the room temperature, until the mist begins to disappear. Then take the temperature again. This will probably be a little higher than the dew point.

It is well to repeat the experiment. After one or two trials, the temperature found going down and that found coming up the scale should be pretty close together—not more than one or two degrees apart. The mean of the two may then be taken as the dew point.

The experimenter must avoid breathing upon the outside of the vessel or holding any damp object near it, while looking for the mist to form. If a thick layer of moisture forms upon the vessel before the reverse process is begun, it should be wiped off. A thin layer can be put on, when it is desired, by a mere breath.

Take the temperature of the air of the room, avoiding the neighborhood of any exceptionally warm or exceptionally cold body. Make a note of the out-of-door weather conditions.

From the dew point and the air temperature find by means of the following table the *relative humidity* of the air of the room, assuming that Boyle's holds, that is, taking the weight of vapor in a given volume at a given temperature as proportional to its pressure.

For example, if the dew point is 5° C., at which temperature the maximum vapor pressure is 0.653 cm., while the temperature of the air of the room as a whole is 15° C., at which temperature the maximum vapor pressure is 1.273 cm., the relative humidity of the air is $0.653 \div 1.273 = 0.51$.

PRESSURE OF SATURATED WATER VAPOR.

(*The pressures given for temperatures below* 0° C. *are those which hold in the presence of ice.*)

Temp.	Pressure (cm. of mercury)	Temp.	Pressure. (cm. of mercury)
−15° C	0.126	10° C	0.918
−10° C	0.197	15° C	1.273
− 5° C	0.303	20° C	1.741
0° C	0.458	25° C	2.355
5° C	0.653	30° C	3.156

EXERCISE 25.

LATENT HEAT OF VAPORIZATION.

Apparatus: Nos. 4, 44, 46, 47, 54, and 57. Snow or ice.

Fill the boiler to a depth of about 5 cm. with water, screw on the top, close the two apertures at the top, place a Bunsen flame beneath and attach the glass "trap," *T*, to the side tube, as in Fig. 35e. To prevent the trap from filling with water from condensation before use, turn it upward and fasten it with a bit of wire alongside the top of the boiler.

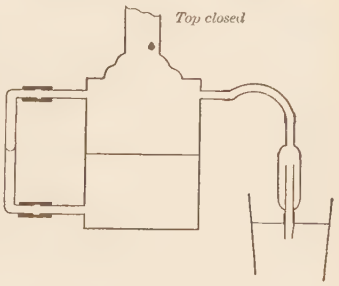


FIG. 35e.

Weigh the calorimeter, and then weigh in it with much care about 300 gm. of water, some fifteen degrees below the temperature of the room. See that the outside of the calorimeter is as dry as may be before the weighing, but after the weighing do not wipe it.

When there is a strong flow of steam through the trap, stir the cold water thoroughly, take and record its temperature, then dip the escape tube of the trap about 2 cm. beneath its surface. The vigorous flow of steam through the trap should continue, and be shown by a noisy and incessant collapse of bubbles in contact with the cold water.

Stir the water from top to bottom with the thermometer as the temperature rises, and when it is about 30 degrees warmer than it was at the start, pull the mouth of the trap quickly away from the

calorimeter, hang the trap up so that the steam from it shall not play on the table top, stir the water quickly and *thoroughly*, take its temperature, and then weigh it carefully without loss of time.

The increase of weight since the previous weighing will, if all has gone well, show the weight of steam condensed.

Read the barometer and find from its reading the temperature of the steam.

From the data thus obtained the *latent heat of vaporization*, which is equal to the amount of heat given out in the *condensation* (not including the subsequent cooling) of 1 gm. of steam is to be calculated.

Record and Calculation of Result:

Let m_c = mass of calorimeter;
m = " " cold water;
m_s = " " steam;
t_w = temperature of cold water;
t_m = " " mixture;
t_s = " " steam;
x = latent heat " "

Call the specific heat of the calorimeter 0.1.

Notice that there are two portions of heat yielded to the cold water, namely, that portion which proceeds from the *condensation* of the steam, and that which is derived from the resulting water, cooling from near 100°, to the final temperature.

With these directions the student should form an equation between the amount of heat given out on the one hand and that taken in on the other and from this equation find the value of x, the quantity to be determined.

The great difficulty of this Exercise—and it is a serious difficulty—is to find accurately the weight of steam condensed. This weight is not large, and an error of 1 gm. affects the result greatly.

If the trap is not used, a considerable amount of *water* trickles into the calorimeter, condensation having occurred in the tube. When the trap is used, a much more serious difficulty *may* be caused by an occasional sudden collapse of the steam in the trap, making a momentary partial vacuum there, into which water rushes back from the calorimeter. This accident is likely to occur if the mouth of the trap is plunged too far beneath the surface of the water, and is probably due to the cooling of the trap itself. It may be prevented by the use of a short rubber tube added to the exit tube of the trap.

Evaporation from the calorimeter may cause perceptible error if the condensation and subsequent weighing take a long time.

The thermometer, if removed several times from the calorimeter, will have carried off enough water to affect the result.

Water may spatter from the calorimeter during the condensation, if the vessel is too full or the tube dips too short a distance beneath the surface.

EXERCISE 26.

VELOCITY OF SOUND IN OPEN AIR.

Apparatus: A pendulum beating seconds. A small spy glass (No. 58). A board. A hammer or stone.

One experimenter strikes the board sharply just when the pendulum bob passes the middle point of its arc. Another places himself at the start about 900 ft. distant from the first, in a line at right angles with the plane in which the pendulum swings, and then, looking through his glass, seeks to place himself at such a distance that the stroke made when the pendulum bob passes through the middle point of its arc in one direction reaches his ear just when the bob passes through the middle point of its arc in the other direction. The distance which the sound travels in one second in one direction is thus roughly determined. As the wind may either help or hinder this movement, the conditions should afterward be reversed by having the experimenter with the glass strike the board, while the other stays at the pendulum and listens for the stroke, signalling to the striker to come nearer or go farther away until coincidence occurs as before. The mean of the two distances now found should be taken as the distance which sound would travel in one second in still air.

This method is not likely to give very accurate results. Its merit lies in its directness and simplicity. A still more direct and simple method is to place two persons a long distance apart, but in sight of each other, and have each in turn discharge a pistol or gun while the other notes, as well as he can, the number of seconds between the sight and the sound of the explosion.

EXERCISE 27.

WAVE LENGTH OF SOUND.

Apparatus: Articles 59 and 60.

Push the piston head along the tube until it is not more than 20 cm. from one end. Strike one prong of the tuning fork upon a piece

of soft wood and then hold it at this end of the tube. While the fork is sounding draw the piston head gradually away from the fork, listening for the rise and subsequent fall of the sound, which occurs when the air column attains and then exceeds the proper length. Find in this way, and mark, one effective position of the piston face.

Find in like manner a second effective position of the piston face, which will give a much longer air column than before.

The distance between the two positions of the piston will be equal to one half of a wave length.

Instead of using a piston in the cylinder, we can vary the length of the vibrating air column by holding the cylinder upright in water at a greater or less depth.

EXERCISE 28.

NUMBER OF VIBRATIONS PER SECOND OF A TUNING FORK.

Apparatus: Nos. 60 and 61.

The method of this Exercise is to draw the smoked glass along in a straight line beneath the style of the vibrating tuning fork, at

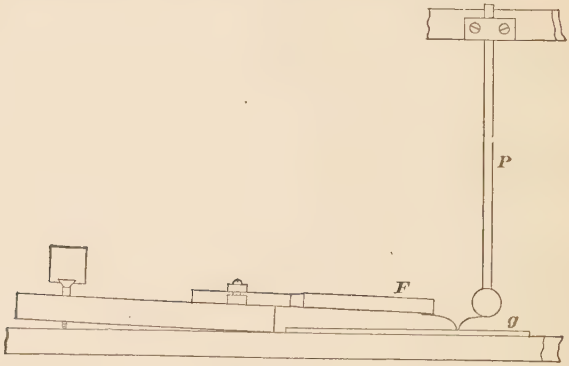


FIG. 36e.

right angles with the direction of the vibrations, thus tracing on the glass a sinuous curve, and simultaneously to cut this curve crosswise by means of traces made by the style of the pendulum.

Arrange the pendulum and fork so that the horizontal part of the pendulum motion shall be parallel to the motion of the fork prongs

and so that the two styles may, when at rest, be in the same vertical plane, with their points as near each other as practicable. See Fig. 36e.

Place the piece of smoked glass under the tuning fork and pendulum, and make such adjustments that the styles will bear lightly upon the glass. Set both pendulum and tuning fork into vibration, and then draw the glass quickly along beneath them in a direction at right angles to the vibrations.

Count the number of vibrations of the pendulum per minute, and find from the smoked plate the number of vibrations of the tuning-fork corresponding to one vibration of the pendulum. Then calculate the number of complete double vibrations of the tuning fork per second.

EXERCISE 29.

USE OF RUMFORD PHOTOMETER.

Apparatus: A 4 c. p. incandescent electric lamp (No. 62) and a common paraffin candle (No. 63). A card-board screen (No. 64). Any opaque rod about 1 cm. in diameter and 10 or 15 cm. tall, supported upright; *e. g.*, a Bunsen burner. A meter rod.

Arrange the apparatus according to the indications of Fig. 37e, in which L, the lamp, and C, the candle, are placed as near the median line on which stands the rod, R, as may be convenient.

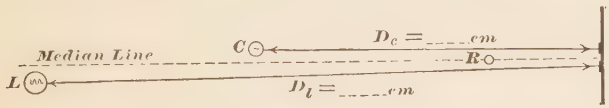


Fig. 37e.

Keep the lamp about 30 cm. farther from the screen than the candle is, and make further adjustments by moving the screen or the rod or both back and forth.

Taking position on the median line, and shielding the eyes from the direct light of the lamp and the candle, so adjust the screen, kept at right angles with the median line, and the rod, that the two shadows will be near each other, but not overlapping, and equally dark at their centres. Then measure the two distances D_c and D_l and record their values on a diagram like Fig. 37e.

Repeat the experiment, with the spiral of the lamp turned sidewise to the screen, make a new diagram and record the new D_c and D_l.

Making use of the *law of inverse square*, find and report the candle-power of the lamp in each of the two aspects described.

EXERCISE 30.

IMAGES IN A PLANE MIRROR.

Apparatus: A mirror (No. 65). A rectangular block (No. 5). Two strips of brass (No. 65a) to hold the mirror to the block. Two straight-edged wooden rulers (No. 28). A measuring stick (No. 2). A sheet of thin white paper about 30 cm. by 50 cm. A small block (No. 66).

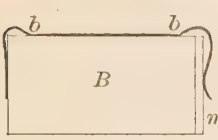


Fig. 38e.

Attach the mirror, *m*, to the large block, *B*, by means of the brass strips in the manner shown by Fig. 38e.

Draw a straight pencil mark across the sheet of paper at its middle, and set the *back* surface of the mirror directly over and parallel to this line, the middle of the mirror being very near the centre of the sheet. See Fig. 39e.

Draw on the sheet of paper in front of the mirror a triangle, each side of which shall be at least 7 cm. long, and no corner of which shall be less than 7 cm. from the mirror. It is well to have one angle of the triangle not directly in front of the mirror, but somewhat to one side, like point No. 1 in the figure.

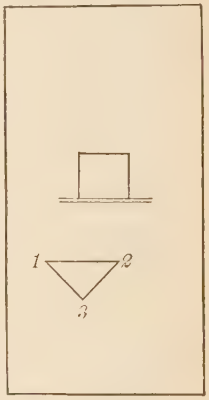


Fig. 39e.

Place the small block in such a position that the vertical pencil-mark which it bears shall be directly over point No. 1 of the triangle. Then lay a straight-edged ruler (Fig. 40e) upon the paper in such a position that one of its long horizontal edges, *PQ*, shall point directly toward the image of the vertical pencil-mark, as seen in the mirror.[1] The ruler should be so placed that the line of sight will strike near one end of the face of

[1] Many persons cannot do this at first unless they are especially instructed. A person who is not near-sighted should hold his eye 20 or 25 cm. distant from *P*, and should then direct the ruler in such a way that the *point P*, the *point Q*, and the image of the vertical pencil-mark seen in the mirror, shall all lie in one straight line. Do not try to look along the vertical *side* of the ruler, but hold the eye high enough to see all the time the top of the ruler.

the mirror. Then with a well sharpened pencil draw upon the paper a fine clear mark alongside that edge of the ruler which lies just beneath the line *PQ* (Fig. 40e), along which the *sight* has been taken. Mark this line 1, because it points toward the image of the vertical pencil-mark when this mark is over point No. 1.

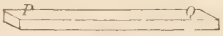


Fig. 40e.

Next, without disturbing anything else, place the ruler in a new position, far removed sidewise from the position just occupied, sight as before, draw another line alongside the ruler, and mark this line also 1.

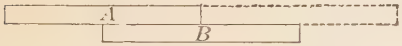


Fig. 41e.

Then with the ruler in a new position, about half way between the first two, if this is convenient, draw a third line in the same way, and mark this also 1.

All this time the small block has remained unmoved, and the pencil mark upon it has pointed straight down at point No. 1.

Now place the small block so that the pencil mark shall point straight down at point No. 2. While it is in this position draw three straight lines, well apart, toward the image, and mark each one of these 2.

Finally, put the pencil mark over point No. 3, draw three straight lines, well apart, toward its image, and mark each of them 3.[1]

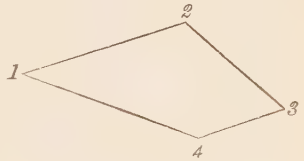


Fig. 42e.

When the three sets of lines have been drawn, remove the two blocks and the mirror from the paper and lengthen[2] each line until it

[1] While drawing all these lines the experimenter should look frequently to see whether the back of the mirror remains in place. It may be thrown out of place by a little blow or by rubbing the paper hard to remove pencil marks.

[2] If a line has to be extended far it is well to use two rulers, *A* and *B*, as shown in Fig. 41e. First *A* is put into position and a line is drawn alongside it. Then, while *A* remains unmoved, *B* is carefully brought close to it, as the figure shows; then *B* is held firmly in place while *A* is pushed forward to the position indicated by the dotted lines. *B* is then removed without disturbing *A*, and again a line is drawn alongside *A*. In this way a line may be continued nearly straight for a considerable distance.

crosses both the others of the same set; that is, prolong each No. 1 line toward or beyond the mirror till it crosses the two other No. 1 lines; then treat the No. 2 set and the No. 3 set in the same way.

After each set of lines has been extended in this way, it will be in order to answer the question whether all the lines of any one set lead to the same point or nearly so, and, if so, where is this point situated with respect to the mirror and to the point whose image it is.

If the image of each point, No. 1, No. 2, and No. 3, can be thus found, connect the image-*points* with each other by straight lines, and thus make an image-*triangle*.

Then fold the sheet of paper carefully along the pencil mark by which the mirror was placed, and holding the folded sheet against a window, so that the light from without will shine through it, compare the size and shape of the two triangles and their relative positions with respect to the line along which the paper is folded.

The general rule for placing the image of any point should be recorded when it is found.

The final result aimed at in this Exercise should be to enable the student to tell, without further experiment, in any new case given him (Fig. 42c, for instance, in which AB is the line upon which the mirror stands), the position of the image of points No. 1, No. 2, No. 3, and No. 4, etc., and so the shape and position of the image of the figure at the corners of which these points lie.

EXERCISE 31.

IMAGES FORMED BY A CONVEX CYLINDRICAL MIRROR.

Apparatus: The mirror (No. 67). A measuring stick (No. 2). Small block (No. 66). A ruler (No. 28). Sheet of white paper. The plane mirror (No. 65) and its supporting block (No. 5).

Hold the curved mirror with its straight edges vertical, and look at the image of your own face in the convex surface. You will see that the image is distorted, appearing too narrow for its length. Hold the mirror with its straight edges horizontal, and the image will be distorted in the opposite way, appearing too wide for its length. The object of the following experiments is to give a better understanding of these curious effects.

Set the curved mirror on the table and bring one end of the plane mirror close to its surface, as in Fig. 43c. Then place the small block

in front of and near both mirrors, in such a position that you can see the block reflected in both mirrors at the same time.

Do the two images thus seen appear to be of the same height?

Do they appear to be of the same width?

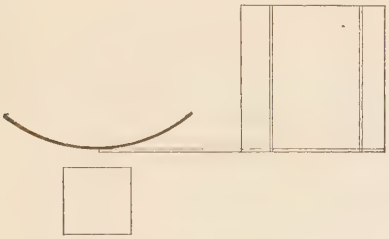


Fig. 43e.

Fill out, if you can, the following statement: *Lines of the object which are parallel to the straight lines of the cylindrical mirror appear (longer? shorter? of the same length?) in the cylindrical mirror . . . . . in the plane mirror.*

Lines of the object which are at right angles with the straight lines of the cylindrical mirror appear (longer? shorter? of same length?) in the cylindrical mirror . . . . . in the plane mirror.

Remove the plane mirror. On the sheet of paper should be found four points previously marked, with stencil or other convenient instrument (see apparatus No. 67a), one to fix the centre of curvature of the convex mirror, *C* in Fig. 44e, the three others to fix the position of the convex surface.

Fig. 44e.

Place the mirror carefully by means of the three points just mentioned and then, holding it firmly in position, make a fine, clear, pencil mark on the paper along the outer curve.

About 5 cm. from the front of the mirror draw an arrow 6 cm. long, marking the ends and the middle as in Fig. 44e, and through each of these three points draw a straight line from *C*. Then place the small block so that the vertical pencil mark which it carries will point straight down at point No. 1.

With the straight-edged ruler draw two lines, well apart, toward

the image of this vertical line as seen in the mirror, using only such parts of the mirror as lie not more than 0.7 cm. to the right or the left from the line C—1. Mark each of these lines 1. Then draw two lines for point No. 2 and two for point No. 3 in the same way, using in each case only a narrow strip of the mirror on either side of the radial line through the object-point in use.

Then clear the paper and prolong each pair of lines till it comes to a crossing-point. The three points thus found will locate the images of object-points No. 1, No. 2, and No. 3, respectively, and a line connecting these three image-points will give an idea of the shape of the image-arrow, whether it is straight or not, and whether its curvature, if it has any, is in the same general direction as the curvature of the mirror or in the opposite direction.

Note how the image points are placed with respect to the radial lines.

Is the image longer or shorter than the object? Is it nearer to, or farther from, the mirror than the object is?

(It must be understood that the pupil is asked these questions only in regard to the particular case that he has tried. He cannot tell without further experiments or further instruction whether the answers he gives in this case would be true for all cases of objects reflected in mirrors such as he is using, for he does not know that the distance of the object from the mirror may not decide all these questions. The fact is, however, that, if he has found correct answers to the questions asked for his one case, the same answers will be true for the same questions in all cases with *convex* cylindrical mirrors. The effects seen with *concave* mirrors are much more complicated.)

EXERCISE 32.

IMAGES FORMED BY A CONCAVE CYLINDRICAL MIRROR.

Apparatus: The same as in the preceding Exercise, and in addition a common pin.

On the paper should be marked in advance (see Apparatus No. 67b) the point C (Fig. 45c), three points to fix position of concave surface of mirror, points for the extremities of A, B, D, and E. With the help of these points complete the figure as it is here shown.

Place the mirror in position as in Fig. 45c, and, keeping the eye about 20 cm. from it, look at the images of A and B.

Do the images of A and B point in the same general direction, from left to right in the figure, as the arrows themselves or in the opposite direction?

Are the images of A and B as thus seen longer or shorter than A and B?

At the centre of A stand a pin upright, and, as in previous exercises, draw on the paper two lines to locate the image, using only a narrow strip of the mirror, 6 or 7 mm., on either side of the radius which runs through the pin. Is the image thus found behind the mirror or in front? Is it, then, a real or is it an unreal image?

By the same method locate the image of the pin when erected at the centre of B, asking and answering the same questions that were asked when the pin was at the centre of A.

By the same method locate the image of each end of D. Is the image of D as thus found real or virtual? Does it point in the same direction as D, or in the opposite direction? Is it longer or shorter than D?

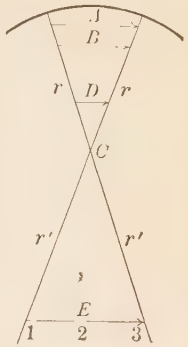


Fig. 45c.

If time permits, treat E as D has been treated.

EXERCISE 33.

INDEX OF REFRACTION OF GLASS.[1]

Apparatus: A piece of plate glass (No. 68). Articles 2, 28A, and 28B. A sheet of paper and two pins.

Place the glass, G (Fig. 46c), on the paper, P. Stick one pin upright at the point 1 close to one of the polished edges[2] of the glass; stick the other pin at (2) close to the other polished edge, making the distance from (2) to the line (1)—S not less than one half or more than three fourths of the width, parallel to (1)—S, of the glass.

Look with one eye along the line (1)—S *through* the whole width of the glass at pin No. 1. Move the eye toward 3, looking all the time through the glass at the pin. It will presently be noticed that the pin seen *through* the glass is not in the same direction from the eye as the same pin seen *over* the glass. That which is seen through the

[1] I owe the plan of this admirable Exercise to Mr. F. M. Gilley of the Chelsea High School. It is described in Gilley's *Principles of Physics*, Allyn & Bacon, Boston.—E. H. H.

[2] In place of pin No. 1 a vertical ink-mark on the glass itself may be used.

glass is an *image* of the real pin, and it is upon this image that the attention should be fixed.

Continue moving the eye in the general direction of 3, keeping it, however, about 30 cm. from the glass, until the image of pin No. 1 is just hidden behind pin No. 2. Then draw along the edge of a straight ruler, in the same straight line with the eye, pin No. 2, and the image of No. 1, the line (3)—(2).

Draw a fine pencil-line upon the paper close to the glass edge touched by pin No. 2. Then remove the glass.

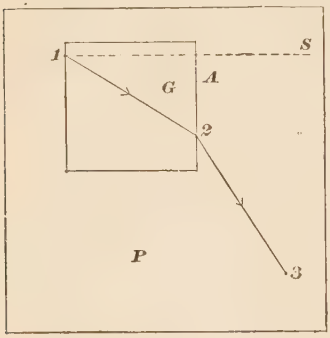


FIG. 46e.

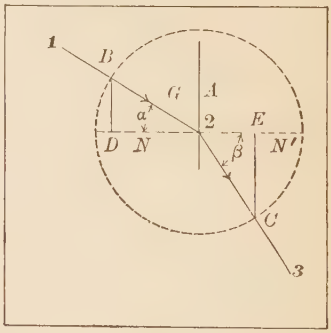


FIG. 47e.

The line last drawn, (2)—*A*, marks the position of the refracting surface. The line (1)—(2), shows the direction, within the glass, of a certain ray from (1). The line (2)—(3) shows the course of the same ray after it leaves the glass at (2). The line NN' (Fig. 47e) is drawn normal to the refracting surface at the point of emergence. From this point equal distances are laid off, to *B* and to *C*. Lines are drawn from *B* and from *C* to the line NN' at right angles.

$$CE \div BD = \textit{the index of refraction from air to glass.}$$

Vary the position of point 2 somewhat and then go through the experiment again.

EXERCISE 34.

FOCAL LENGTH OF A CONVERGING LENS.

Apparatus: The lens (No. 69) mounted on a block (No. 69a). A meter rod. A small block bearing a white cardboard screen (No. 70). A common pin.

First Method.—Place the lens and the screen upon the rod, as in Fig. 48e, and point the rod at some distant object, seen against the sky, in such a way that the light from this object will pass through the lens and then fall upon the screen. Move the screen back and forth until that part of the image[1] which lies on or near the principal axis of the lens is made as distinct as possible. Then by means of the graduations of the meter rod, or by an independent measuring stick if this is preferred, note the distance from this part of the image to the nearer face of the lens. This is the *focal length.*

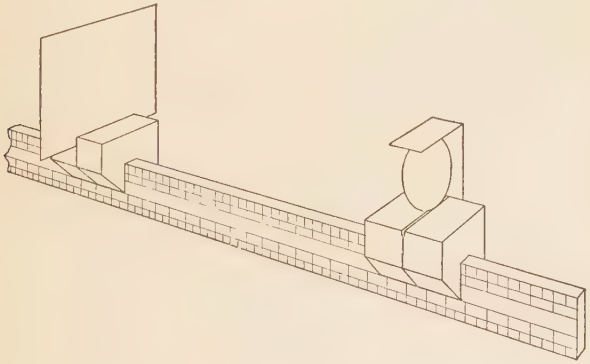


Fig. 48e.

Second Method.—Remove the screen from its block and put the pin upright in its place. Let the pin, thus mounted, be placed on the meter rod, about as far from the end of the rod as the pupil usually holds a book from his eyes when reading. Place the lens somewhat farther from the same end of the rod.

Place the eye at this end of the rod and, looking sharply at the *pin*, direct the rod and adjust the lens in such a way that the light from some distant object will pass through the lens and form an image *in the air* close to the pin. To decide whether the image is nearer the eye than the pin is, move the eye to and fro, to the right and the left,

[1] The image is formed because light coming from any one small spot of the object is brought to a small spot again by the lens. The image is made up of such small spots each in its own place. For the purposes of this Exercise the *distant* object need not be more than 150 or 200 ft. from the experimenter. The images on the screen will be much more distinct if the apparatus is used in the back part of the room, well away from the windows.

watching the pin and the image.[1] If the pin is more distant than the image, it will, when the eye is moved toward the right, appear to move across the image toward the right. If the pin is nearer than the image, it will, when the eye is moved toward the right, appear to move across the image toward the left. The rod should not be held in the hands during this test, but should be placed on some steady support.

Continue the adjustments until the test described fails to show which of the two, the pin or the image, is nearer the eye. Then measure the distance from the pin to the lens. It should be the focal length of the lens.

Compare the values of the focal length given by the two methods.

The second method is more difficult, but it is instructive, and it can be used in cases where the image is too faint to show clearly upon the screen.

EXERCISE 35.

RELATION OF IMAGE-DISTANCE TO OBJECT-DISTANCE: CONJUGATE FOCI OF A LENS.

Apparatus: The same lens that was used in Exercise 34. A meter rod. Supports (No. 72) for rod. Small block with a cardboard screen (No. 70). An incandescent electric lamp (No. 71) and an asbestos cylinder (No. 71a).

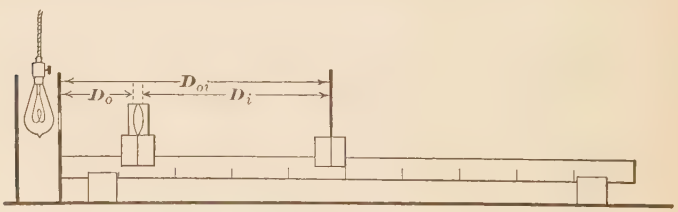


FIG. 49e.

Arrange the apparatus according to Fig. 49e. Take the + mark, on the tracing-cloth that covers the hole in the asbestos cylinder, at the same height as the middle of the lens, for the *object*, the image of which is to be received upon the screen.

[1] To see the reason of the test here described, close one eye and hold the two forefingers, some inches apart, in line with the other eye, so that one finger hides the other. Then move the eye to the right and left and notice the apparent movement of the fingers with respect to each other.

Place the screen at first at a distance from the object about equal to three times the focal length of the lens. Then move the lens back and forth on the rod between the object and the screen, and see whether in any position it gives upon the screen a clear image of the object. If it does, measure the distance from the lens in this position to the object, and write this distance as the first number in a record column headed D_o (object-distance). Measure also the distance from the lens to the *screen*, and put this distance as the first number in a record column headed D_i (image-distance). Measure finally the distance D_{oi} (Fig. 49c), which exceeds $D_o + D_i$ by the thickness of the lens, and put this distance as the first number in a record column headed D_{oi}.

If, with the present position of the screen and object, there is *no* position of the lens that will cause a distinct image of the object to fall upon the screen, move the screen about 1 cm. farther from the object, and then try again to get a good image. If still none is found, move the screen still farther away, continuing the trial till a distinct image is obtained. Then measure and record the D_o, D_i, and D_{oi}, as already described.

Then at one move place the screen about 10 cm. farther still from the object, find a position of the lens that will give a distinct image, measure and record D_o, D_i, and D_{oi} as before. Without moving the screen, see whether there is any other position of the lens that will give a distinct image; if there is, measure and record the D_o and the D_i for this position of the lens.

Move the screen 10 cm. farther away, and then do exactly as before.

If there is time, move the screen two or three more times, adjusting the lens, measuring, and recording, each time. It is better to make a moderate number of settings and readings well than a large number carelessly, but an error of 1 or 2 mm. in these readings will be of no great consequence.

How many times the focal length, F, of the lens is the least distance D_{oi} which gives a distinct image?

Find how well the data obtained in this exercise agree with the formula.

$$\frac{1}{F} = \frac{1}{D_o} + \frac{1}{D_i}, \text{ or } D_o \times D_i = F\,(D_o + D_i).$$

EXERCISE 36.

SHAPE AND SIZE OF A REAL IMAGE FORMED BY A LENS.

Apparatus: The lens (No. 69). Measuring stick (No. 2). Block of No. 70, carrying in the narrow slot on its top a piece of wire extending first horizontally and then downward (see Fig. 51e). Two rulers (No. 28). Block (No. 66). A sheet of paper about 30 cm. wide and 1 m. long, having near one end an arrow 8 cm. long, drawn at right angles with a pencil mark about 30 cm. long, and marked, or numbered, as shown by Fig. 50e. Weights to hold the corners of this sheet in place on the table.

Arrange the apparatus as shown by Fig. 51e, the centre of the lens over a point on the long pencil mark, at a distance from the centre of the arrow about equal to one and a half times the focal length of the

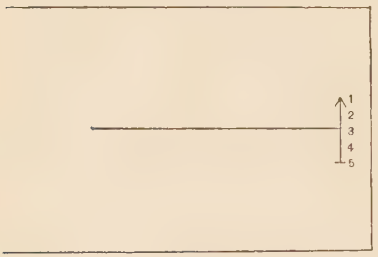


FIG. 50e.

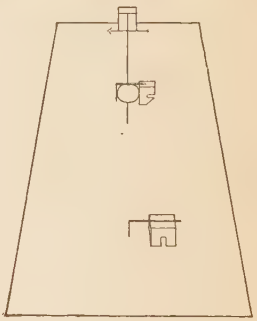


FIG. 51e.

lens, and block No. 66 in such a position that the vertical mark upon its face points straight down to point No. 3 of the arrow. This vertical mark will now cross the principal axis of the lens, if the lens is *accurately* placed.

Place the other block near the other end of the paper in such position that the vertical part of the wire it carries shall be near the principal axis of the lens. Keep the eye 20 or 30 cm. distant from this part of the wire, on a level with the centre of the lens, and in line with the centre of the lens and the vertical part of the wire. Look at this part of the wire so as to see it *distinctly*, and note whether you can see at the same time, near the wire, the *image* of the pencil-

mark on the farther block. If so, find out by moving the eye to the right or left, as in Exercise 34, whether this image is more or less distant from the eye than the vertical wire is. Then move the block carrying the wire into such a position that the image and the wire seem to keep close together when the eye is moved a considerable distance to the right or left. When this adjustment is made, put a dot on the paper just beneath the vertical wire and mark this dot 3. It represents the image of object-point No. 3.

Find in a similar manner the image-points 1, 2, 4, 5, corresponding to the object-points 1, 2, 4, 5. The experimenter must take care not to let any idea he may have as to the position where an image-point *ought* to be affect his judgment in deciding where it is.

After all the five image-points are found, connect them, No. 1 to No. 2, No. 2 to No. 3, etc., by means of straight lines, thus getting a rough representation of the whole image.

Draw a straight line from each object-point to the corresponding image-point, and note the point where these lines cross each other.

If the length of a straight line drawn from image-point 1 to image-point 5 is called L_i, the length of the image, and the shortest distance from the lens to this line is called D_i, while the length of the object is called L_o and the shortest distance from the object to the lens is called D_o, how does the ratio $(L_i \div L_o)$ compare with the ratio $(D_i \div D_o)$?

EXERCISE 37.

SHAPE AND SIZE OF A VIRTUAL IMAGE FORMED BY A LENS.

Apparatus: The same as for the preceding Exercise, except that the sheet of paper need not be more than one-half as long, and that the arrow upon it should be 4 cm. long and about 20 cm. distant from one end of the sheet.

Place the lens between the arrow and the (to the arrow) nearer end of the sheet of paper, at a distance from the arrow equal to about two thirds of its focal length, and in such a position that its principal axis extends over the middle point of the arrow. Place the small block with its vertical pencil mark pointing straight down at the middle point, No. 3, of the arrow. Turn the vertical part of the wire on the other block so that it will point up instead of down, and place this block some distance behind the other one.

Holding the eye 20 or 30 cm. from the lens, look *through* the lens at the *image* of the vertical pencil mark, and at the same time *over* the lens at the vertical part of the wire. Bring the wire into line with the image, and then by the usual test find which of them is the more distant. Move the wire back and forth until it coincides in position with the image. Then mark with a figure 3 the point just under the vertical part of the wire. This represents the image of object-point No. 3.

In a similar manner locate the images of points 1, 2, 4, and 5.

Connect the image-points by straight lines, from 1 to 2, from 2 to 3, etc., thus forming an image of the arrow.

Draw a straight line from each image-point to its corresponding object-point, and note where these lines will cross each other if continued.

Make a comparison like that called for at the end of Exercise 36.

EXERCISE 38.

ESSENTIALS OF TELESCOPE AND COMPOUND MICROSCOPE.

First Part: The Telescope.

Apparatus: Two convex lenses, one (No. 74) of 30 or 40 cm., the other (No. 75) of 4 or 5 cm., focal length. A screen (No. 76). A meter rod and holders, like those of Fig. 49e, for the lenses and the screen.

Find and record the focal length of each lens. Mount the short-focus lens on the meter rod close to one end, and beyond it mount the screen, at such a distance that the details of its surface will appear clearly to the eye held close to the lens. Beyond the screen place the long-focus lens and move it back and forth until it throws upon the screen, so as to be seen by the eye looking through the other lens, a clear image of some distant, distinctly outlined, small object.

Measure, and record on a diagram, the distance from each lens to the screen.

Remove the screen without disturbing the lenses, and observe that the image of the object is still to be seen through the short-focus lens, which we may call the *eye-piece*, the other lens being the *objective*, of the rudimentary telescope which has thus been constructed.

Looking with one eye through the eye piece at the image and with

the other eye directly at the object, make a rough estimate of the ratio of the apparent width of the image to the apparent width of the object. Record this ratio as the approximate magnifying power of the telescope.

Second Part: The Microscope.

Apparatus: Two convex lenses, each 4 or 5 cm. in focal length, two screens, one like that of the FIRST PART, the other a thin piece of cardboard with a pin hole through it. A meter rod, with supports for the lenses and screens. An incandescent electric lamp.

Measure and record the focal length of each lens.

Arrange the eye piece and the translucent screen just as in the FIRST PART. Place the other lens about 25 cm. beyond the screen. Darken the room and then place the other screen, with the lamp behind it so as to illuminate the pin hole in it, at such a distance beyond the second lens that a distinct image of the pin hole will fall upon the translucent screen and be seen there through the eye piece.

Make a diagram of the apparatus as now arranged, marking on it the distances between the screens and the lenses.

Remove the translucent screen and observe that the image is still visible through the eye piece.

EXERCISE 39.[1]

LINES OF FORCE NEAR A BAR MAGNET.

Apparatus: A bar magnet (No. 77). A small compass (No. 78). A sheet of paper about 50 cm. square.

Fasten the sheet of paper on a table, and lay the magnet on the middle of the sheet, the north-seeking pole pointing north. Place the small compass at the extreme northeast corner of this magnet, and then move it away in the exact direction in which the compass needle points. Continue this movement, changing direction in such a way as to follow continually the changing indication of the compass needle, until the path reaches the edge of the paper or returns to the magnet. Trace upon the paper the line thus followed by the middle of the compass, putting arrow heads at several points

[1] Jarring magnets or touching like poles together may weaken the magnetism. When the magnets are not in use, it is well to keep them, well apart, in a kind of rack, the north-seeking ends pointing *downward*.

to indicate the direction in which the north-seeking end of the compass needle points at these places.

Then place the compass a very little distance from the northeast corner of the large magnet toward the middle, and, starting anew, trace another line, and mark it as before.

Then, beginning still farther toward the middle of the magnet, do as before.

Finally, start not more than 3 or 4 cm. from the middle of the magnet and trace a line.

Trace a line beginning at the southeast corner.

Trace an equal number of lines on the western side of the magnet.

Trace upon the paper the outline of the magnet as used, marking the position of its north-seeking end. Draw also an arrow showing the direction *north*.

Study the diagram thus obtained, and try to explain all its peculiar features in accordance with the general properties of magnets and the familiar action of the earth upon them. Does any line of force turn in such a way as to cross itself or lead back to its own starting point? Does any line cross another line?

Nails or screws in the table-top may affect the lines in places.

EXERCISE 40.

SINGLE-FLUID GALVANIC CELL.

Apparatus: Materials for a small copper-zinc cell (without porous cup), using clean dilute sulphuric acid, about 20 parts in volume of water to 1 part in volume of concentrated acid.[1] A galvanoscope (No. 80, Fig. 52e). The zinc is at first unamalgamated.

(In this and every other Exercise in which galvanic cells are used, whenever wires have to be connected, take care to make the connection firm, with good metallic contact.)

Place the galvanoscope so that each circle of wire upon it shall be in a vertical north and south plane according to the magnetic needle.

Fill the glass with the acid to a level about 1.5 cm. below the top.

Scour the strip of copper with emery-paper till so much of it as will be immersed in the acid is clean and bright.

Adjust the zinc strip and the copper strip in the clamps attached

[1] In mixing *pour the acid slowly into the water*, stirring the mixture. Pouring water upon the acid is dangerous. A small quantity of acid sinking at once in a large mass of water is speedily cooled in spite of the strong chemical action. A small quantity of water floating on the top of a large mass of acid is heated to boiling, and then spatters the acid.

to the block at the top of the glass jar, so that each strip shall reach straight downward almost to the bottom of the jar.

Zinc Unamalgamated; *Circuit Open.*—Observe for a short time and record what happens at the surface of each strip, all metallic connection between them being avoided.

Circuit Closed.—Then put the two strips into metallic connection with each other through the 15 turns of the galvanoscope, and again observe and record what happens at the surface of each strip. Note, too, the behavior of the galvanoscope needle, and record the position in which it comes to rest, tapping the instrument lightly to prevent the needle from being detained by friction in a wrong position.

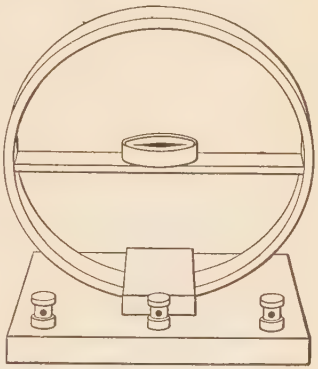


Fig. 52e.

Zinc Amalgamated; *Circuit Open.*—Remove the zinc strip from the cell and *amalgamate*[1] its surface by dipping it for an instant in mercury. Shake off or wipe off the superfluous mercury that clings to the strip and then replace the zinc in the cell. Take care that no mercury shall touch the copper.

Again observe and record the action at the surface of the zinc, the wires remaining unconnected.

Circuit Closed.—Again connect the strips through the 15 turns of the galvanoscope, read and record the position of rest of the needle as soon as convenient, and read and record every minute thereafter for three minutes.

Observe what occurs at the metal surfaces

Optional.—If no decided change in the size of the deflection is noticed in the course of three minutes, take out the copper strip, which, *if the acid solution is in the right condition,* will still look bright, rinse it, and again scour it with emery-paper. Then replace it in the cell,[2] take a reading as soon as convenient, and continue reading for several minutes.

[1] Or exchange it for one of the same size already amalgamated, thus avoiding continual sacrifice of zinc strips.

[2] It is better to use a fresh solution for this part of the Exercise; otherwise confusing results may follow.

EXERCISE 41.

STUDY OF A TWO-FLUID GALVANIC CELL.

Apparatus and Materials: The solid parts of a small Daniell cell, the zinc well amalgamated. Saturated solution of sulphate of copper, and a sulphuric acid solution of the same strength as that used in Exercise 40. A galvanoscope. A platform balance [1] (No. 4).

Place the zinc in the porous cup and place this cup and the copper in the glass jar. Fill the jar with sulphate of copper solution to a level about 2 cm. from the top and fill the porous cup with the dilute acid to the same level, or a little higher. See Fig. 53e.

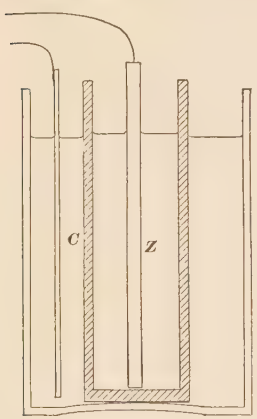


FIG. 53e.

Lift the copper plate from the solution, let it drip 15 seconds, then weigh it carefully with all the liquid that goes with it. Then replace it in the cell. Wipe the liquid from the pan of the balance.

Lift the zinc from the acid, let it drip 15 seconds, then weigh it carefully with all the liquid that goes with it. Replace the zinc in the cell and wipe the balance dry.

Connect the poles of the cell with the terminals of the 5-turn section of the galvanoscope, and note the time at which this is done. Read and record the position of the needle as soon as it comes to rest, tapping the galvanoscope lightly as in Exercise 40.

Read and record the position of the needle every five minutes thereafter for half an hour, if the time of the Exercise permits.

Break the circuit, noting the time; then weigh each of the metal plates, just as before, after allowing each to drip 15 seconds.

Find how much each plate has lost or gained in weight per second of the time during which the circuit was closed. From the change of weight of the copper per second find the strength of the current in amperes. See § 338.

NOTE.—Each plate carries about the same amount of liquid to

[1] If the pans of this balance are of metal, the zinc and copper plates from the cell must not be placed directly upon them. Saucers or plates of glass should protect the pans.

the balance both times, so that the difference of its two weighings should tell, pretty nearly, its own change of weight. Wiping the plates might rub off some loose metal, and it is therefore not advisable. The zinc should not be freshly amalgamated at the beginning of this Exercise; for freshly amalgamated zinc carries superfluous mercury, which would be likely to drop off during the Exercise, making the weighings of the zinc useless.

If the current increases in strength for several minutes after the circuit is closed, this is probably due to the gradual driving out of obstructing air from the pores of the clay cup. If the cell is to be put to any use requiring a very constant current, the porous cup, kept outside the jar, should be filled some little time before the work begins. When a film of moisture appears on the outside of the cup it may be placed in the jar.

EXERCISE 42.

LINES OF MAGNETIC FORCE ABOUT THE GALVANOSCOPE.

Apparatus: The galvanoscope with detachable compass. A Daniell cell. A commutator (No. 82). Short pieces of wire for making connections.

The lines of force to be examined in this Exercise are not due entirely to the current in the galvanoscope. They represent the combined action of the earth's magnetism and the current, and the character of the lines must depend, to some extent, upon the position of the galvanoscope with respect to the earth's lines of magnetic force. Something might be said in favor of studying the lines when the windings are placed north and south, as in the ordinary use of the instrument, but, on the whole, it seems better to place the windings east and west for this Exercise.

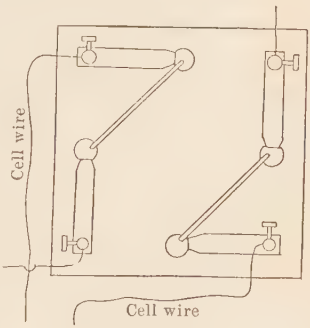


FIG. 54e

At some points the force due to the current will be in exactly the same direction as the horizontal force due to the earth. At such points the resultant force will be greater than that due to the earth alone, and the needle, if disturbed, will vibrate *more quickly* than it would if the current were not acting.

At other points the force due to the current is exactly opposite in direction to the horizontal magnetic force of the earth. At such points, if the force due to the current is greater than the other, the needle will be reversed and point south. If the earth's magnetic force prevails, the needle will still point north, but will vibrate *less quickly* than if the current were not acting.

In the diagram of the lines, points where the lines run north are to be marked *strong* or *weak*, if the resultant force at these points is greater or less than the earth's horizontal magnetic force.

Put the Daniell cell in circuit, through the commutator (Fig. 54c), with 15 turns of the galvanoscope, and place the latter so that the plane of its circle shall be east and west.

Make in the note book two lines, *E* and *W* (Fig. 55e), to represent half-size a horizontal cut through the middle of the galvanoscope.

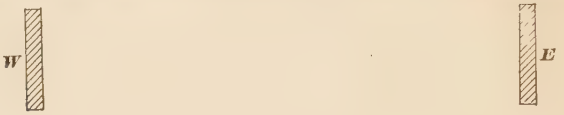


Fig. 55e.

Adjust the commutator so that the current, that is, what we call the positive current, the current coming from the cell by way of the copper plate, shall flow from east to west on the top of the galvanoscope circle.

Hold the compass about 20 cm. south of the centre of the circle on the same level, and record with an arrow-head on the diagram the direction of pointing of the needle. Move the compass north eight stages of 5 cm. each, recording on the diagram the direction of the needle at each stopping-place.

Then place the compass inside the circle, near to the eastern side, and mark the direction of the needle. Follow the needle, as in Exercise 39, and mark its direction at three or four other points several centimeters apart, until the line followed appears to be completed.

Make similar observations and record with respect to the western side of the circle.

Find, and record in words, the direction of pointing just over and just under the top of the circle.

Reverse the current through the galvanoscope, by operating the commutator, and then repeat all the observations just indicated.

Make a special observation just east of the eastern side and one just west of the western side of the circle, if the compass in the use already prescribed does not *lead* to these points.

EXERCISE 43.

MEASUREMENT OF RESISTANCE WITH THE WHEATSTONE BRIDGE.

Apparatus: A Daniell cell. A Wheatstone bridge (No. 87). An astatic galvanometer (No. 88). The 20-m copper coil (No. 85). The German-silver coils (No. 83 and 84). A box of resistance coils (No. 89), with values given in ohms, or at least some known resistance of about 5 ohms. Screw calipers (No. 90).

First Part.—Introduce the copper coil, the resistance of which is to be measured, as J, between the binding posts d and e in Fig. 56c. Introduce a known resistance as K, between the posts f and g, in the

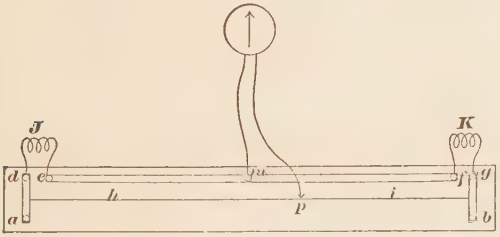


Fig. 56c

same figure. Connect one wire from the galvanometer (Fig. 57c) to the binding post m. Connect the other galvanometer wire to the piece that makes sliding contact with the straight G.-s. wire, setting the contact at first at the middle point of this wire.

Attach one of the cell wires to the binding post a. *Touch* the other cell wire for an instant to the post b, and note the effect upon the galvanometer. If the needle does not move, it is probable that the contact of the slider with the G.-s. wire is not good. If the needle does move, observe the direction and magnitude of its motion.

Move the sliding contact to some new position, and again note the effect upon the galvanometer when the cell wire touches b for an instant. Experiment in this way till some point p is found which

gives no perceptible current when contact is made and continued for a second or two at b. (*Prolonged* contact at b will allow the copper coil to be heated by the current and suffer change of resistance.) It is best, after a little experimenting, to introduce such a resistance at K that the equilibrium point p will be not far from the middle of the wire.

The distances h and i are now read. Observing that the resistances of the thick copper strips of the bridge are very small compared with J and K, we have

$$\frac{J}{K} = \frac{h}{i}, \text{ or } J = K \times \frac{h}{i}.$$

The resistance of the copper wire is thus found in ohms. To find its specific resistance we must take account of its *length*, 20m., and its diameter, which may be found by the screw calipers.

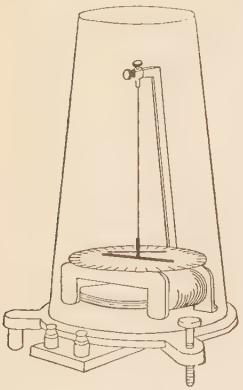


Fig. 57e.

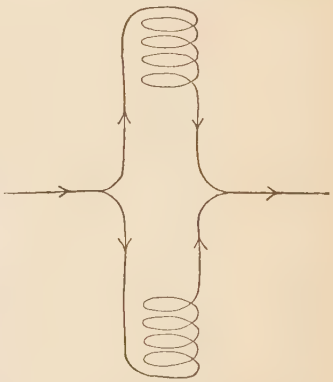


Fig. 58e.

Second Part.—A profitable extension of this Exercise is the following:

Measure the resistance of two wires, for example 200 cm. of No. 30 G.-s. wire and 120 cm. of the same kind of wire, singly.

Measure the resistance of the same two wires connected with each other in multiple, as in Fig. 58e.

From the measured single resistances calculate what the multiple resistance should be, and compare the result with the value of the multiple resistance found by direct measurement.

EXERCISE 44.

TEMPERATURE-COEFFICIENT OF RESISTANCE.

Apparatus: A Daniell cell. A Wheatstone bridge. An astatic galvanometer. A known resistance of 1 or 2 ohms. A "temperature-coil" (No. 91). A vessel of ice water. The boiler (No. 44) without the top.

The unknown resistance J (see preceding Exercise) is now the "temperature coil," which is attached to the Wheatstone bridge in the manner indicated by Fig. 59e.

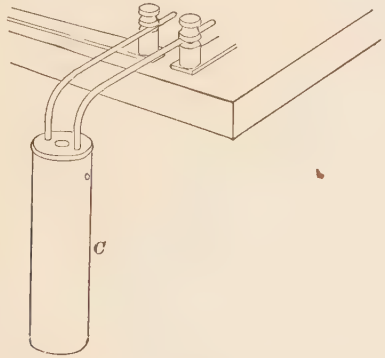


FIG. 59e.

Measure carefully the resistance of this coil while it is in ice water. Measure the resistance of the coil again with equal care while it is in boiling water.

Calculate the temperature-coefficient of the resistance.

Instead of using ice water one may measure the resistance, R_1, of the coil in water at any convenient low temperature, t_1 and later its resistance, R_2, in water boiling at the temperature t_2. These data give two equations, $R_1 = R_0(1+\gamma t_1)$ and $R_2 = R_0(1+\gamma t_2)$, from which the value of γ, the temperature-coefficient, can be found.

EXERCISE 45.

BATTERY RESISTANCE.

Apparatus: Two similar Daniell cells, with copper plates not more than 3 cm. wide. A galvanoscope. A commutator. A resistance box.

Allow the pores of the cups to fill with liquid (see the last paragraph of Exercise 41) before the cups are placed in the jars.

Variation of Effective Plate-Area.—Put one of the cells in circuit, through the commutator, with the 5-turn section of the galvanoscope.

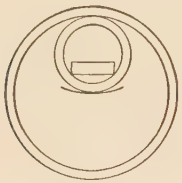


Fig. 60e.

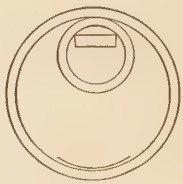


Fig. 61e.

(1) Keeping the porous cup in contact with one side of the jar, and the copper and zinc as near together as may be (see Fig. 60e), make and record the reading, then reverse the commutator and read and record again.

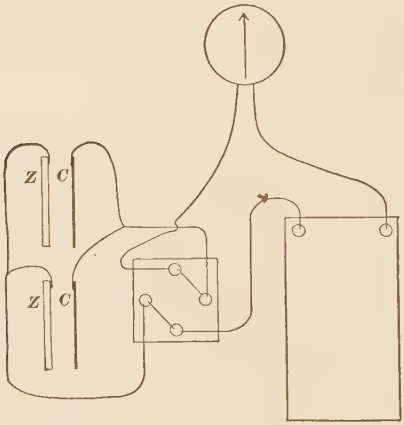


Fig. 62e.

(2) Without changing the distance between the copper and the zinc, raise both of them till they dip about 0.5 cm. only below the surface of the liquids. While they are in this position, read, reverse, and read again. Compare with (1).

(3) *Variation of Distance Between Plates.*—Keeping the porous cup in contact with the side of the jar, put the plates as far apart as may be (see Fig. 61e), and while they are in this position read, reverse, and read again. Compare with (1).

Cells in Parallel.—Join the two cells in parallel, zinc to zinc and copper to copper (see Fig. 62e), and put them thus joined into the circuit, through the commutator and the resistance box, with the 5-turn section of the galvanoscope.

(4) Adjust the resistance box so that none of its coils shall be in the circuit; read, reverse, and read again.

(5) Introduce a resistance of 10 ohms, read, etc.

Cells in Series.—Join the two cells in series, other arrangements being as before.

(6) Take readings with no coils of the box in circuit.

(7) Take readings with the same coils in circuit that were used with the cells in parallel.

Under what conditions does the multiple, or parallel, arrangement of the cells give a stronger current than the series arrangement?

Under what conditions does the series arrangement give a stronger current than the multiple arrangement?

EXERCISE 46.

CALL-BELL ARRANGEMENT.

Apparatus: An ordinary electric call-bell (No. 92). All the parts and materials of a Leclanché, or some other "sal-ammoniac," cell (No. 93), but with the ammonium chloride ("sal-ammoniac") not yet placed in the water. Two ordinary push-buttons (No. 94) or simple substitutes therefor. Wire for the connections. A small screw-driver (No. 95). A platform balance. A thermometer.

Make a simple sketch of the solid parts of the cell.

After making sure that the top of the glass jar is clean and dry, dip it to a depth of 1 or 2 cm. in melted paraffin at a temperature near 75° C., taking care to have the inside as well as the outside become coated. The object of this treatment is to leave a layer of wax around the mouth of the jar in order to prevent *creeping* of the sal-ammoniac solution to the outside of the vessel. The glass should be immersed long enough, perhaps half a minute, in the paraffin bath to bring out a liquid, transparent, coating of the wax.

Weigh the jar.[1] Put into the jar enough water to fill it to a level about 2 cm. below the *shoulder* when the carbon and zinc cylinders are in place, then weigh the glass and the water and so find the weight of the water alone. Take the temperature of the water and then put into it about one seventh part of its own weight of sal-ammoniac; stir the mixture thoroughly with the thermometer, noting and recording the lowest temperature reached; note approximately the length of time required to dissolve the sal-ammoniac.

"Assemble" the cell; that is, put the solid parts in place in the jar containing the solution. Connect the cell, the call-bell, and the push-buttons in such a way that the bell will ring when either button is pushed. Make a diagram of the connections.

Make a simple sketch of the bell, illustrating particularly the device by means of which the current makes the hammer vibrate.

EXERCISE 47.

TELEGRAPHIC SOUNDER AND KEY.

Put together, "assemble," the parts of a very simple telegraphic key and sounder (No. 96), the latter sensitive enough to be operated by the current from a single Daniell cell.

The key may be something like Fig. 63e, where *b* is a strip of spring-brass, and the sounder may be correspondingly simple.

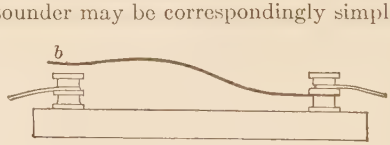


FIG. 63e.

EXERCISE 48.

ELECTRIC MOTOR.

Put together, "assemble," the parts of a small electric motor (No. 97), to be driven by a single Daniell cell.

EXERCISE 49.

THE DYNAMO.

Put together the parts of a dynamo (No. 98) which, driven by the motor of Exercise 48, will give a current capable of affecting, at least, an astatic or a d'Arsonval galvanometer.

[1] An error of 2 or 3 grams in the weighings of this Exercise is not important.

EXERCISE 50.

EFFICIENCY[1] TEST OF AN ELECTRIC MOTOR.

Apparatus: A direct-current motor (No. 99) of 0.1 horse-power or more, suited to the available voltage and having at one end of the shaft a grooved pulley about 5 cm. in diameter. An ammeter (No. 100). A voltmeter (No. 101). Two spring-balances, one of which

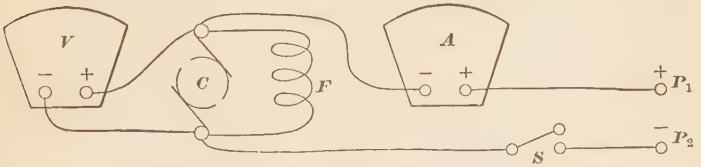


Fig. 64e.

should have a capacity of 500 gm. and the other a capacity of 2 or more kgm. A speed-counter (No. 102). A watch or clock. Connecting wires and string.

Connect the apparatus in general accordance with the indications given in the diagram Fig. 64e, in which C represents the commutator of the motor, F the "field coils," A the ammeter, V the voltmeter, P_1 and P_2 the points of connection with the source of current, and S the *switch*, or break-circuit.

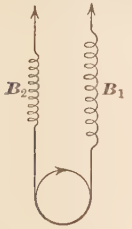


Fig. 65e.

If the pulley of the motor is to revolve in the

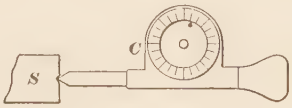


Fig. 66e.

direction of the circle arrow of Fig. 65e, arrange the two spring-balances according to the indication of this Figure, connecting their hooks by means of a string passing under the pulley and placing the stronger balance, B_1, where its pull will oppose the revolution of the pulley. B_1 may well be suspended from any convenient support, but B_2 should be so held that the pull upon it can be easily varied.

[1] I am indebted to the Turner and Hersey Manual (Knott Apparatus Company) for evidence that an exercise like this is practicable for schools having the requisite apparatus.—E. H. H.

First Case, High Speed.—Close the switch S, thus putting on the current, and pull gently on B_2 so as to press the string against the pulley and reduce the speed perceptibly but not very greatly. Keeping the pull as constant as may be, find by means of the speed-counter, C (Fig. 66e), applied to a small cavity or pit in the end of the armature shaft, S, the number of revolutions of the pulley in 30 seconds, and meanwhile have readings of the ammeter, the voltmeter, and both spring balances made. These may be recorded as follows:

1st case	N Rev. per sec.	I Amperes	D Volts	P_1 Pull on B_1	P_2 Pull on B_2
	$\ldots \div 30 = \ldots$	...	...	... gm.	... gm.

With a slender string or a piece of thread measure the circumference of the pulley at the bottom of the groove, and call this length O.

We have now the data needed for calculating the rate of inflow of electrical energy to the motor, the number of *watts*. Find what fraction of a horse power this is, and call this fraction $(H.\,P.)_i$.

We have next to find the rate of output of mechanical work, done against the friction of the string on the pulley. Here, as always, we have work $=$ *force* $\times$ *distance*. The force is resistance of the string, and this is evidently equal to $(P_1 - P_2)$ gm. The distance per second which the pulley goes against this resistance is $N \times O$ cm. The product of these two factors, *force* and *distance per second*, will give the work in gm.-cm. per second. To find what fraction of a horse-power this rate of work is, we have to remember that one horse-power is 550 ft. lbs. per second and that 1 ft. lb. $=$ 13,826 gm.-cm., so that one horse power $=$ 7,605,000 gm. cm. per second. Find what fraction of a horse power the mechanical out-put of the motor is and call this $(H.\,P.)_o$.

Find the *efficiency*, E, of the motor in the case described; that is, $(H.\,P.)_o \div (H.\,P.)_i = E$.

Second Case, Medium Speed.—Repeat, pulling now more strongly on B_2, so as to lessen the speed considerably. Make record and calculations as before.

Third Case, Low Speed.—Repeat, pulling still more strongly on B_2, so as to make the speed still less. Make record and calculations as before.

APPENDIX I.

A FEW EQUIVALENTS IN THE ENGLISH AND METRIC SYSTEMS.

1 meter	=1.0936 yards.
1 "	=3.2809 ft.
1 "	=39.3705 in.
1 kilom.	=0.6214 mile.
1 gram	=0.0353 oz.
1 kilo.	=2.2046 lbs. avoir.
1 yard	=0.9144 meter.
1 foot	=0.3048 "
1 inch	=0.0254 "
1 mile	=1.6093 kilom.
1 lb. avoir.	=0.4536 kgm.
1 ounce	=28.35 gm.
1 sq. m.	=10.765 sq. ft.
1 sq. cm.	=0.155 sq. in.
1 sq. ft.	=0.0929 sq. m.
1 sq. inch	=6.452 sq. cm.
1 cu. m.	=35.31 cu. ft.
1 cu. cm.	=0.06103 cu. in.
1 cu. ft.	=0.02832 cu. m.
1 cu. inch	=16.387 cu. cm.

The following are approximate equivalents:

1 decimeter	=4 in.
1 meter	=1.1 yards.
1 kilometer	=0.62 mile.
1 kilogram	=2.2 lbs.
1 liter	=1.06 quart.
1 kgm.-meter	=7.22 ft.-lbs.

APPENDIX II.

TABLE OF APPROXIMATE VALUES OF PHYSICAL CONSTANTS.

(All the data which involve very high temperatures are somewhat uncertain. Breaking-strength and electrical resistance vary greatly with change of physical state or chemical purity.)

Material	Breaking-strength, kgm. per sq. cm.	Density, grams per cu. cm.	Mean Coefficient of Cubical Expansion from 0° to 100° C.	Mean Specific Heat between 0° and 100° C.	Melting-point, Centigrade.	Latent Heat of Melting, gm.-deg. C.	Boiling-point, Centigrade.	Electrical Resistance, silver the standard.
Copper	4000	8.9	0.000051	0.092	1070	30?	2100	1.04
Gold	.....	19.3	0.000044	0.032	1060	.....	....	1.36
Iron	6000	7.8	0.000036	0.110	1600?	35?	....	6.4
Lead	200	11.3	0.000088	0.031	330	5.6	1500	12.3
Mercury	.....	13.6	0.000182	0.033	−39.4	2.8	357	60
Platinum	3000	21.5	0.000027	0.032	1770	27	....	7
Silver	2500	10.6	0.000058	0.056	960	21	....	1.0
Tin	300	7.2	0.000082	0.058	230	14	1700	8
Zinc	350	7.0	0.000116	0.096	415	28	1040	3
Brass (cast):	1200	8.3	0.00005	0.094	900	.....	....	4
" (hard-drawn).	7000	8.5	0.000057	0.09	900	.....	....	4
Steel (cast).	5000	7.8	0.000034	0.12	1400	.....	....	.....
" (wire).	11,000 to 23,600	7.9	0.000037	0.12	.....	.....	....	.....
Glass: (crown).	450	2.5	0.000025	0.19	.....	.....	....	.....
" (flint).	.....	3.5	0.000025	0.19?	.....	.....	....	.....

APPENDIX III.

VAPOR PRESSURES AT VARIOUS TEMPERATURES, EXPRESSED IN MEGADYNES[1] PER SQUARE CENTIMETER.

Temperature Centigrade	Alcohol	Bisulphide of carbon	Ether	Mercury	Water
0°	0.017	0.17	0.25	0.0000	0.006
10°	0.032	0.27	0.38	0.0000	0.012
20°	0.059	0.40	0.58	0.0000	0.023
30°	0.10+	0.58	0.85	0.0000	0.042
40°	0.18	0.82	1.2	0.0000	0.073
50°	0.29	1.14	1.7	0.0000	0.123
60°	0.47	1.6	2.3	0.0001	0.198
70°	0.72	2.1	3.1	0.0001	0.310
80°	1.08	2.7	4.0	0.0002	0.472
90°	1.6	3.5	5.2	0.0004	0.701
100°	2.3	4.4	6.6	0.0006	1.014
110°	3.2	5.5	8.3	0.0010	1.44
150°	9.8	12.	.....	0.0045	4.78
190°	.....	.....	.....	0.018	12.38

[1] The megadyne is 1,000,000 dynes. A pressure of one standard atmosphere is 1,033,000 dynes per square centimeter, nearly. To find, approximately, the pressure in centimeters of mercury column, multiply the pressure in megadynes per square centimeter by 75.

APPENDIX IV.

VALUE IN MILLIMETERS OF BROWN & SHARPE WIRE GAUGE NUMBERS.

Number	Diameter mm.	Number	Diameter mm
1	7.348	17	1.150
2	6.544	18	1.024
3	5.827	19	0.912
4	5.189	20	0.812
5	4.621	21	0.723
6	4.115	22	0.644
7	3.656	23	0.573
8	3.264	24	0.511
9	2.906	25	0.455
10	2.582	26	0.405
11	2.305	27	0.361
12	2.053	28	0.321
13	1.828	29	0.286
14	1.628	30	0.255
15	1.459	31	0.227
16	1.291	32	0.202

APPENDIX V.

Proof that $\alpha_1 = \alpha_2$ *(see* § 170*).*

In Fig. 21a each point of the line T_1T_1 represents by its distance from OP the volume which a given body of gas will have, at temperature T_1, under a pressure indicated by the height of the same point above OV; and the product p_1v_1 is the same for all points of this line (Boyle's Law). T_1T_1 is called the *isothermal line* T_1. T_2T_2 is another isothermal, for the temperature T_2, of the same body of gas. The product p_2v_2, for this line, is also a constant.

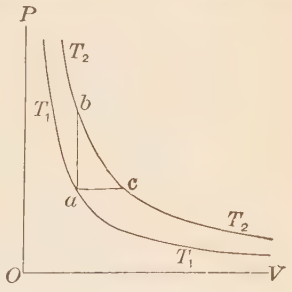


Fig. 21a.

Let us suppose that the line T_1T_1 is for 0° C. Let a certain body of gas be at first in a condition indicated by the point a, in which condition we will call its pressure p_a and its volume v_a. If this body of gas is heated at constant volume to the temperature T_2 at b, where its pressure may be called p_b, we shall have, by the definition of α_1,

$$p_b = p_a[1 + \alpha_1(T_2 - T_1)], \text{ or } p_bv_a = p_av_a[1 + \alpha_1(T_2 - T_1)]. \quad (1)$$

If, on the other hand, the gas is heated at constant pressure to the temperature T_2 at c, where its volume may be called v_c, we shall have, by the definition of α_2

$$v_c = v_a[1 + \alpha_2(T_2 - T_1)], \text{ or } p_av_c = p_av_a[1 + \alpha_2(T_2 - T_1)]. \quad (2)$$

But p_bv_a is p_2v_2 for the point b, and p_av_c is p_2v_2 for the point c; hence $p_bv_a = p_av_c$, and accordingly, from (1) and (2),

$$p_av_a[1 + \alpha_1(T_2 - T_1)] = p_av_a[1 + \alpha_2(T_2 - T_1)],$$

whence $\alpha_1 = \alpha_2$.

Proof that $\frac{pv}{T}$ = *constant* (*see* § 173).

Let T_1T_1, (Fig. 21a), be any isothermal line, not necessarily for 0° C., and T_2T_2 any higher isothermal. According to eq. 3, (§ 173), we have

$$\frac{p_a}{p_b} = \frac{T_1}{T_2}, \text{ or, since } v_a = v_b, \frac{p_a v_a}{p_b v_b} = \frac{T_1}{T_2}. \tag{3}$$

But $p_a v_a = p_1 p_1$, and $p_b v_b = p_2 v_2$, and so from (3)

$$\frac{p_1 v_1}{p_2 v_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}. \tag{4}$$

APPENDIX VI.

ELECTRO-STATIC INDUCTION MACHINE.

(Continued from § 327.)

Every such machine has one part, or more than one, which corresponds to the resinous plate of the electrophorus, and is, like this plate, charged at the beginning of operations, by friction or otherwise. Such parts are called *armatures*.

Every such machine has something corresponding to the movable metal plate of the electrophorus. This something is usually a glass plate, sometimes provided with strips or disks of metal, mounted so as to revolve readily upon a horizontal axis. We shall call this the *carrier*.

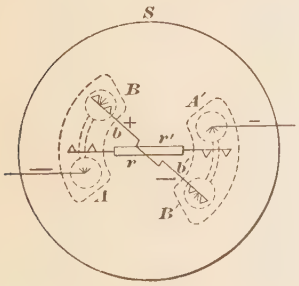


Fig. 22a.

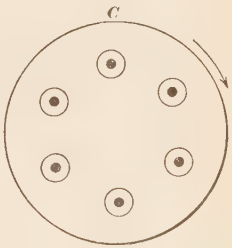


Fig. 23a.

The particular form of machine which will now be described is called sometimes the Voss, and sometimes the Toepler-Holtz, machine. It is equivalent to two electrophoruses.

It has two armatures, AB and $A'B'$ (Fig. 22a), each consisting of a strip of tin foil, protected by a covering of paper, fastened to the back of a stationary glass plate, S, fixed in a vertical position.

A small metallic brush in *front* of the plate is in metallic connection, around the edge of the plate, with A, and a similar brush is similarly connected with A'.

Just above the first brush there is a horizontal metal rod with projecting short wires, which is called a comb. Just below the other brush there is a similar comb. These two combs are both supported by a rod of hard rubber, rr', but are not metallically connected with each other.

The left-hand comb is metallically connected with the inner coating of a small Leyden jar, J (Fig. 26a); the other comb is similarly connected with the inner coating of the jar J'.

The brass rod bb' has at each end both a brush and a comb. One end of this rod is placed in front of B, the other in front of B'.

The filaments of all the brushes and the teeth of all the combs are turned *toward* the fixed plate S, but are at such a distance from it that the revolving carrier, glass plate C (Fig. 23a), is placed between, the axle on which it revolves passing through a hole in the centre of the plate S.

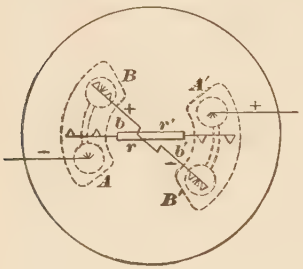


Fig. 24a.

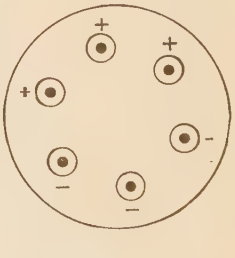


Fig. 25a.

On the front side of C are six disks of tin foil, each having a small boss at the centre. Each of these bosses is touched by each of the four brushes at every revolution of the plate C.

The process of charging the machine for operation is, or may be, as follows: Separate the balls, b and b' (Fig. 26a), two or three centimeters, and then rub the paper of the armature AB (Fig. 22a) gently with a catskin, the plate C meanwhile remaining at rest.

If now, before C is started, different metal parts of the apparatus are tested by touching them with the small insulated "proof-ball," b (Fig. 233), and then bringing this ball near a charged gold-leaf electroscope, the sign of the charge given to the ball at any one of the several points touched will probably be that indicated by the sign + or − placed near that point in Fig. 22a. At most of these

points the charge is an induced charge, which can be traced back to the charge imparted to AB by the rubbing.

When C is now started, each carrier-disk in turn touches the brush connected with A and thus acquires a $-$ charge. It then passes behind the comb just above A and gives off to this comb, as the cover on the electrophorus gives off to the finger, its $-$ charge, and takes on a $+$ charge. At the comb opposite B the disk may have this $+$ charge increased. Then, going on to the brush connected with A', the disk imparts a $+$ charge to the left-hand armature.

After a few revolutions of C, if the machine is in good condition, the process of charging is complete, and frequent sparks pass between the balls b and b' (Fig. 26a).

The state of charge can now be tested while the carrier C is in motion, by means of the proof ball, provided for the occasion with a

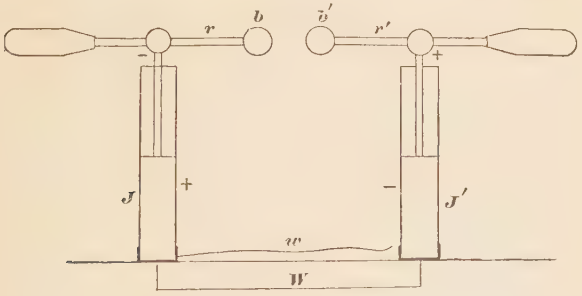


Fig. 26a.

small metal brush. This state is probably that indicated by the $+$ and $-$ signs in Figs. 24a and 25a, although it is not easy to determine by experiment whether the magnitudes of the $+$ and $-$ *charges* on the carrier correspond to the magnitudes of the $+$ and $-$ *signs* here shown.

Exploration with the proof ball after the carrier is stopped is likely to give a false idea of the state of things which exists when the carrier is in motion.

It has been already stated that a pair of Leyden jars (see Fig. 26a) make a part of this machine. The outer coatings of these jars are connected together by a wire W. The inner coatings are connected with the balls b and b', the distance between which can be varied.

When the charge upon the jars becomes great enough, a spark leaps between the balls, uniting the opposite charges of the inner

coatings. When this occurs, the positive and negative charges on the outer coatings, which charges have been kept apart by the a tractions of the inner charges, rush together along the wire *W*.

So great is the rush of this external discharge, that if a second wire, *w*, is made to touch one of the outer coatings and almost to touch the other outer coating, leaving a narrow gap, some of the current will pass by this wire at every discharge, and a small spark will occur at the gap. This fact has attracted much attention, and is referred to in connection with the discussion of protection from lightning (§ 329).

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